Essay 103 : The Three Dimensional Conic Sections

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These are the mathematical objects that describe all orbits in classical dynamics provided that the force of attraction between an orbiting mass m and a central mass M is the inverse square law of universal gravitation. They are generated from the two dimensional conic sections by replacing the plane polar by the spherical polar coordinates in the kinetic energy of the hamiltonian and lagrangian. There are many more 3D conic sections than 2D conic sections. Both the 3D and 2D conic section scan be classified by the eccentricity. In 2D the classification consists of the circle (0), ellipse (0 to 1), hyperbola (greater than 1) and parabola (1) where the numbers in brackets denote the eccentricity. It was Bernoulli who first demonstrated that the conic section in 2D gives the orbits of the inverse square law of attraction between m and M. In 3D these values of the eccentricity lead to three dimensional orbits that can take on many different shapes.

The 2D conic sections are exactly equivalent to the hamiltonian, provided that the half right latitude and the eccentricity are defined in terms of the two conserved quantities of 2D theory, the total energy E and L sub Z, the Z component of the angular momentum. In 2D theory only L sub Z is considered and the plane polar coordinates, r and phi, are used. In 3D theory there are three components of angular momentum, and the magnitude of the total angular momentum is denoted L. The spherical polar coordinates are used, r, phi and theta. The inverse square law of attraction between m and M remains the same in 2D and 3D, but the kinetic energy is changed from 2D to 3D. This change result in phi of the 2D conic sections being replaced by an angle denoted beta of the 3D conic sections. The latter are named the beta conic sections.

In 3D theory the beta conic sections are again exactly equivalent to the hamiltonian, and the half right latitude and eccentricity in 3D are again expressed in terms of the conserved quantities: the total energy E and the total angular momentum L. So the L sub Z of the 2D theory is replaced by L of the 3D theory in the definition of the half right latitude and the eccentricity. The dependence of r on phi in the 2D conic section is replaced by the dependence of r on beta in the 3D conic section. These simple changes result in an entirely new type of cosmology on the classical level, as described in UFT269 ff. on www.aias.us. They also affect special relativistic effects such as the Thomas precession or rotation of the Minkowski metric of special relativity.

By using a lagrangian analysis, relations can be deduced between the angle beta and phi, between the angle beta and theta, between phi and theta, between the angle beta and a combination of phi and theta, and so on. They result in a wealth of new information as graphed in various incisive ways by co author Horst Eckardt in UFT269 to UFT276 and on the diary or blog of www.aias.us. The classifications of the 3D conic sections depend on the eccentricity, but now the various choices of eccentricity lead to many more orbital possibilities. In a well defined limit the beta conic section manifests itself as a precessing ellipse whose perihelion precession is L / L sub Z. So the orbits of the solar system, so long thought to be two dimensional, are manifestations of a three dimensional orbit in a given limit defined by the ratio of L to L sub Z slightly greater than one. In general the precession of the perihelion depends on all the angles of the three dimensional system, beta, phi and theta. So the precession in general is clearly three dimensional and can become an intricate phenomenon.

The beta conic sections are by far the clearest manifestation of three dimensional orbits, but the theory can be developed in Cartesian coordinates in order to demonstrate the possibilities, for example spherical orbits, hyperboloidal, ellipsoidal or cigar shaped orbits,

parabaloidal and so on. The entire analysis can be repeated for any observed three dimensional orbit, which can be analysed with the three dimensional Binet equation to give the force law needed for that orbit. In general a three dimensional orbit is any function of r, phi and theta. So an entirely new cosmology can be developed. All that has been done is to replace the plane polar by the spherical polar coordinates, so all can agree on this development.