Essay 23: THE FERMION EQUATION

The fermion equation emerged from ECE theory in about 2009 (UFT 129 and 130) and is developed in UFT 172 onwards. The fermion equation could only have been inferred from a successful generally covariant unified field theory, and so settles in the affirmative the question of whether or not ECE has advantages - it is clear to everyone that it has major advantages which we are only just beginning to explore. The fermion is named after Enrico Fermi and can be thought of as a particle with half integral spin. An example is the electron, another is its anti particle the positron. The term Ahalf integral spin@ is an obscure way of describing spectral properties that were first noticed over a hundred years ago in the Zeeman effect, the effect of a magnetic field on atomic spectra. The spectra were anomalous, and could not be explained easily without the introduction of a new type of angular momentum denoted S, the spin angular momentum. Up to that time angular momentum was thought of in the classical sense of vector product r and p, position and momentum, and denoted L. The new type of total angular momentum was denoted L + 2S and this is still how chemists learn the subject today.

When Erwin Schroedinger inferred his famous equation in the mid nineteen twenties the spin angular momentum was not incorporated. So the Schroedinger equation describes certain features of the atomic spectrum of hydrogen (H) but not others. The latter are known as Afine structure@. Certain lines in the spectrum are split, and this splitting needed a new explanation. Stern and Gerlach had carried out an experiment in which an electron beam was passed between the poles of a powerful magnet. The electron beam was split into two components, showing that the electron had a property that was hitherto unknown up to that time. It was realized that this property was related to the anomalous Zeeman effect in atomic spectra and was due to spin angular momentum S. Wolfgang Pauli realized that the half integral spin could be described by 2 x 2 matrices with certain properties called commutator properties in mathematics. The commutator contained the factor 1 / 2 needed to explain half integral spin. When the Pauli matrices are incorporated into the Schroedinger equation the anomalous Zeeman effect is explained through what is known as the Lande factor or g = 2 factor of the electron. The latter can have spin half or minus half, explaining the Stern Gerlach experiment with the Pauli matrices.

Paul Dirac began to think of how to incorporate these results, and the then new matrix quantum mechanics of Werner Heisenberg, into a quantum mechanics that fitted in with the special relativity of Albert Einstein. The latter=s energy equation, inferred in about 1905, is quadratic in the total relativistic energy denoted by the symbol E. It has the format E squared = c squared p squared + m squared c fourth, where E0 = m c squared is the rest energy. The problem that faced Dirac was how to incorporate the quantum mechanical operator format of p and E into special relativity. This had to be done while retaining intact Max Born=s probability interpretation of the wave function. The probability had to be rigorously non-negative and also properly relativistic, the scalar part of the probability four-current. Dirac=s intention was to obtain a first order differential equation similar to the time dependent Schroedinger equation. He solved the problem mathematically with use of nineteenth century Clifford algebra to obtain an equation in which was incorporated a set of four 4 x 4 matrices known as the Dirac matrices. The wave function of the Dirac equation had to be a column vector with four entries, the right and left Pauli spinors placed on top of each other.

To Heisenberg this procedure looked artificial, and Heisenberg reacted strongly against the Dirac equation, calling it a Alow point in physics@. However it soon became clear that the Dirac equation=s hamiltonian produced the Lande factor and the relativistic Thomas factor inferred about 1926 or 1927 by Llewelyn Thomas, and also gave a rigorously non

negative Born probability. Furthermore, the probability four current was properly conserved. The Thomas factor appears in spectra through spin orbit interaction which causes lines to be split. However the Dirac equation had one major drawback - it appeared to produce negative energy. This concept has plagued theoretical physics for eighty years until the emergence of the ECE fermion equation.

The fermion equation is a first order differential equation in 2 x 2 Pauli matrices, and gives all that the 4 x 4 matrix Dirac equation does without the problem of negative energy. This means that the fermion equation is the first true single particle equation of the electron and positron, and all other fermions in particle physics. The advantages of the fermion equation are multifold and only just beginning to be realized. Notably, the fermion equation has no negative energy eigenvalues, so there is no need for a multi particle interpretation, no need for a Dirac sea and no need for any development in quantum field theory based on the Dirac sea. The fermion equation is part of a generally covariant unified field theory, and was inferred from the ECE theory which showed that the fermion spinor had to be a 2 x 2 matrix and not a column vector. This 2 x 2 matrix is an example of a Cartan tetrad in the complex space of two dimensions. The fermion equation is therefore generally covariant, while the Dirac equation is Lorentz covariant only. As shown by Lewis Ryder, the Dirac equation is a way of expressing the Lorentz transformation of the two Pauli spinors, right and left, and so the fermion equation achieves this elegant result in a simpler and more powerful way.

Therefore the whole of relativistic quantum mechanics and quantum field theory can be redeveloped on the basis of the fermion equation and within the context of a generally covariant unified field theory that is far simpler and much more powerful than standard model attempts at a unified field theory in the twentieth century. The solutions of the fermion equation can be understood much more clearly than those of the Dirac equation, because there are no eigenfunctions of the fermion equation that lead to negative energy eigenvalues, backward in time motion and so forth. These were mathematical artifices of an incomplete and over-complicated theory. In computational quantum chemistry the fermion equation has a simpler structure and can be solved numerically in a simpler way. The unification of the fermion field with the gravitational, electromagnetic, weak and strong fields has been achieved on the basis of Cartan geometry because all the eigenfunctions are tetrads of various kinds in the general spacetime. Finally the minimal prescription is easily incorporated into the fermion equation, which can be solved straightforwardly with the new half-operator method explained in UFT 173. The minimal prescription can be used for any field.

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