ESSAY 41 : THE FUNDAMENTAL THEOREM OF GEOMETRY

The fundamental theorem of geometry is the appellation given to the equation which defines the geometrical connection of Christoffel in terms of the metric of Riemann. The latter inferred the concept of metric in the frst half of the nineteenth century essentially as a measure of non Euclidean space. The connection was inferred in 1867 by Christoffel. The theorem is regarded as being as fundamental as that of Pythagoras of Samos, inferred 2,500 years ago, the archetypical theorem of geometry. The Pythagoras theorem holds in a two dimensional space. The propositions of Euclid hold in a three dimensional space. The three dimensional space of Euclid is one where the connection vanishes and in which the metric is the three by three diagonal unit matrix. In the nineteenth century it was realized that the Euclidean space can be made into a more general concept of geometry, known loosely as curved space. In such a mathematical space the metric is no longer the unit matrix, and there exists an object known as the connection. The latter is defined by the way in which the derivative of a vector in a curved space differs from the derivative in flat space. The term flat space is used to define a space without a connection.

In the late nineteenth century mathematicians were interested in the problem of how to define the Christoffel connection given the Riemann metric. Levi - Civita and Ricci, pioneers of tensor analysis, made an attempt to solve the problem in about 1900 using three equations of metric compatibility in cyclic permutation. The equation of metric compatibility asserts that the covariant derivative of the metric vanishes, and the covariant derivative is the type of derivative that must be used in a curved space. Unfortunately, Levi - Civita and Ricci used the wrong symmetry for the geometrical connection. The latter is not a tensor, because it does not transform as tensor, and is an object with three indices, one upper and two lower. Levi - Civita and Ricci used a connection that was symmetric in its lower two indices, and that allowed them to define the connection uniquely given the metric. This became known as the fundamental theorem of Riemann geometry. This is a misappellation because Riemann never inferred the idea of the geometrical connection.

As described in previous essays it is known clearly now that the connection must be antisymmetric in its lower two indices. The reason for this is that the connection is isolated and defined as being unequivocally antisymmetric by the commutator of covariant derivatives acting on any tensor in any space in any dimension. In other words the connection is defined in precisely the same way as the Riemann torsion and Riemann curvature, names given to the two fundamental tensors of geometry. The consequences of using the wrong connection symmetry are disastrous for standard physics. Essentially everything in Einsteinian general relativity rested on this choice, the second Bianchi identity for example. Einstein based his field equation directly on that identity. A symmetric connection means no torsion, and indeed the subject of general relativity in the twentieth century was developed almost axiomatically as being torsion free. It is now known that any symmetric connection must vanish, so the older type of general relativity must be discarded as meaningless.

ECE theory was developed from the beginning as a subject with torsion included. It gradually became clear that almost every idea in the foggy dogma (or fogma) of the twentieth century needed an overhaul. On July 2nd 2011 the fundamental theorem itself was developed into one based on the right connection symmetry. The new fundamental theorem of geometry is proven by starting in the same way as Levi-Civita and Ricci, by using three metric compatibility equations in cyclic permutation. Two equations are subtracted from a third in cyclic permutation. Antisymmetry is applied, and two of the resulting three equations added. The result is a relatively simple and unique relation between the general metric and the antisymmetric connection. For a diagonal metric the fundamental theorem assumes a simple

format, easily worked out by hand or by computer. The hand calculations were checked by computer algebra. The theorem has also been named AThe Theorem of the Antisymmetric Connection@ and was derived on July 2nd, 2011.

After more than 110 years therefore the correct relation between the Riemann metric and the Christoffel connection has been established. Having achieved this, cosmology may at last be developed as a rigorously correct subject using a combination of the fundamental theorem and Evans identity, discarding dark matter, big bang, black holes and string theory as being fogmatic idols of the cave. It has been known for half a century that the obsolete Einstein field equation was a total failure outside the solar system, where it gave only a mirage of accuracy. The so called ASchwarzschild metric@ was not derived by him, its actual origin is a mystery. The real origin for such a metric is now known to be spherical symmetry of spacetime. When put to the test of the new torsion based cosmology it fails except in the limit of infinite r.

So we are the start of a completely new era in geometry and cosmology.