

```
(%i1)
```

```
/* define special summation function */
f(i,j) := sum(R[i,j,sigma,0]*gContr[i,sigma]*gContr[j,0],sigma,0,3)
        + sum(R[i,j,sigma,1]*gContr[i,sigma]*gContr[j,1],sigma,0,3)
        + sum(R[i,j,sigma,2]*gContr[i,sigma]*gContr[j,2],sigma,0,3)
        + sum(R[i,j,sigma,3]*gContr[i,sigma]*gContr[j,3],sigma,0,3);
```

```
(%o1) f(i,j) := sum(Ri,j,σ,0 gContri,σ gContrj,0,σ,0,3) +
```

```
sum(Ri,j,σ,1 gContri,σ gContrj,1,σ,0,3) +
```

```
sum(Ri,j,σ,2 gContri,σ gContrj,2,σ,0,3) +
```

```
sum(Ri,j,σ,3 gContri,σ gContrj,3,σ,0,3)
```

```
(%i2) /* define coordinate vector */
```

```
array(x, 3);
[x[0],x[1],x[2],x[3]]: [t, r, theta, phi];
```

```
(%o2) x
```

```
(%o3) [ t , r , θ , φ ]
```

```
(%i4) /* g1 is symm. metric with indices 1...4 */
```

```
g1: matrix(
  [-(1+a*r^2),0,0,0],
  [0,(1-3*a*r^2)^(2/3)/((1+3*a*r^2)^(2/3)-b*r^2),0,0],
  [0,0,r^2,0],
  [0,0,0,r^2*sin(theta)^2]
);
```

```
(%o4) 
$$\begin{bmatrix} -a r^2 - 1 & 0 & 0 & 0 \\ 0 & \frac{(1 - 3 a r^2)^{2/3}}{(3 a r^2 + 1)^{2/3} - b r^2} & 0 & 0 \\ 0 & 0 & r^2 & 0 \\ 0 & 0 & 0 & r^2 \sin(\theta)^2 \end{bmatrix}$$

```

```
(%i5) /* contravariant g is inverse of g */
```

```
gContr1: ratsimp(invert(g1));
```

$$(\%o5) \begin{bmatrix} -\frac{1}{a r^2 + 1} & 0 & 0 & 0 \\ 0 & \frac{(3 a r^2 + 1)^{2/3} - b r^2}{(1 - 3 a r^2)^{2/3}} & 0 & 0 \\ 0 & 0 & \frac{1}{r^2} & 0 \\ 0 & 0 & 0 & \frac{1}{r^2 \sin(\theta)^2} \end{bmatrix}$$

(%i6)

```
/* g1 and gContr1 are transformed to g and gContr (indices 0...3) */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    g      [mu,nu]: g1      [mu+1, nu+1],
    gContr[mu,nu]: gContr1[mu+1, nu+1]
}}$
```

```
(%i7) /* computation of Christoffel symbols Gamma^sigma_mu_nu */
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Gamma[sigma,mu,nu] :
    /* rho sum by function call: */
    sum(
        1/2 * gContr[sigma,rho]*(
            diff(g[nu,rho],x[mu] ) +
            diff(g[rho,mu],x[nu] ) -
            diff(g[mu,nu] ,x[rho])),
        rho, 0, 3),
    /* evaluate differentiation dy/dr */
    Gamma[sigma,mu,nu]: ev(Gamma[sigma,mu,nu],diff)
}}}$
```

```
(%i8) /* display Gamma's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
    if Gamma[i,j,k] # 0 then {
        display(Gamma[i,j,k])
    }}}}$
```

$$\Gamma_{0,0,1} = \frac{a r}{a r^2 + 1}$$

$$\Gamma_{0,1,0} = \frac{a r}{a r^2 + 1}$$

$$\Gamma_{1,0,0} = \frac{a r \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$\Gamma_{1,1,1} =$$

$$\frac{\left((3 a r^2 + 1)^{2/3} - b r^2 \right) \left(- \frac{4 a r}{(1 - 3 a r^2)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)} - \frac{(1 - 3 a r^2)^{2/3} \left(\frac{4 a r}{(3 a r^2 + 1)^{1/3}} - 2 b r \right)}{\left((3 a r^2 + 1)^{2/3} - b r^2 \right)^2} \right)}{2 (1 - 3 a r^2)^{2/3}}$$

$$\Gamma_{1,2,2} = - \frac{r \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$\Gamma_{1,3,3} = - \frac{r \left((3 a r^2 + 1)^{2/3} - b r^2 \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3}}$$

$$\Gamma_{2,1,2} = \frac{1}{r}$$

$$\Gamma_{2,2,1} = \frac{1}{r}$$

$$\Gamma_{2,3,3} = - \cos(\theta) \sin(\theta)$$

$$\Gamma_{3,1,3} = \frac{1}{r}$$

$$\Gamma_{3,2,3} = \frac{\cos(\theta)}{\sin(\theta)}$$

$$\Gamma_{3,3,1} = \frac{1}{r}$$

$$\Gamma_{3,3,2} = \frac{\cos(\theta)}{\sin(\theta)}$$

```
(%i9) /* compute Riemann tensor elements */
for rho:0 thru 3 do {
for sigma:0 thru 3 do {
for mu:0 thru 3 do {
for nu:0 thru 3 do {
  R[rho,sigma,mu,nu] :
    diff(Gamma[rho,nu,sigma],x[mu]) -
    diff(Gamma[rho,mu,sigma],x[nu]) +
    /* lambda sums by function call: */
    sum(
      Gamma[rho,mu,lambda] * Gamma[lambda,nu,sigma] -
      Gamma[rho,nu,lambda] * Gamma[lambda,mu,sigma],
      lambda, 0, 3)
}}}}$
```

```
(%i10) /* display R's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
for k:0 thru 3 do {
for l:0 thru 3 do {
R[i,j,k,l] : /*ratsimp*/(factor(R[i,j,k,l])),
if R[i,j,k,l] # 0 then display(R[i,j,k,l])
}}}}$
```

$$R_{0,1,0,1} =$$

$$\frac{a \left(a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1 \right)}{(a r^2 + 1)^2 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{0,1,1,0} = -$$

$$\frac{a \left(a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1 \right)}{(a r^2 + 1)^2 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{0,2,0,2} = - \frac{a r^2 \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)}$$

$$R_{0,2,2,0} = \frac{a r^2 \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)}$$

$$R_{0,3,0,3} = - \frac{a r^2 \left((3 a r^2 + 1)^{2/3} - b r^2 \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)}$$

$$R_{0,3,3,0} = \frac{a r^2 \left((3 a r^2 + 1)^{2/3} - b r^2 \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)}$$

$$R_{1,0,0,1} =$$

$$\frac{a \left(a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{1,0,1,0} = -$$

$$\frac{a \left(a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{1,2,1,2} = \frac{r^2 \left(a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a \right)}{(1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{1,2,2,1} = - \frac{r^2 \left(a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a \right)}{(1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{1,3,1,3} = \frac{r^2 \left(a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{1,3,3,1} = - \frac{r^2 \left(a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

$$R_{2,0,0,2} = - \frac{a \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$R_{2,0,2,0} = \frac{a \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$R_{2,1,1,2} = - \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{(3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{2,1,2,1} = \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{(3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{2,3,2,3} = - \frac{\left((3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2 \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3}}$$

$$R_{2,3,3,2} = \frac{\left((3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2 \right) \sin(\theta)^2}{(1 - 3 a r^2)^{2/3}}$$

$$R_{3,0,0,3} = - \frac{a \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$R_{3,0,3,0} = \frac{a \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3}}$$

$$R_{3,1,1,3} = - \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{(3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{3,1,3,1} = \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{(3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}$$

$$R_{3,2,2,3} = \frac{(3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2}{(1 - 3 a r^2)^{2/3}}$$

$$R_{3,2,3,2} = - \frac{(3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2}{(1 - 3 a r^2)^{2/3}}$$

```
(%i11) /* Ricci tensor Ric[mu,nu] */
for mu:0 thru 3 do {
for nu:0 thru 3 do {
    Ric[mu,nu]: sum(R[lambda,mu,lambda,nu], lambda, 0, 3)
}}$
```

```
(%i12) /* display Ric's being different from zero */
for i:0 thru 3 do {
for j:0 thru 3 do {
    Ric[i,j] : /*ratsimp*/(factor(Ric[i,j])),
    if Ric[i,j] # 0 then display(Ric[i,j])
}}$
```

$$Ric_{0,0} = - (a (7 a^2 b r^6 (3 a r^2 + 1)^{1/3} + 7 a b r^4 (3 a r^2 + 1)^{1/3} - 4 b r^2 (3 a r^2 + 1)^{1/3} - 18 a^3 r^6 - 23 a^2 r^4 + 6 a r^2 + 3)) / ((1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3})$$

$$Ric_{1,1} = (3 a^3 b r^6 (3 a r^2 + 1)^{1/3} + 5 a^2 b r^4 (3 a r^2 + 1)^{1/3} - 4 a b r^2 (3 a r^2 + 1)^{1/3} - 2 b (3 a r^2 + 1)^{1/3} + 3 a^3 r^4 + 20 a^2 r^2 + 9 a) / ((a r^2 + 1)^2 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3} ((3 a r^2 + 1)^{2/3} - b r^2))$$

$$Ric_{2,2} = (3 a^2 r^4 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 2 a r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} - (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 7 a^2 b r^6 (3 a r^2 + 1)^{1/3} + a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 18 a^3 r^6 - 5 a^2 r^4 + 6 a r^2 + 1) / ((1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3})$$

$$Ric_{3,3} = ((3 a^2 r^4 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 2 a r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} - (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 7 a^2 b r^6 (3 a r^2 + 1)^{1/3} + a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 18 a^3 r^6 - 5 a^2 r^4 + 6 a r^2 + 1) \sin(\theta)^2) / ((1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3})$$

```
(%i13) /* Ricci Scalar */
RicSc: sum(gContr[0,lambda]*Ric[lambda,0], lambda, 0, 3)
+ sum(gContr[1,lambda]*Ric[lambda,1], lambda, 0, 3)
+ sum(gContr[2,lambda]*Ric[lambda,2], lambda, 0, 3)
+ sum(gContr[3,lambda]*Ric[lambda,3], lambda, 0, 3)
;
```

$$RicSc = (2 (3 a^2 r^4 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 2 a r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} - (1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} + 7 a^2 b r^6 (3 a r^2 + 1)^{1/3} + a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 18 a^3 r^6 - 5 a^2 r^4 + 6 a r^2 + 1)) / (r^2 (1 - 3 a r^2)^{2/3} (a r^2 + 1) (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}) + (3 a^3 b r^6 (3 a r^2 + 1)^{1/3} +$$

$$\frac{5 a^2 b r^4 (3 a r^2 + 1)^{1/3} - 4 a b r^2 (3 a r^2 + 1)^{1/3} - 2 b (3 a r^2 + 1)^{1/3} + 3 a^3 r^4 + 20 a^2 r^2 + 9 a}{((1 - 3 a r^2)^{2/3} (a r^2 + 1)^2 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}) + (a (7 a^2 b r^6 (3 a r^2 + 1)^{1/3} + 7 a b r^4 (3 a r^2 + 1)^{1/3} - 4 b r^2 (3 a r^2 + 1)^{1/3} - 18 a^3 r^6 - 23 a^2 r^4 + 6 a r^2 + 3))} \cdot \frac{1}{((1 - 3 a r^2)^{2/3} (a r^2 + 1)^2 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3})}$$

```
(%i14) ratsimp(RicSc);
```

```
(%o14) ((3 a r^2 + 1)^{1/3}
(24 a^3 b r^8 + (1 - 3 a r^2)^{2/3} (6 a^3 r^6 + 10 a^2 r^4 + 2 a r^2 - 2) + 28 a^2 b r^6 - 10 a b r^4 - 6 b r^2)
- 54 a^4 r^8 - 66 a^3 r^6 + 28 a^2 r^4 + 26 a r^2 + 2) / ((1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3}
(3 a^3 r^8 + 5 a^2 r^6 + a r^4 - r^2))
```

```
(%i15)
```

```
/* Test for R^q */
for mu: 0 thru 3 do (
for sigma:0 thru 3 do (
for nu: 0 thru 3 do (
for rho: 0 thru 3 do (
R_q: R[mu,sigma,nu,rho] + R[mu,rho,sigma,nu] + R[mu,nu,rho,sigma],
if R_q # 0 then (
display("=====Einstein equation R^q=0 not fulfilled! "),
display(mu,sigma,nu,rho),
display(R_q)
)
))));
```

```
(%o15) done
```

```
(%i16) /* Raising of indices,
contravarinat metric el. is g^x^x(contr.) = 1/g_x_x(cov.) */
/*print("Riemann elements R^0_1^0^1, R^0_2^0^2, R^0_3^0^3:");*/

R0101: f(0,1);
R0202: f(0,2);
R0303: f(0,3);
```

```
(%o16) -
```

$$\frac{a \left(a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)^3 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

```
(%o17) \frac{a \left( (3 a r^2 + 1)^{2/3} - b r^2 \right)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)^2}
```

$$(\%o18) \quad \frac{a((3ar^2+1)^{2/3}-br^2)}{(1-3ar^2)^{2/3}(ar^2+1)^2}$$

(%i19) R0101: factor(R0101);
 R0202: factor(R0202);
 R0303: factor(R0303);

(%o19) -

$$\frac{a(a^2br^6(3ar^2+1)^{1/3}+3abr^4(3ar^2+1)^{1/3}-2br^2(3ar^2+1)^{1/3}-5a^2r^4+4ar^2+1)}{(1-3ar^2)^{2/3}(ar^2+1)^3(3ar^2-1)(3ar^2+1)^{1/3}}$$

$$(\%o20) \quad \frac{a((3ar^2+1)^{2/3}-br^2)}{(1-3ar^2)^{2/3}(ar^2+1)^2}$$

$$(\%o21) \quad \frac{a((3ar^2+1)^{2/3}-br^2)}{(1-3ar^2)^{2/3}(ar^2+1)^2}$$

(%i22) R1010: f(1,0);
 R1212: f(1,2);
 R1313: f(1,3);

(%o22) (a

$$\left(a^2br^6(3ar^2+1)^{1/3}+3abr^4(3ar^2+1)^{1/3}-2br^2(3ar^2+1)^{1/3}-5a^2r^4+4ar^2+1\right) \\ \left((3ar^2+1)^{2/3}-br^2\right) / \left((1-3ar^2)^{4/3}(ar^2+1)^2(3ar^2-1)(3ar^2+1)^{1/3}\right)$$

$$(\%o23) \quad \frac{\left(abr^2(3ar^2+1)^{1/3}-b(3ar^2+1)^{1/3}+4a\right)\left((3ar^2+1)^{2/3}-br^2\right)}{(1-3ar^2)^{4/3}(3ar^2-1)(3ar^2+1)^{1/3}}$$

$$(\%o24) \quad \frac{\left(abr^2(3ar^2+1)^{1/3}-b(3ar^2+1)^{1/3}+4a\right)\left((3ar^2+1)^{2/3}-br^2\right)}{(1-3ar^2)^{4/3}(3ar^2-1)(3ar^2+1)^{1/3}}$$

(%i25) R1010: factor(R1010);
 R1212: factor(R1212);
 R1313: factor(R1313);

(%o25) - (a

$$\left(a^2br^6(3ar^2+1)^{1/3}+3abr^4(3ar^2+1)^{1/3}-2br^2(3ar^2+1)^{1/3}-5a^2r^4+4ar^2+1\right) \\ \left((3ar^2+1)^{2/3}-br^2\right) / \left((1-3ar^2)^{1/3}(ar^2+1)^2(3ar^2-1)^2(3ar^2+1)^{1/3}\right)$$

$$(\%o26) \quad - \frac{\left(abr^2(3ar^2+1)^{1/3}-b(3ar^2+1)^{1/3}+4a\right)\left((3ar^2+1)^{2/3}-br^2\right)}{(1-3ar^2)^{1/3}(3ar^2-1)^2(3ar^2+1)^{1/3}}$$

$$(\%o27) \quad - \frac{\left(abr^2(3ar^2+1)^{1/3}-b(3ar^2+1)^{1/3}+4a\right)\left((3ar^2+1)^{2/3}-br^2\right)}{(1-3ar^2)^{1/3}(3ar^2-1)^2(3ar^2+1)^{1/3}}$$


```
(%i28) R2020: f(2,0);
R2121: f(2,1);
R2323: f(2,3);
```

$$(\%o28) \quad - \frac{a((3ar^2+1)^{2/3} - br^2)}{r^2(1-3ar^2)^{2/3}(ar^2+1)}$$

$$(\%o29) \quad \frac{ab r^2(3ar^2+1)^{1/3} - b(3ar^2+1)^{1/3} + 4a}{r^2(1-3ar^2)^{2/3}(3ar^2-1)(3ar^2+1)^{1/3}}$$

$$(\%o30) \quad - \frac{(3ar^2+1)^{2/3} - (1-3ar^2)^{2/3} - br^2}{r^4(1-3ar^2)^{2/3}}$$

```
(%i31) R2020: factor(R2020);
R2121: factor(R2121);
R2323: factor(R2323);
```

$$(\%o31) \quad - \frac{a((3ar^2+1)^{2/3} - br^2)}{r^2(1-3ar^2)^{2/3}(ar^2+1)}$$

$$(\%o32) \quad \frac{ab r^2(3ar^2+1)^{1/3} - b(3ar^2+1)^{1/3} + 4a}{r^2(1-3ar^2)^{2/3}(3ar^2-1)(3ar^2+1)^{1/3}}$$

$$(\%o33) \quad - \frac{(3ar^2+1)^{2/3} - (1-3ar^2)^{2/3} - br^2}{r^4(1-3ar^2)^{2/3}}$$

```
(%i34) R3030: f(3,0);
R3131: f(3,1);
R3232: f(3,2);
```

$$(\%o34) \quad - \frac{a((3ar^2+1)^{2/3} - br^2)}{r^2(1-3ar^2)^{2/3}(ar^2+1)\sin(\theta)^2}$$

$$(\%o35) \quad \frac{ab r^2(3ar^2+1)^{1/3} - b(3ar^2+1)^{1/3} + 4a}{r^2(1-3ar^2)^{2/3}(3ar^2-1)(3ar^2+1)^{1/3}\sin(\theta)^2}$$

$$(\%o36) \quad - \frac{(3ar^2+1)^{2/3} - (1-3ar^2)^{2/3} - br^2}{r^4(1-3ar^2)^{2/3}\sin(\theta)^2}$$

```
(%i37) R3030: factor(R3030);
R3131: factor(R3131);
R3232: factor(R3232);
```

$$(\%o37) \quad - \frac{a((3ar^2+1)^{2/3} - br^2)}{r^2(1-3ar^2)^{2/3}(ar^2+1)\sin(\theta)^2}$$

$$(\%038) \quad \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \sin(\theta)^2}$$

$$(\%039) \quad - \frac{(3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2}{r^4 (1 - 3 a r^2)^{2/3} \sin(\theta)^2}$$

(%i40) /* Coulomb law */
DivE : R0101 + R0202 + R0303;

$$(\%040) \quad \frac{2 a ((3 a r^2 + 1)^{2/3} - b r^2)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)^2} -$$

$$\frac{a (a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1)}{(1 - 3 a r^2)^{2/3} (a r^2 + 1)^3 (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

(%i41) ratsimp(DivE);

$$(\%041) \quad ((1 - 3 a r^2)^{1/3} (3 a r^2 + 1)^{1/3} (7 a^3 b r^6 + 7 a^2 b r^4 - 4 a b r^2) + (1 - 3 a r^2)^{1/3} (-18 a^4 r^6 - 23 a^3 r^4 + 6 a^2 r^2 + 3 a)) / ((3 a r^2 + 1)^{1/3} (9 a^5 r^{10} + 21 a^4 r^8 + 10 a^3 r^6 - 6 a^2 r^4 - 3 a r^2 + 1))$$

(%i42) /* J[r] */
Jr : -(R1010 + R1212 + R1313);

(%042) (a

$$\begin{aligned} & (a^2 b r^6 (3 a r^2 + 1)^{1/3} + 3 a b r^4 (3 a r^2 + 1)^{1/3} - 2 b r^2 (3 a r^2 + 1)^{1/3} - 5 a^2 r^4 + 4 a r^2 + 1) \\ & ((3 a r^2 + 1)^{2/3} - b r^2) / ((1 - 3 a r^2)^{1/3} (a r^2 + 1)^2 (3 a r^2 - 1)^2 (3 a r^2 + 1)^{1/3}) \\ & + \frac{2 (a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a) ((3 a r^2 + 1)^{2/3} - b r^2)}{(1 - 3 a r^2)^{1/3} (3 a r^2 - 1)^2 (3 a r^2 + 1)^{1/3}} \end{aligned}$$

(%i43) ratsimp(Jr);

$$(\%043) \quad - ((3 a r^2 + 1)^{2/3} (3 a^3 b^2 r^8 + 5 a^2 b^2 r^6 - 4 a b^2 r^4 - 2 b^2 r^2) + (3 a r^2 + 1)^{1/3} (-9 a^4 b r^8 - 15 a^3 b r^6 + 27 a^2 b r^4 + 19 a b r^2 + 2 b) - 9 a^4 r^6 - 63 a^3 r^4 - 47 a^2 r^2 - 9 a) / ((1 - 3 a r^2)^{1/3} (3 a r^2 + 1)^{2/3} (9 a^4 r^8 + 12 a^3 r^6 - 2 a^2 r^4 - 4 a r^2 + 1))$$

(%i44) /* J[theta] */
Jtheta : -(R2020 + R2121 + R2323);

$$(\%044) \quad \frac{(3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2}{r^4 (1 - 3 a r^2)^{2/3}} + \frac{a ((3 a r^2 + 1)^{2/3} - b r^2)}{r^2 (1 - 3 a r^2)^{2/3} (a r^2 + 1)} -$$

$$\frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3}}$$

(%i45) ratsimp(Jtheta);

(%o45)
$$- \left((3 a r^2 + 1)^{1/3} \right. \\ \left. \left(7 a^2 b r^6 + (1 - 3 a r^2)^{2/3} (3 a^2 r^4 + 2 a r^2 - 1) + a b r^4 - 2 b r^2 \right) - 18 a^3 r^6 - 5 a^2 r^4 \right. \\ \left. + 6 a r^2 + 1 \right) / \left((1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} (3 a^2 r^8 + 2 a r^6 - r^4) \right)$$

(%i46) /* J[phi] */
Jphi : -(R3030 + R3131 + R3232);

(%o46)
$$\frac{(3 a r^2 + 1)^{2/3} - (1 - 3 a r^2)^{2/3} - b r^2}{r^4 (1 - 3 a r^2)^{2/3} \sin(\theta)^2} + \frac{a \left((3 a r^2 + 1)^{2/3} - b r^2 \right)}{r^2 (1 - 3 a r^2)^{2/3} (a r^2 + 1) \sin(\theta)^2} - \\ \frac{a b r^2 (3 a r^2 + 1)^{1/3} - b (3 a r^2 + 1)^{1/3} + 4 a}{r^2 (1 - 3 a r^2)^{2/3} (3 a r^2 - 1) (3 a r^2 + 1)^{1/3} \sin(\theta)^2}$$

(%i47) ev(ratsimp(Jphi),r);

(%o47)
$$- \left((3 a r^2 + 1)^{1/3} \right. \\ \left(7 a^2 b r^6 + (1 - 3 a r^2)^{2/3} (3 a^2 r^4 + 2 a r^2 - 1) + a b r^4 - 2 b r^2 \right) - 18 a^3 r^6 - 5 a^2 r^4 \\ \left. + 6 a r^2 + 1 \right) / \left((1 - 3 a r^2)^{2/3} (3 a r^2 + 1)^{1/3} (3 a^2 r^8 + 2 a r^6 - r^4) \sin(\theta)^2 \right)$$

(%i48) DivE_p: ev(at(DivE,[a=1,b=1]));

(%o48)
$$\frac{2 \left((3 r^2 + 1)^{2/3} - r^2 \right)}{(1 - 3 r^2)^{2/3} (r^2 + 1)^2} - \\ \frac{r^6 (3 r^2 + 1)^{1/3} + 3 r^4 (3 r^2 + 1)^{1/3} - 2 r^2 (3 r^2 + 1)^{1/3} - 5 r^4 + 4 r^2 + 1}{(1 - 3 r^2)^{2/3} (r^2 + 1)^3 (3 r^2 - 1) (3 r^2 + 1)^{1/3}}$$

(%i49) Jr_p: ev(at(Jr,[a=1,b=1]));

(%o49)
$$\frac{\left(r^6 (3 r^2 + 1)^{1/3} + 3 r^4 (3 r^2 + 1)^{1/3} - 2 r^2 (3 r^2 + 1)^{1/3} - 5 r^4 + 4 r^2 + 1 \right) \left((3 r^2 + 1)^{2/3} - r^2 \right)}{(1 - 3 r^2)^{1/3} (r^2 + 1)^2 (3 r^2 - 1)^2 (3 r^2 + 1)^{1/3}} + \\ \frac{2 \left(r^2 (3 r^2 + 1)^{1/3} - (3 r^2 + 1)^{1/3} + 4 \right) \left((3 r^2 + 1)^{2/3} - r^2 \right)}{(1 - 3 r^2)^{1/3} (3 r^2 - 1)^2 (3 r^2 + 1)^{1/3}}$$

(%i50) Jtheta_p: ev(at(Jtheta,[a=1,b=1]));

```
(%o50)
```

$$\frac{(3r^2+1)^{2/3} - (1-3r^2)^{2/3} - r^2}{r^4(1-3r^2)^{2/3}} + \frac{(3r^2+1)^{2/3} - r^2}{r^2(1-3r^2)^{2/3}(r^2+1)} - \frac{r^2(3r^2+1)^{1/3} - (3r^2+1)^{1/3} + 4}{r^2(1-3r^2)^{2/3}(3r^2-1)(3r^2+1)^{1/3}}$$

```
(%i51) Jphi_p: ev(at(Jphi,[a=1,b=1,theta=%pi/2]));
```

```
(%o51)
```

$$\frac{(3r^2+1)^{2/3} - (1-3r^2)^{2/3} - r^2}{r^4(1-3r^2)^{2/3}} + \frac{(3r^2+1)^{2/3} - r^2}{r^2(1-3r^2)^{2/3}(r^2+1)} - \frac{r^2(3r^2+1)^{1/3} - (3r^2+1)^{1/3} + 4}{r^2(1-3r^2)^{2/3}(3r^2-1)(3r^2+1)^{1/3}}$$

```
(%i52) assume (1-3*r^2 > 0);
```

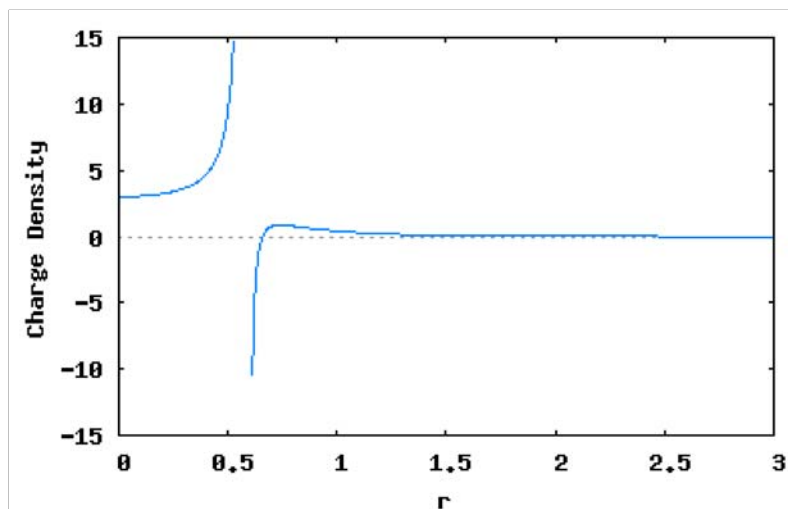
```
(%o52) [ r^2 < 1/3 ]
```

```
(%i53)
```

```
wxplot2d([DivE_p], [r,0,3],[y,-15,15], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "Charge Density"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

```
(%t53)
```

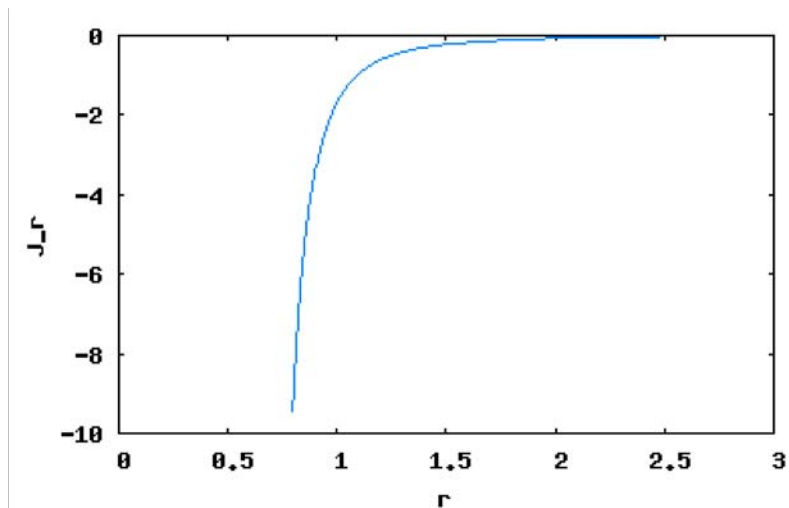


```
(%i54)
```

```
wxplot2d([Jr_p], [r,0,3],[y,-10,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_r"])$
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t54)

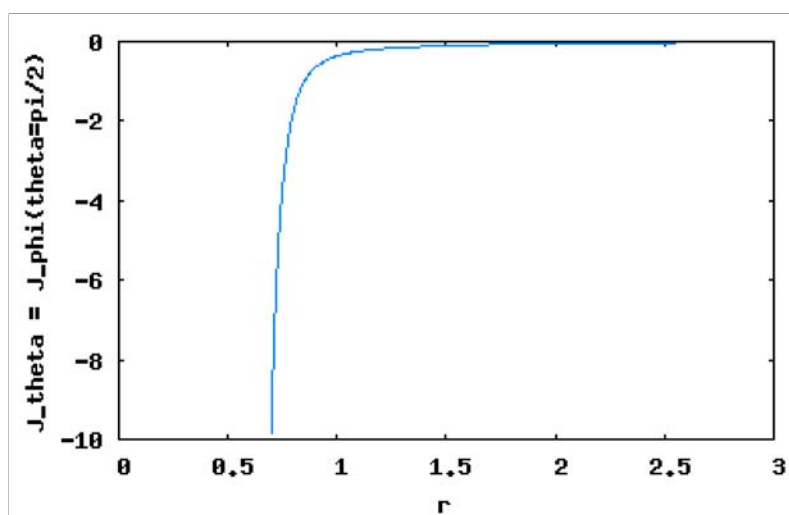


(%i55)

```
wxplot2d([Jtheta_p], [r,0,3],[y,-10,0], [gnuplot_preamble, "set zeroaxis;"],
[xlabel, "r"], [ylabel, "J_theta = J_phi(theta=pi/2)"])
```

Output file "C:/Documents and Settings/Administrator/maxout.png".

(%t55)



(%i56)