ECE Engineering Model

The Basis for Electromagnetic and Mechanical Applications

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ECE Field Equations I

- Field equations in mathematical form notation

$$D \wedge \tilde{T}^a = \tilde{R}^a{}_b \wedge q^b$$

$$D \wedge T^a = R^a{}_b \wedge q^b$$

- with
  - ^: antisymmetric wedge product
  - T^a: antisymmetric torsion form
  - $R^a{}_b$: antisymmetric curvature form
  - q^a: tetrad form (from coordiate transformation)
  - ~: Hodge dual transformation
  - *D* operator and *q* are 1-forms, *T* and *R* are 2-forms
  - summation over same upper and lower indices

ECE Axioms

- Geometric forms T^a, q^a are interpreted as physical quantities
- 4-potential A is proportional to Cartan tetrad q:

  $A^a=A^{(0)}q^a$
- Electromagnetic/gravitational field is proportional to torsion:

  $F^a=A^{(0)}T^a$
- a: index of tangent space
- $A^{(0)}$: constant with physical dimensions

ECE Field Equations II

- Field equations in tensor form

$$\partial_\mu \tilde{F}^{a\mu\nu} = A^{(0)}\left(\tilde{R}^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b}\tilde{T}^{b\mu\nu}\right) =: \mu_0 j^{a\nu}$$

$$\partial_\mu F^{a\mu\nu} = A^{(0)}\left(R^{a}{}_{\mu}{}^{\mu\nu} - \omega^{a}{}_{\mu b}T^{b\mu\nu}\right) =: \mu_0 J^{a\nu}$$

- with
  - *F*: electromagnetic field tensor, $\tilde{T}$ its Hodge dual, see later
  - *ω*: spin connection
  - *J*: charge current density
  - *j*: „homogeneous current density“, „magnetic current“
  - *a,b*: polarization indices
  - *μ,ν*: indexes of spacetime *(t,x,y,z)*

Properties of Field Equations

- *J* is not necessarily external current, is defined by spacetime properties completely
- *j* only occurs if electromagnetism is influenced by gravitation, or magnetic monopoles exist, otherwise =0
- Polarization index „*a*“ can be omitted if tangent space is defined equal to space of base manifold

Electromagnetic Field Tensor

- F and $\tilde{F}$ are antisymmetric tensors, related to vector components of electromagnetic fields (polarization index omitted)
- Cartesian components are $E_x = E^1$ etc.

$$F^{\mu\nu} = \begin{pmatrix} F^{00} & F^{01} & F^{02} & F^{03} \\ F^{10} & F^{11} & F^{12} & F^{13} \\ F^{20} & F^{21} & F^{22} & F^{23} \\ F^{30} & F^{31} & F^{32} & F^{33} \end{pmatrix} = \begin{pmatrix} 0 & -E^1 & -E^2 & -E^3 \\ E^1 & 0 & -cB^3 & cB^2 \\ E^2 & cB^3 & 0 & -cB^1 \\ E^3 & -cB^2 & cB^1 & 0 \end{pmatrix}$$

$$\tilde{F}^{\mu\nu} = \begin{pmatrix} 0 & -cB^1 & -cB^2 & -cB^3 \\ cB^1 & 0 & E^3 & -E^2 \\ cB^2 & -E^3 & 0 & E^1 \\ cB^3 & E^2 & -E^1 & 0 \end{pmatrix}$$

Potential with polarization directions

- Potential matrix:

$$\begin{pmatrix} \Phi^{(0)} & \Phi^{(1)} & \Phi^{(2)} & \Phi^{(3)} \\ 0 & A_1^{(1)} & A_1^{(2)} & A_1^{(3)} \\ 0 & A_2^{(1)} & A_2^{(2)} & A_2^{(3)} \\ 0 & A_3^{(1)} & A_3^{(2)} & A_3^{(3)} \end{pmatrix}$$

- Polarization vectors:

$$\mathbf{A}^{(1)} = \begin{pmatrix} A_1^{(1)} \\ A_2^{(1)} \\ A_3^{(1)} \end{pmatrix}, \quad \mathbf{A}^{(2)} = \begin{pmatrix} A_1^{(2)} \\ A_2^{(1)} \\ A_3^{(2)} \end{pmatrix}, \quad \mathbf{A}^{(3)} = \begin{pmatrix} A_1^{(3)} \\ A_2^{(3)} \\ A_3^{(3)} \end{pmatrix}$$

ECE Field Equations – Vector Form

$$\nabla \cdot \mathbf{B}^a = \mu_0 \rho_{eh}{}^a = \rho_{eh}{}^a{}' = 0$$ Gauss Law

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}_{eh}{}^a = \mathbf{j}_{eh}{}^a{}' = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E}^a = \frac{\rho_e{}^a}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}_e{}^a$$ Ampère-Maxwell Law

„Material“ Equations

$$\mathbf{D}^a = \varepsilon_r \varepsilon_0 \mathbf{E}^a$$ Dielectric Displacement

$$\mathbf{B}^a = \mu_r \mu_0 \mathbf{H}^a$$ Magnetic Induction

ECE Field Equations – Vector Form without Polarization Index

$$\nabla \cdot \mathbf{B} = \mu_0 \rho_{eh} = \rho_{eh}' = 0$$ Gauss Law

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = \mu_0 \mathbf{j}_{eh} = \mathbf{j}_{eh}' = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \mu_0 \mathbf{J}_e$$ Ampère-Maxwell Law

„Material“ Equations

$$\mathbf{D} = \varepsilon_r \varepsilon_0 \mathbf{E}$$ Dielectric Displacement

$$\mathbf{B} = \mu_r \mu_0 \mathbf{H}$$ Magnetic Induction

Physical Units

$$[\mathbf{E}] = \frac{V}{m}$$

$$[\mathbf{B}] = T = \frac{V \cdot s}{m^2} = \frac{N}{A \cdot m}$$

$$[\mathbf{D}] = \frac{C}{m^2}, \quad [\mathbf{H}] = \frac{A}{m}$$

$$[\Phi] = V$$

$$[\mathbf{A}] = \frac{Vs}{m} = Tm$$

$$[\omega_0] = \frac{1}{s}$$

$$[\boldsymbol{\omega}] = \frac{1}{m}$$

Charge Density/Current

$$[\rho_e] = C / m^3$$

$$[\mathbf{J}_e] = A / m^2 = C / (m^2 s)$$

„Magnetic“ Density/Current

$$[\rho_{eh}] = \frac{A}{m^2}$$

$$[\rho_{eh}'] = \frac{Vs}{m^3}$$

$$[\mathbf{j}_{eh}] = \frac{A}{ms}$$

$$[\mathbf{j}_{eh}'] = \frac{V}{m^2}$$

Field-Potential Relations I
Full Equation Set

$$\mathbf{E}^a = -\nabla\Phi^a - \frac{\partial \mathbf{A}^a}{\partial t} - \omega_0{}^a{}_b \mathbf{A}^b + \boldsymbol{\omega}^a{}_b \Phi^b$$

$$\mathbf{B}^a = \nabla \times \mathbf{A}^a - \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b$$

Potentials and Spin Connections

$\mathbf{A}^a$: Vector potential

Φ^a: scalar potential

$\boldsymbol{\omega}^a{}_b$: Vector spin connection

$\omega_0{}^a{}_b$: Scalar spin connection

Please observe the Einstein summation convention!

ECE Field Equations in Terms of Potential I

Gauss Law :

$$\nabla \cdot (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b) = 0$$

Faraday Law of Induction :

$$-\nabla \times (\omega_0{}^a{}_b \mathbf{A}^b) + \nabla \times (\boldsymbol{\omega}^a{}_b \Phi^b) - \frac{\partial (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b)}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}^a}{\partial t} - \Delta \Phi^a - \nabla \cdot (\omega_0{}^a{}_b \mathbf{A}^b) + \nabla \cdot (\boldsymbol{\omega}^a{}_b \Phi^b) = \frac{\rho_e{}^a}{\varepsilon_0}$$

Ampère-Maxwell Law :

$$\nabla(\nabla \cdot \mathbf{A}^a) - \Delta \mathbf{A}^a - \nabla \times (\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b)$$

$$+ \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{A}^a}{\partial t^2} + \frac{\partial (\omega_0{}^a{}_b \mathbf{A}^b)}{\partial t} + \nabla \frac{\partial \Phi^a}{\partial t} - \frac{\partial (\boldsymbol{\omega}^a{}_b \Phi^b)}{\partial t} \right) = \mu_0 \mathbf{J}_e{}^a$$

Antisymmetry Conditions of ECE Field Equations I

Electric antisymmetry constraints:

$$\nabla \Phi^a - \frac{\partial \mathbf{A}^a}{\partial t} - \omega_0{}^a{}_b \mathbf{A}^b - \boldsymbol{\omega}^a{}_b \Phi^b = 0$$

Magnetic antisymmetry constraints:

$$\frac{\partial A^a{}_3}{\partial x_2} + \frac{\partial A^a{}_2}{\partial x_3} + \omega^a{}_{b,2} A^b{}_3 + \omega^a{}_{b,3} A^b{}_2 = 0$$

$$\frac{\partial A^a{}_3}{\partial x_1} + \frac{\partial A^a{}_1}{\partial x_3} + \omega^a{}_{b,1} A^b{}_3 + \omega^a{}_{b,3} A^b{}_1 = 0$$

$$\frac{\partial A^a{}_2}{\partial x_1} + \frac{\partial A^a{}_1}{\partial x_2} + \omega^a{}_{b,1} A^b{}_2 + \omega^a{}_{b,2} A^b{}_1 = 0$$

Or simplified Lindstrom constraint (not exact):

$$\nabla \times \mathbf{A}^a + \boldsymbol{\omega}^a{}_b \times \mathbf{A}^b = 0$$

Field-Potential Relations II
One Polarization only

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} + \boldsymbol{\omega}\Phi$$

$$\mathbf{B} = \nabla \times \mathbf{A} - \boldsymbol{\omega} \times \mathbf{A}$$

Potentials and Spin Connections

$\mathbf{A}$: Vector potential

Φ: scalar potential

$\boldsymbol{\omega}$: Vector spin connection

ω_0: Scalar spin connection

ECE Field Equations in Terms of Potential II

Gauss Law :

$$\nabla \cdot (\boldsymbol{\omega} \times \mathbf{A}) = 0$$

Faraday Law of Induction :

$$-\nabla \times (\omega_0 \mathbf{A}) + \nabla \times (\boldsymbol{\omega} \Phi) - \frac{\partial (\boldsymbol{\omega} \times \mathbf{A})}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \Delta \Phi - \nabla \cdot (\omega_0 \mathbf{A}) + \nabla \cdot (\boldsymbol{\omega} \Phi) = \frac{\rho_e}{\varepsilon_0}$$

Ampère-Maxwell Law :

$$\nabla (\nabla \cdot \mathbf{A}) - \Delta \mathbf{A} - \nabla \times (\boldsymbol{\omega} \times \mathbf{A})$$

$$+ \frac{1}{c^2} \left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \frac{\partial (\omega_0 \mathbf{A})}{\partial t} + \nabla \frac{\partial \Phi}{\partial t} - \frac{\partial (\boldsymbol{\omega} \Phi)}{\partial t} \right) = \mu_0 \mathbf{J}_e$$

Antisymmetry Conditions of ECE Field Equations II

Electric antisymmetry constraints:

$$\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} - \omega_0 \mathbf{A} - \boldsymbol{\omega}\Phi = 0$$

Magnetic antisymmetry constraints:

$$\frac{\partial A_3}{\partial x_2} + \frac{\partial A_2}{\partial x_3} + \omega_2 A_3 + \omega_3 A_2 = 0$$

$$\frac{\partial A_3}{\partial x_1} + \frac{\partial A_1}{\partial x_3} + \omega_1 A_3 + \omega_3 A_1 = 0$$

$$\frac{\partial A_2}{\partial x_1} + \frac{\partial A_1}{\partial x_2} + \omega_1 A_2 + \omega_2 A_1 = 0$$

Or simplified
Lindstrom constraint (not exact):

$$\nabla \times \mathbf{A} + \boldsymbol{\omega} \times \mathbf{A} = 0$$

All these relations appear in addition to the ECE field equations and are constraints of them. They replace Lorenz Gauge invariance and can be used to derive special properties.

Relation between Potentials and Spin Connections derived from Antisymmetry Conditions

$$\omega_0 \mathbf{A} = \boldsymbol{\omega}\Phi = \frac{1}{2}(-\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi)$$

Thus spin connections can be calculated from the potentials:

$$\boldsymbol{\omega} = \frac{1}{2\Phi}(-\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi)$$

$$\omega_0 = \frac{\Phi}{A^2}\boldsymbol{\omega}\cdot\mathbf{A} = \frac{1}{2A^2}(-\frac{\partial \mathbf{A}}{\partial t} + \nabla\Phi)\cdot\mathbf{A}$$

Denominators have to be given attention:

$A \neq 0$

$\Phi \neq 0$

Alternative I: ECE Field Equations with Alternative Current Definitions (a)

$$\text{Standard ECE definition of currents (Maxwell-like):}$$

$$\partial_\mu \tilde{F}^{a\mu\nu} = A^{(0)}(\tilde{R}^a{}_\mu{}^{\mu\nu} - \omega^a{}_{\mu b}\tilde{T}^{b\mu\nu}) =: \mu_0 j^{a\nu}$$

$$\partial_\mu F^{a\mu\nu} = A^{(0)}(R^a{}_\mu{}^{\mu\nu} - \omega^a{}_{\mu b}T^{b\mu\nu}) =: \mu_0 J^{a\nu}$$

$$\text{Alternative definition (covariant derivative maintained):}$$

$$D_\mu \tilde{F}^{a\mu\nu} = \partial_\mu \tilde{F}^{a\mu\nu} + \omega^a{}_{\mu b}\tilde{F}^{b\mu\nu} = A^{(0)}\tilde{R}^a{}_\mu{}^{\mu\nu} =: \mu_0 j_A{}^{a\nu}$$

$$D_\mu F^{a\mu\nu} = \partial_\mu F^{a\mu\nu} + \omega^a{}_{\mu b}F^{b\mu\nu} = A^{(0)}R^a{}_\mu{}^{\mu\nu} =: \mu_0 J_A{}^{a\nu}$$

Alternative I: ECE Field Equations with Alternative Current Definitions (b)

$$\nabla \cdot \mathbf{B}^a = \mu_0 \rho_{Aeh}{}^a = \rho_{Aeh}{}^{a\prime} = 0$$ Gauss Law

$$\nabla \times \mathbf{E}^a + \frac{d\mathbf{B}^a}{dt} = \mu_0 \mathbf{j}_{Aeh}{}^a = \mathbf{j}_{Aeh}{}^{a\prime} = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E}^a = \frac{\rho_{Ae}{}^a}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B}^a - \frac{1}{c^2}\frac{d\mathbf{E}^a}{dt} = \mu_0 \mathbf{J}_{Ae}{}^a$$ Ampère-Maxwell Law

with

$$\frac{d}{dt} = \frac{\partial}{\partial t} + \mathbf{v} \cdot \nabla$$

v is relative velocity between observer and detector

(http://www.angelfire.com/sc3/elmag/files/phipps/phipps01.pdf)

Alternative II: ECE Field Equations with currents defined by curvature only

Coulomb Laws:

$$-\nabla\cdot\frac{\partial \mathbf{A}}{\partial t}-\Delta\Phi=\frac{\rho_{e0}}{\varepsilon_0}$$

$$-\nabla\cdot(\omega_0\mathbf{A})+\nabla\cdot(\boldsymbol{\omega}\Phi)=\frac{\rho_{e1}}{\varepsilon_0}$$

Ampère-Maxwell Laws:

$$\nabla(\nabla\cdot\mathbf{A})-\Delta\mathbf{A}+\frac{1}{c^2}\left(\frac{\partial^2\mathbf{A}}{\partial t^2}+\nabla\frac{\partial\Phi}{\partial t}\right)=\mu_0\mathbf{J}_{e0}$$

$$-\nabla\times(\boldsymbol{\omega}\times\mathbf{A})+\frac{1}{c^2}\left(\frac{\partial(\omega_0\mathbf{A})}{\partial t}-\frac{\partial(\boldsymbol{\omega}\Phi)}{\partial t}\right)=\mu_0\mathbf{J}_{e1}$$

ρ_{e0}, J_{e0}: normal charge density and current

ρ_{e1}, J_{e1}: “cold“ charge density and current

Field-Potential Relations III
Linearized Equations

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\omega}_E$$

$$\mathbf{B} = \nabla \times \mathbf{A} + \boldsymbol{\omega}_B$$

Potentials and Spin Connections

A: Vector potential
Φ: scalar potential
$\boldsymbol{\omega}_E$: Vector spin connection of electric field
$\boldsymbol{\omega}_B$: Vector spin connection of magnetic field

ECE Field Equations in Terms of Potential III

Gauss Law :

$$\nabla \cdot \boldsymbol{\omega}_B = 0$$

Faraday Law of Induction :

$$\nabla \times \boldsymbol{\omega}_E + \frac{\partial \boldsymbol{\omega}_B}{\partial t} = 0$$

Coulomb Law :

$$-\nabla \cdot \frac{\partial \mathbf{A}}{\partial t} - \Delta\Phi + \nabla \cdot \boldsymbol{\omega}_E = \frac{\rho_e}{\varepsilon_0}$$

Ampère-Maxwell Law :

$$\nabla(\nabla \cdot \mathbf{A}) - \Delta\mathbf{A} + \nabla \times \boldsymbol{\omega}_B$$

$$+ \frac{1}{c^2}\left(\frac{\partial^2 \mathbf{A}}{\partial t^2} + \nabla \frac{\partial \Phi}{\partial t} - \frac{\partial \boldsymbol{\omega}_E}{\partial t}\right) = \mu_0 \mathbf{J}_e$$

Antisymmetry Conditions of ECE Field Equations III

Define additional vectors
$\boldsymbol{\omega}_{E1}$, $\boldsymbol{\omega}_{E2}$, $\boldsymbol{\omega}_{B1}$, $\boldsymbol{\omega}_{B2}$:

$$\boldsymbol{\omega}_E = -(\boldsymbol{\omega}_{E1} - \boldsymbol{\omega}_{E2})$$
$$\boldsymbol{\omega}_B = -(\boldsymbol{\omega}_{B1} - \boldsymbol{\omega}_{B2})$$

Electric antisymmetry constraints:

$$\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} + \boldsymbol{\omega}_{E1} + \boldsymbol{\omega}_{E2} = 0$$

Magnetic antisymmetry constraints:

$$\begin{pmatrix} \frac{\partial A_3}{\partial x_2} + \frac{\partial A_2}{\partial x_3} \\ \frac{\partial A_1}{\partial x_3} + \frac{\partial A_3}{\partial x_1} \\ \frac{\partial A_2}{\partial x_1} + \frac{\partial A_1}{\partial x_2} \end{pmatrix} + \boldsymbol{\omega}_{B1} + \boldsymbol{\omega}_{B2} = 0$$

Curvature Vectors

Orbital curvature (electric field):

$$\mathbf{R}_E{}^a{}_b = \mathbf{R}^a{}_b(orbital) = \frac{1}{c}\left(-\nabla \omega_0{}^a{}_b - \frac{\partial \boldsymbol{\omega}^a{}_b}{\partial t} - \omega_0{}^a{}_c \boldsymbol{\omega}^c{}_b + \omega_0{}^c{}_b \boldsymbol{\omega}^a{}_c \right)$$

without polarisation:

$$\mathbf{R}_E = \mathbf{R}(orbital) = \frac{1}{c}\left(-\nabla \omega_0 - \frac{\partial \boldsymbol{\omega}}{\partial t} \right)$$

Spin curvature (magnetic field):

$$\mathbf{R}_B{}^a{}_b = \mathbf{R}^a{}_b(spin) = \nabla \times \boldsymbol{\omega}^a{}_b - \boldsymbol{\omega}^a{}_c \times \boldsymbol{\omega}^c{}_b$$

without polarisation:

$$\mathbf{R}_B = \mathbf{R}(spin) = \nabla \times \boldsymbol{\omega}$$

Units:

$$[\mathbf{R}_E{}^a{}_b] = [\mathbf{R}_B{}^a{}_b] = \frac{1}{m^2}$$

Geometrical Definition of Electric Charge/Current Densities

With polarization:

Charge density:

$$\rho_e{}^a = \varepsilon_0\left(\boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b - c\mathbf{A}^b \cdot \mathbf{R}_E{}^a{}_b\right)$$

Electric current:

$$\mathbf{J}_e{}^a = \varepsilon_0 \omega_0{}^a{}_b \mathbf{E}^b + \frac{1}{\mu_0}\left(\boldsymbol{\omega}^a{}_b \times \mathbf{B}^b - \frac{1}{c}\Phi^b \cdot \mathbf{R}_E{}^a{}_b - \mathbf{A}^b \times \mathbf{R}_B{}^a{}_b\right)$$

Without polarization:

Charge density:

$$\rho_e = \varepsilon_0\left(\boldsymbol{\omega} \cdot \mathbf{E} - c\mathbf{A} \cdot \mathbf{R}_E\right)$$

Electric current:

$$\mathbf{J}_e = \varepsilon_0 \omega_0 \mathbf{E} + \frac{1}{\mu_0}\left(\boldsymbol{\omega} \times \mathbf{B} - \frac{1}{c}\Phi \cdot \mathbf{R}_E - \mathbf{A} \times \mathbf{R}_B\right)$$

Geometrical Definition of Magnetic Charge/Current Densities

With polarization:

Homogeneous charge density:

$$\rho_{eh}{}^{a}{}' = \boldsymbol{\omega}^{a}{}_{b} \cdot \mathbf{B}^{b} - \mathbf{A}^{b} \cdot \mathbf{R}_{B}{}^{a}{}_{b}$$

Homogeneous current:

$$\mathbf{J}_{eh}{}^{a}{}' = -\omega_{0}{}^{a}{}_{b}\mathbf{B}^{b} - \boldsymbol{\omega}^{a}{}_{b} \times \mathbf{E}^{b} + \Phi^{b} \cdot \mathbf{R}_{B}{}^{a}{}_{b} + c\mathbf{A}^{b} \times \mathbf{R}_{E}{}^{a}{}_{b}$$

Without polarization:

Homogeneous charge density:

$$\rho_{eh}' = \boldsymbol{\omega} \cdot \mathbf{B} - \mathbf{A} \cdot \mathbf{R}_{B}$$

Homogeneous current:

$$\mathbf{J}_{eh}' = -\omega_{0}\mathbf{B} - \boldsymbol{\omega} \times \mathbf{E} + \Phi \cdot \mathbf{R}_{B} + c\mathbf{A} \times \mathbf{R}_{E}$$

Additional Field Equations due to Vanishing Homogeneous Currents

With polarization:

$$\boldsymbol{\omega}^a{}_b \cdot \mathbf{B}^b = \mathbf{A}^b \cdot \mathbf{R}_B{}^a{}_b$$

$$\boldsymbol{\omega}^a{}_b \times \mathbf{E}^b - \omega_0{}^a{}_b \mathbf{B}^b = -\Phi^b \cdot \mathbf{R}_B{}^a{}_b + c\mathbf{A}^b \times \mathbf{R}_E{}^a{}_b$$

$$\nabla \cdot \left(\boldsymbol{\omega}^a{}_b \times \mathbf{A}^b \right) = 0$$

Without polarization:

$$\boldsymbol{\omega} \cdot \mathbf{B} = \mathbf{A} \cdot \mathbf{R}_B$$

$$\boldsymbol{\omega} \times \mathbf{E} - \omega_0 \mathbf{B} = -\Phi \cdot \mathbf{R}_B + c\mathbf{A} \times \mathbf{R}_E$$

$$\nabla \cdot \left(\boldsymbol{\omega} \times \mathbf{A} \right) = 0$$

Resonance Equation of Scalar Torsion Field

With polarization:

$$\frac{\partial T^{a0}}{\partial t} + \omega_0{}^a{}_b T^{b0} = cR^a$$

Without polarization:

$$\frac{\partial T^0}{\partial t} + \omega_0 T^0 = cR$$

Physical units:

$$[T^0] = \frac{1}{m}$$

$$[R] = \frac{1}{m^2}$$

Axioms of ECE2

- Alternative, curvature-based definitions
  - Compatible to torsion-based axioms
- 4-potential *A* is proportional to Cartan tetrad *q*:
  $A^a = A^{(0)} q^a$
- Electromagnetic/gravitational field is proportional to torsion and curvature 2-forms:
  $F^a = A^{(0)} T^a$, $F^a{}_b = W^{(0)} R^a{}_b$
- *a, b*: indices of tangent space, can be removed
- $A^{(0)}$, $W^{(0)}$: constants with physical dimensions,
  $[A^{(0)}]$=T*m=V*s/m, $[W^{(0)}]$=V*s

Electromagnetic Fields of ECE2

Orbital curvature (electric field) :

$$\mathbf{E}^a{}_b = cW^{(0)}\mathbf{R}_E{}^a{}_b = cW^{(0)}\mathbf{R}^a{}_b(orbital)$$

with polarisation removed :

$$\mathbf{E} = cW^{(0)}\mathbf{R}_E = cW^{(0)}\mathbf{R}(orbital)$$

Spin curvature (magnetic field) :

$$\mathbf{B}^a{}_b = W^{(0)}\mathbf{R}_B{}^a{}_b = W^{(0)}\mathbf{R}^a{}_b(spin)$$

with polarisation removed :

$$\mathbf{B} = W^{(0)}\mathbf{R}_B = W^{(0)}\mathbf{R}(spin)$$

Curvature vectors are defined as in slide 24.
Charge/current densities are defined as in slides 25/26.

Geometrical Definition of Electric Charge/Current Densities in ECE2

With polarization:

Charge density:

$$\rho_e{}^a = \varepsilon_0 \left(\boldsymbol{\omega}^a{}_b \cdot \mathbf{E}^b - \frac{1}{W^{(0)}} \mathbf{A}^b \cdot \mathbf{E}^a{}_b \right)$$

Electric current:

$$\mathbf{J}_e{}^a = \varepsilon_0 \omega_0{}^a{}_b \mathbf{E}^b + \frac{1}{\mu_0} \left(\boldsymbol{\omega}^a{}_b \times \mathbf{B}^b - \frac{1}{c^2 W^{(0)}} \Phi^b \mathbf{E}^a{}_b - \frac{1}{W^{(0)}} \mathbf{A}^b \times \mathbf{B}^a{}_b \right)$$

Without polarization:

Charge density:

$$\rho_e = 2\varepsilon_0 \left(\frac{1}{W^{(0)}} \mathbf{A} - \boldsymbol{\omega} \right) \cdot \mathbf{E}$$

Electric current:

$$\mathbf{J}_e = 2 \left[-\varepsilon_0 \omega_0 \mathbf{E} + \frac{1}{\mu_0} \left(\frac{1}{c^2 W^{(0)}} \Phi \mathbf{E} + \left(\frac{1}{W^{(0)}} \mathbf{A} - \boldsymbol{\omega} \right) \times \mathbf{B} \right) \right]$$

Geometrical Definition of Magnetic Charge/Current Densities in ECE2

With polarization:

Homogeneous charge density:

$$\rho_{eh}{}^{a}{}' = \boldsymbol{\omega}^{a}{}_{b} \cdot \mathbf{B}^{b} - \frac{1}{W^{(0)}} \mathbf{A}^{b} \cdot \mathbf{B}^{a}{}_{b}$$

Homogeneous current:

$$\mathbf{J}_{eh}{}^{a}{}' = -\omega_{0}{}^{a}{}_{b} \mathbf{B}^{b} + \frac{1}{W^{(0)}} \Phi^{b} \mathbf{B}^{a}{}_{b} - \boldsymbol{\omega}^{a}{}_{b} \times \mathbf{E}^{b} + \frac{1}{W^{(0)}} \mathbf{A}^{b} \times \mathbf{E}^{a}{}_{b}$$

Without polarization:

Homogeneous charge density:

$$\rho_{eh}{}' = 2\left(\frac{1}{W^{(0)}} \mathbf{A} - \boldsymbol{\omega} \right) \cdot \mathbf{B}$$

Homogeneous current:

$$\mathbf{J}_{eh}{}' = 2\left[\left(\omega_{0} - \frac{1}{W^{(0)}} \Phi \right) \mathbf{B} + \left(\boldsymbol{\omega} - \frac{1}{W^{(0)}} \mathbf{A} \right) \times \mathbf{E} \right]$$

ECE2 Field Equations – Vector Form

$$\nabla \cdot \mathbf{B}^a = \mu_0 \rho_{eh}{}^a = \rho_{eh}{}^a{}' = 0$$ Gauss Law

$$\nabla \times \mathbf{E}^a + \frac{\partial \mathbf{B}^a}{\partial t} = \mu_0 \mathbf{j}_{eh}{}^a = \mathbf{j}_{eh}{}^a{}' = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E}^a = \frac{\rho_e{}^a}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B}^a - \frac{1}{c^2}\frac{\partial \mathbf{E}^a}{\partial t} = \mu_0 \mathbf{J}_e{}^a$$ Ampère-Maxwell Law

Currents as defined in preceding slides

ECE2 Field Equations – Vector Form with Wave Vectors

$$\nabla \cdot \mathbf{B} = \boldsymbol{\kappa} \cdot \mathbf{B} = \rho_{eh}' = 0$$ Gauss Law

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = -\left(c\kappa_0 \mathbf{B} + \boldsymbol{\kappa} \times \mathbf{E}\right) = \mathbf{j}_{eh}' = 0$$ Faraday Law of Induction

$$\nabla \cdot \mathbf{E} = \boldsymbol{\kappa} \cdot \mathbf{E} = \frac{\rho_e}{\varepsilon_0}$$ Coulomb Law

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = \frac{\kappa_0}{c}\mathbf{E} + \boldsymbol{\kappa} \times \mathbf{B} = \mu_0 \mathbf{J}_e$$ Ampère-Maxwell Law

with

$$\kappa_0 = \frac{2}{c}\left(\frac{1}{W^{(0)}}\Phi - \omega_0\right)$$

$$\boldsymbol{\kappa} = 2\left(\frac{1}{W^{(0)}}\mathbf{A} - \boldsymbol{\omega}\right)$$

Field Equations without Magnetic Currents

$\nabla \cdot \mathbf{B} = 0$ Gauss Law

$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$ Faraday Law of Induction

$\nabla \cdot \mathbf{E} = \boldsymbol{\kappa} \cdot \mathbf{E}$ Coulomb Law

$\nabla \times \mathbf{B} - \frac{1}{c^2} \frac{\partial \mathbf{E}}{\partial t} = \boldsymbol{\kappa} \times \mathbf{B}$ Ampère-Maxwell Law

with

$$\boldsymbol{\kappa} = 2\left(\frac{1}{W^{(0)}} \mathbf{A} - \boldsymbol{\omega} \right)$$

$$\boldsymbol{\kappa} \perp \mathbf{B}, \quad \boldsymbol{\kappa} \parallel \mathbf{E}, \quad \kappa_0 = 0$$

ECE2 Fields in Terms of Potentials

$$\mathbf{E} = -\nabla\Phi - \frac{\partial \mathbf{A}}{\partial t} + 2(\omega_0 \mathbf{A} - \Phi \boldsymbol{\omega})$$

$$\mathbf{B} = \nabla \times \mathbf{A} + 2\boldsymbol{\omega} \times \mathbf{A}$$

Maxwell form with W potentials:

$$\mathbf{E} = -\nabla\Phi_W - \frac{\partial \mathbf{W}}{\partial t}$$

$$\mathbf{B} = \nabla \times \mathbf{W}$$

with

$$\Phi_W = W^{(0)}\omega_0 = cW_0$$

$$\mathbf{W} = W^{(0)}\boldsymbol{\omega}$$

Equations of the Free Electromagnetic Field/Photon

Field equations: $\nabla \cdot \mathbf{B} = 0$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = 0$$

Spin equations: $\boldsymbol{\omega} \cdot \mathbf{B} = 0$

$$\boldsymbol{\omega} \times \mathbf{E} - \omega_0 \mathbf{B} = 0$$

$$\boldsymbol{\omega} \cdot \mathbf{E} = 0$$

$$\boldsymbol{\omega} \times \mathbf{B} + \frac{1}{c^2}\omega_0 \mathbf{E} = 0$$

$$\omega_0 = c\kappa$$

$\kappa = \text{wave number}$

$$\boldsymbol{\omega} = \boldsymbol{\kappa}$$

$\boldsymbol{\kappa} = \text{wave vector}$

$$\mathbf{p} = \hbar\boldsymbol{\kappa} = \hbar\boldsymbol{\omega}$$

$\mathbf{p} = \text{momentum}$

$$\mathrm{E} = \hbar\omega = \hbar\omega_0$$

$E = \text{energy}$

$\omega = \text{time frequency}$

Beltrami Solutions of the Free Electromagnetic Field

Field equations:

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{E} = 0$$

$$\nabla \times \mathbf{B} - \frac{1}{c^2}\frac{\partial \mathbf{E}}{\partial t} = 0$$

Beltrami equations:

$$\nabla \times \mathbf{B} = \kappa \mathbf{B}$$

$$\nabla \times \mathbf{E} = \kappa \mathbf{E}$$

$$\nabla \times \mathbf{A} = \kappa \mathbf{A}$$

$$\nabla \times \boldsymbol{\omega} = \kappa \boldsymbol{\omega}$$

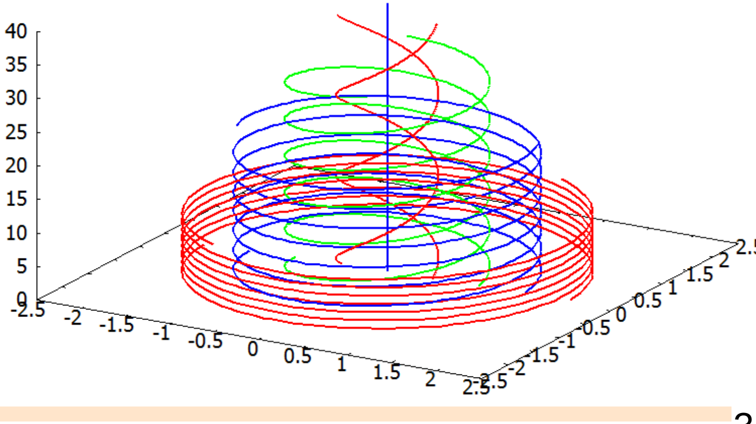
$$\nabla \times \mathbf{J} = \kappa \mathbf{J}$$

Boundary conditions for quasi-static free field:

$$\mathbf{B} = \frac{\mu_0}{\kappa^2}\nabla \times \mathbf{J} = \frac{\mu_0}{\kappa}\mathbf{J}$$

wave number:

$$\kappa = \frac{\omega}{c} = \frac{2\pi f}{c}$$



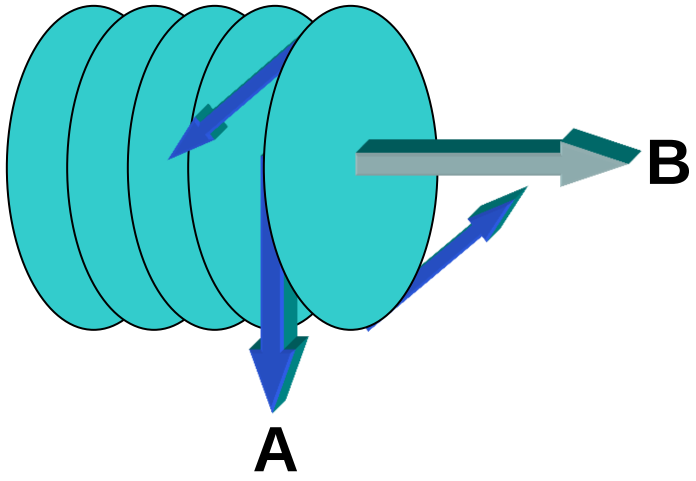
Properties of ECE Equations

- The ECE equations in potential representation define a well-defined equation system (8 equations with 8 unknows), can be reduced by antisymmetry conditions and additional constraints
- There is much more structure in ECE than in standard theory (Maxwell-Heaviside)
- There is no gauge freedom in ECE theory
- In representation by the potential, the Gauss and Faraday law do not make sense in standard theory (see red fields)
- Resonance structures (self-enforcing oscillations) are possible in Coulomb and Ampère-Maxwell law

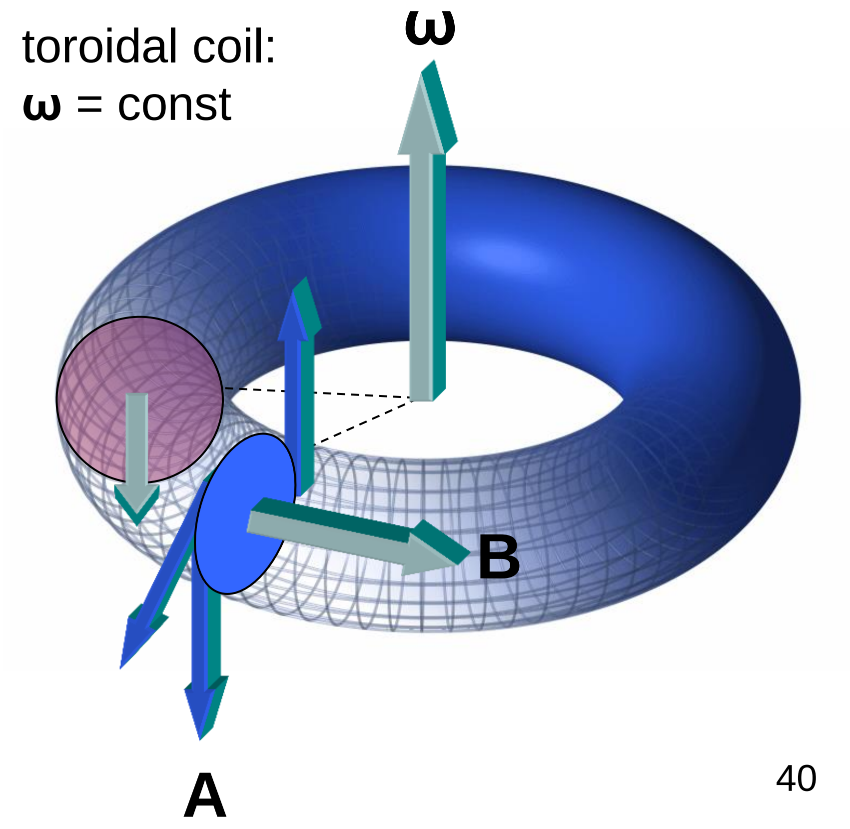
Examples of Vector Spin Connection

Vector spin connection **ω** represents rotation of plane of **A** potential

linear coil:
ω = 0



toroidal coil:
ω = const



ECE Field Equations of Dynamics

$$\nabla \cdot \mathbf{h} = 4\pi G \rho_{mh} = 0$$ (Equivalent of Gauss Law)

$$\nabla \times \mathbf{g} + \frac{1}{c}\frac{\partial \mathbf{h}}{\partial t} = \frac{4\pi \mathrm{G}}{\mathrm{c}} \mathbf{j}_{mh} = 0$$ Gravito-magnetic Law

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m$$ Newton's Law (Poisson equation)

$$\nabla \times \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi \mathrm{G}}{\mathrm{c}} \mathbf{J}_m$$ (Equivalent of Ampère-Maxwell Law)

Only Newton‘s Law is known in the standard model.

ECE Field Equations of Dynamics Alternative Form with $\mathbf{\Omega}$

$$\nabla \cdot \mathbf{\Omega} = \frac{4\pi \mathrm{G}}{\mathrm{c}} \rho_{mh} = 0$$ (Equivalent of Gauss Law)

$$\nabla \times \mathbf{g} + \frac{\partial \mathbf{\Omega}}{\partial t} = \frac{4\pi \mathrm{G}}{\mathrm{c}} \mathbf{j}_{mh} = 0$$ Gravito-magnetic Law

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m$$ Newton's Law (Poisson equation)

$$\nabla \times \mathbf{\Omega} - \frac{1}{c^2} \frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi \mathrm{G}}{\mathrm{c}^2} \mathbf{J}_m$$ (Equivalent of Ampère-Maxwell Law)

Alternative gravito-magnetic field: $\mathbf{\Omega} = \frac{\mathbf{h}}{c}$

Only Newton's Law is known in the standard model.

Fields, Currents and Constants

Fields and Currents

g: gravity acceleration

ρ_m: mass density

J_m: mass current

Ω, **h**: gravito-magnetic field

ρ_{mh}: gravito-magn. mass density

j_{mh}: gravito-magn. mass current

Constants

G: Newton's gravitational constant

c: vacuum speed of light, required for correct physical units

Force Equations

$$\mathbf{F} = m\mathbf{g}$$ Newtonian Force Law

$$\mathbf{F} = E_0\mathbf{T}$$ Torsional Force Law

$$\mathbf{F}_{\mathrm{L}} = mc\mathbf{v}\times\mathbf{h}$$ Lorentz Force Law

$$\mathbf{M} = \frac{\partial \mathbf{L}}{\partial t} - \mathbf{\Theta}\times\mathbf{L}$$ Torque Law

Physical quantities and units

F [N]	Force
M [Nm]	Torque
T [1/m]	Torsion
g, h [m/s^2]	Acceleration
m [kg]	Mass
v [m/s]	Mass velocity
$E_0=mc^2$ [J]	Rest energy
$\mathbf{\Theta}$ [1/s]	Rotation axis vector
L [Nms]	Angular momentum

Field-Potential Relations

$$\mathbf{g} = -\frac{\partial \mathbf{Q}}{\partial t} - \nabla \Phi - \omega_0 \mathbf{Q} + \boldsymbol{\omega} \Phi$$

$$\boldsymbol{\Omega} = \frac{\mathbf{h}}{c} = \nabla \times \mathbf{Q} - \boldsymbol{\omega} \times \mathbf{Q}$$

Potentials and Spin Connections

$\mathbf{Q}=c\mathbf{q}$: Vector potential

Φ: Scalar potential

$\boldsymbol{\omega}$: Vector spin connection

ω_0: Scalar spin connection

Physical Units

Fields

$$[\mathbf{g}] = [\mathbf{h}] = \frac{m}{s^2}$$

$$[\mathbf{\Omega}] = \frac{1}{s}$$

Potentials

$$[\Phi] = \frac{m^2}{s^2}$$

$$[\mathbf{Q}] = \frac{m}{s}$$

Spin Connections

$$[\omega_0] = \frac{1}{s}$$

$$[\boldsymbol{\omega}] = \frac{1}{m}$$

Constants

$$[G] = \frac{m^3}{kg\, s^2}$$

Mass Density/Current

$$[\rho_m] = \frac{kg}{m^3}$$

$$[J_m] = \frac{kg}{m^2 s}$$

„Gravito-magnetic“ Density/Current

$$[\rho_{mh}] = \frac{kg}{m^3}$$

$$[j_m] = \frac{kg}{m^2 s}$$

Antisymmetry Conditions of ECE Field Equations of Dynamics

Relations for

classical and ECE Potenitals:

$$\nabla \Phi = \frac{\partial \mathbf{Q}}{\partial t}$$

$$\frac{\partial Q_1}{\partial x_2} = -\frac{\partial Q_2}{\partial x_1}$$

$$\frac{\partial Q_1}{\partial x_3} = -\frac{\partial Q_3}{\partial x_1}$$

$$\frac{\partial Q_2}{\partial x_3} = -\frac{\partial Q_3}{\partial x_2}$$

Relations for

spin connections:

$$\omega_0 \mathbf{Q} = -\boldsymbol{\omega} \Phi$$

$$\omega_1 Q_2 = -\omega_2 Q_1$$

$$\omega_1 Q_3 = -\omega_3 Q_1$$

$$\omega_2 Q_3 = -\omega_3 Q_2$$

ECE2 Field Equations of Dynamics

$$\nabla \cdot \mathbf{\Omega} = \boldsymbol{\kappa} \cdot \mathbf{\Omega} = \frac{4\pi \mathrm{G}}{\mathrm{c}} \rho_{mh} = 0 \qquad \text{(Gauss Law)}$$

$$\nabla \times \mathbf{g} + \frac{\partial \mathbf{\Omega}}{\partial t} = -\left(c\kappa_0 \mathbf{\Omega} + \boldsymbol{\kappa} \times \mathbf{g}\right) = \frac{4\pi \mathrm{G}}{\mathrm{c}} \mathbf{j}_{mh} = 0 \qquad \text{(Gravito-magnetic Law)}$$

$$\nabla \cdot \mathbf{g} = \boldsymbol{\kappa} \cdot \mathbf{g} = 4\pi G \rho_m \qquad \text{Newton's Law}$$

$$\nabla \times \mathbf{\Omega} - \frac{1}{c^2} \frac{\partial \mathbf{g}}{\partial t} = \frac{\kappa_0}{c} \mathbf{g} + \boldsymbol{\kappa} \times \mathbf{\Omega} = \frac{4\pi \mathrm{G}}{\mathrm{c}^2} \mathbf{J}_m \qquad \text{(Ampère-Maxwell Law)}$$

Potentials:

$$\mathbf{g} = -\nabla \Phi - \frac{\partial \mathbf{Q}}{\partial t} + 2\left(\omega_0 \mathbf{Q} - \Phi \boldsymbol{\omega}\right)$$

$$\mathbf{\Omega} = \nabla \times \mathbf{Q} + 2\boldsymbol{\omega} \times \mathbf{Q}$$

Wave numbers:

$$\kappa_0 = \frac{2}{c}\left(\frac{A^{(0)}}{W^{(0)}c}\Phi - \omega_0\right)$$

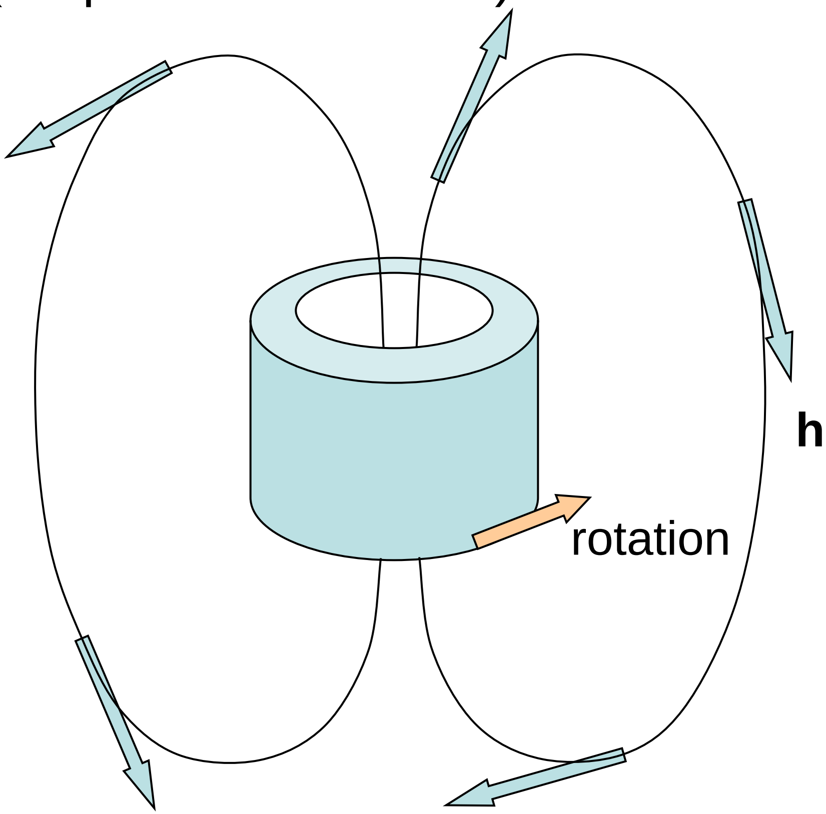
$$\boldsymbol{\kappa} = 2\left(\frac{A^{(0)}}{W^{(0)}c}\mathbf{Q} - \boldsymbol{\omega}\right)$$

Properties of ECE Equations of Dynamics

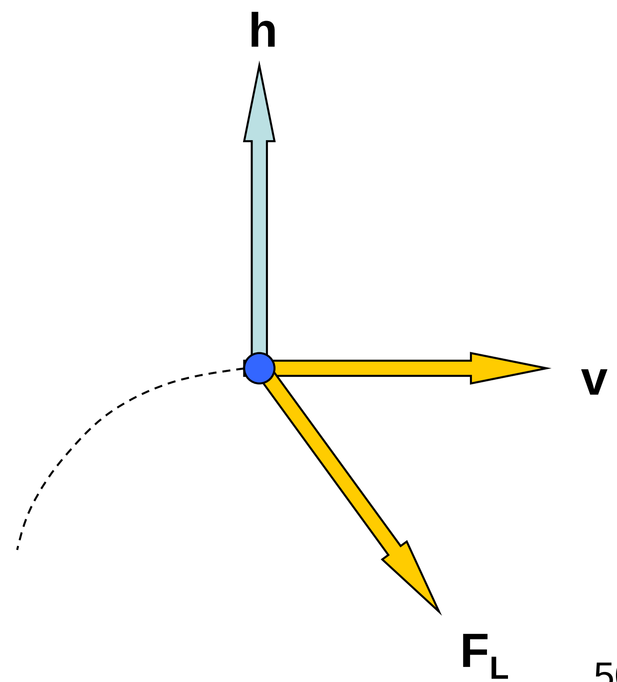
- Fully analogous to electrodynamic case
- Only the Newton law is known in classical mechanics
- Gravito-magnetic law is known experimentally (ESA experiment)
- There are two acceleration fields **g** and **h**, but only **g** is known today
- **h** is an angular momentum field and measured in m/s^2 (units chosen the same as for **g**)
- Mechanical spin connection resonance is possible as in electromagnetic case
- Gravito-magnetic current occurs only in case of coupling between translational and rotational motion

Examples of ECE Dynamics

Realisation of gravito-magnetic field **h** by a rotating mass cylinder (Ampere-Maxwell law)



Detection of **h** field by mechanical Lorentz force $\mathbf{F}_L$
v: velocity of mass m



Polarization and Magnetization

Electromagnetism

P: Polarization
M: Magnetization

$$D = \varepsilon_0 E + P$$

$$[P] = \frac{C}{m^2}$$

$$B = \mu_0 (H + M)$$

$$[M] = \frac{A}{m}$$

Dynamics

p_m: mass polarization
m_m: mass magnetization

$$g = g_0 + p_m$$

$$[p_m] = \frac{m}{s^2}$$

$$h = h_0 + m_m$$

$$[m_m] = \frac{m}{s^2}$$

Note: The definitions of p_m and m_m, compared to g and h, differ from the electrodynamic analogue concerning constants and units.

Field Equations for Polarizable/Magnetizable Matter

Electromagnetism

D: electric displacement
H: (pure) magnetic field

$$\nabla \cdot \mathbf{B} = 0$$

$$\nabla \times \mathbf{E} + \frac{\partial \mathbf{B}}{\partial t} = 0$$

$$\nabla \cdot \mathbf{D} = \rho_e$$

$$\nabla \times \mathbf{H} - \frac{\partial \mathbf{D}}{\partial t} = \mathbf{J}_e$$

Dynamics

g: mechanical displacement
h_0: (pure) gravito-magnetic field

$$\nabla \cdot \mathbf{h}_0 = 0$$

$$\nabla \times \mathbf{g}_0 + \frac{1}{c}\frac{\partial \mathbf{h}_0}{\partial t} = 0$$

$$\nabla \cdot \mathbf{g} = 4\pi G \rho_m$$

$$\nabla \times \mathbf{h} - \frac{1}{c}\frac{\partial \mathbf{g}}{\partial t} = \frac{4\pi \mathrm{G}}{c}\mathbf{J}_m$$

ECE Field Equations of Dynamics in Momentum Representation

$$\nabla \cdot \mathbf{S} = \frac{1}{2} c V \rho_{hm} = 0$$ (Equivalent of Gauss Law)

$$\nabla \times \mathbf{L} + \frac{1}{c}\frac{\partial \mathbf{S}}{\partial t} = \frac{1}{2} V \mathbf{j}_m = 0$$ Gravito-magnetic Law

$$\nabla \cdot \mathbf{L} = \frac{1}{2} c V \rho_m = \frac{1}{2} mc$$ Newton's Law (Poisson equation)

$$\nabla \times \mathbf{S} - \frac{1}{c}\frac{\partial \mathbf{L}}{\partial t} = \frac{1}{2} V \mathbf{J}_m = \frac{1}{2}\mathbf{p}$$ (Equivalent of Ampère-Maxwell Law)

None of these Laws is known in the standard model.

Physical Units

Fields and Currents

L: orbital angular momentum **S**: spin angular momentum

p: linear momentum

ρ_m: mass density ρ_{mh}: gravito-magn. mass density

J_m: mass current j_{mh}: gravito-magn. mass current

V: volume of space [m^3] m: mass=integral of mass density

Fields

$$[\mathbf{L}] = [\mathbf{S}] = \frac{kg \cdot m^2}{s}$$

$$[\mathbf{p}] = \frac{kg \cdot m}{s}$$

Mass Density/Current

$$[\rho_m] = \frac{kg}{m^3}$$

$$[J_m] = \frac{kg}{m^2 s}$$

„Gravito-magnetic“ Density/Current

$$[\rho_{mh}] = \frac{kg}{m^3}$$

$$[j_m] = \frac{kg}{m^2 s}$$