Physical Theory of the Levitron

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1.0 Introduction

The Levitron is an anti-gravity device. It consists of a magnetized top, and a base magnet. The top will levitate above the base, when the top is spinning at a proper rpm. This levitation property has defied quantitatively accurate description. In [1], Berry focused on mechanical model principles, which were dependant on the structure of the magnetic field of the base. In [2], the Levitron dynamics is used to demonstrate aspects of the new ECE-Theory. A general quantitative description of Levitron dynamics is the focus of this paper. The requirement for the top to spin was not rigorously explained in [1], or by conventional physics methods. This is addressed in [2], by the use of ECE-Theory. However, a proper numerical description of the Levitron will quantify the dynamics, and also analytically address the spin requirement.

From [2], we have the illustration of a generic Levitron device, Fig. 1. The top (s) has an attached ring magnet (M_1), and a magnetic base (M_2). The spinning top (i.e. the rotating M_1) can float stably above M_2, the magnetic base. Items M_1 (i.e. M_L) and M_2 (i.e. M_B) can be magnetic devices other than permanent magnets. Figure 2 shows the precession of the top, as its spin degrades.

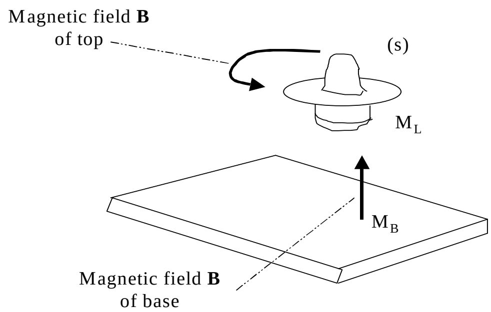

Fig. 1 Generic LEVITRON

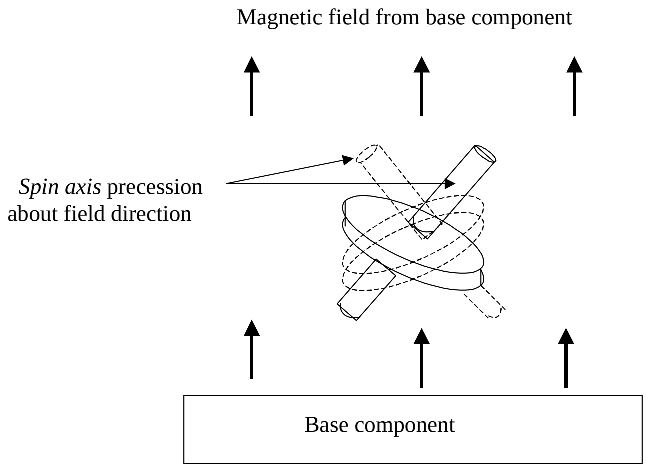

Fig. 2 Precession of Top

The numerical calculations, in this paper, show the Levitron can be explained in a straightforward manner. The dynamics of the Levitron demonstrate physical principles (such as levitation) which can be applied to areas of engineering, such as vehicular transport. The methods presented in this paper should aid in this, and related areas.

2.0 Physical principles behind the Levitron

Simple analytical calculation

The Levitron or spinning top is a small rotating magnetic dipole which is held at its position by the magnetic field of the base component. Dipole field and base field push off one another, both fields have the same magnetic polarity working against each other. The top stays at that position where its gravity force is counteracted exactly by the magnetic repelling force.

The physical description of the Levitron starts with definition of two magnetic moments: $\boldsymbol{\mu}_1$ is the moment of the spinning top and $\boldsymbol{\mu}_2$ is the moment of the base. The magnetic moment is defined as the integral over the magnetization **M** which is a magnetic moment density:

$$\boldsymbol{\mu} = \int \mathbf{M}(\mathbf{x}) d^3x. \tag{1}$$

This approach is valid if there is no feedback effect on the magnetization by other magnetic effects, i.e. the magnets are built from a “hard” magnetic material. This prerequisite is normally fulfilled for the Levitron.

Next the behaviour of a magnetic dipole (with mass) in an externally given magnetic field of the Levitron base has to be described. The energy U of a dipole $\boldsymbol{\mu}$ in a magnetic field **B** is

$$U = -\boldsymbol{\mu} \cdot \mathbf{B}. \tag{2}$$

Since the Levitron base is an (extended) dipole too, we are dealing with dipoles only. The simplest way to describe the field of the base is by a magnetic scalar potential Φ_M, see [3]. If we assume a homogeneous magnetization in the base magnet, Φ_M depends on a surface integral only. In analogy to electrostatics, the magnetic field lines start perpendicular from the the suface of the magnet. Therefore we assume a constant value of $\Phi_\mathrm{M}(\mathrm{A})$ at the surface. We need not further consider the interior of the magnet and obtain the potential in the magnetization-free region as the solution of the Laplace equation:

$$\nabla^2\Phi_{\mathrm{M}} = 0. \tag{3}$$

The magnetic field then simply is

$$\mathbf{B} = -\mu_0\boldsymbol{\nabla}\Phi_{\mathrm{M}}, \tag{4}$$

again in analogy to electrostatics. In cases where the dimensions of the base magnet can be neglected, it can be considered as a magnetic dipole like the spinning top. Denoting the magnetic moment of the base by $\boldsymbol{\mu}_2$, the scalar potential takes the simple form

$$\Phi_{\mathrm{M}}(\mathbf{x}) = \frac{\boldsymbol{\mu}_2 \cdot \mathbf{x}}{4\pi r^3} \tag{5}$$

where r is the absolute value of the coordinate vector **x** measured from the dipole center. The magnetic dipole field of this potential is

$$\mathbf{B}(\mathbf{x}) = \frac{\mu_0}{4\pi}\left(\frac{3\mathbf{n}(\mathbf{n}\cdot\boldsymbol{\mu}_2)-\boldsymbol{\mu}_2}{|\mathbf{x}|^3} + \frac{8\pi}{3}\boldsymbol{\mu}_2\delta(\mathbf{x})\right) \tag{6}$$

where **n** is a unit vector in direction of **x** and $\delta(\mathbf{x})$ is the Dirac delta function.

The potential energy of the top is

$$U = -\boldsymbol{\mu}_1 \cdot \mathbf{B} \tag{7}$$

with $\boldsymbol{\mu}_1$ being its magnetic dipole moment. So far we have dealt with magnetic properties only. In order to describe the dynamics and equilibrium conditions of the top, we have to apply mechanics. With the top having mass *m*, the equation of motion is

$$m\frac{\partial^2\mathbf{x}}{\partial t^2} = -\boldsymbol{\nabla}U - m\mathbf{g} = \boldsymbol{\nabla}\left(\boldsymbol{\mu}_1 \cdot \mathbf{B}\right) - m\mathbf{g}. \tag{8}$$

We assume that $\boldsymbol{\mu}_1$ is not precessing, so is parallel to the earth' gravitational force and keeps its direction during its path of motion, i.e. movement is in vertical direction only. Mechanical rotation (angular momentum) is only required to maintain the vertical alignment. The top is in equilibrium when

$$m\mathbf{g} = \boldsymbol{\nabla}(\mu_1 \cdot B) \tag{9}$$

where **g** is the gravitational acceleration. Assuming that **g** is parallel to the z axis of the coordinate system, the last equation reduces to one dimension, giving with Eq. (6):

$$mg = \frac{\partial}{\partial z}(\mu_1 \cdot B_z) = \mu_1\frac{\partial}{\partial z}\left(\frac{\mu_0}{4\pi}\cdot\frac{2\mu_2}{z^3}\right) = -\frac{3\mu_0\mu_1\mu_2}{2\pi z^4}\,. \tag{10}$$

The z value of equilibrium then is

$$z_0 = \sqrt[4]{-\frac{3\mu_0\mu_1\mu_2}{2\pi gm}}\,. \tag{11}$$

The magnetic moments of top and base have to have opposed signs so that the expression in the quartic root is positive. This simply means that both moments have to repel each other. In Fig. 3 some examples for the dependence of z_0 on $\boldsymbol{\mu}_1$ are shown (all constants set to unity). For a heavier mass *m*, the top sinks down in equilibrium position. A larger dipole moment lifts it up as expected.

A more realistic calculation

A more elaborate calculation beyond the simple dipole approximation requires a solution for the magnetic vector potential **A** from which the magnetic field can be derived by

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}. \tag{12}$$

Such a solution obtained by a Finite Element program is shown in Figs. 4 and 5. The repulsive nature of the fields can be seen nicely. This calculation was performed for a fixed position of the top. In order to calculate the equilibrium position the force field has to be considered. This is derived from the Maxell stress tensor [4] and can be transformed to a surface integral. The force distribution is [5]

$$\mathbf{f_s} = \frac{1}{2}(\mu_2 - \mu_1)\left(H_{t1}^2 + \frac{\mu_1}{\mu_2} H_{n1}^2\right)\mathbf{n} \tag{13}$$

where the index 2 refers to the object and the index 1 to the surrounding space. Please note that μ_1 and μ_2 here represent the magnetic permeabilities. The field quantities involved are the tangential and normal components of the **H** field outside the object, and **n** is the normal vector of the surface in outside direction. For practical purposes it is easier to refer to the field inside the object which leads to the force density [5]

$$\mathbf{f_s} = \frac{1}{2}(\mu_2 - \mu_1)\left(H_{t2}^2 + \frac{\mu_2}{\mu_1} H_{n2}^2\right)\mathbf{n}. \tag{14}$$

Determining the equilibrium position would require more elaborate Finite Element calculations. As long as there is no precession of the top, its motion can be described by the center of mass and the downward motion can be simulated by a simple time discretization of the equation

$$m\frac{d^2z}{dt^2} = m\mathrm{g} + \mathrm{f_z}\,. \tag{15}$$

Next we consider a spinning top with a non-vertical spin axis. In the mass point approximation, the top experiences a torque of strength

$$\mathbf{N} = \boldsymbol{\mu}_1 \times \mathbf{B}. \tag{16}$$

This is zero if $\boldsymbol{\mu}_1$ is parallel to **B** as was assumed before. **N** effects a precession of $\boldsymbol{\mu}_1$ around the direction of **B**. This leads to the known unstable behavior of the top. The precession effect can be calculated by classical mechanics. In the coordinate system of the rotating top the total mechanical torque is

$$\mathbf{N}_{tot} = \mathbf{N}_{Newton} - \boldsymbol{\Omega} \times \mathbf{L} = \frac{\mathrm{d}\mathbf{L}}{\mathrm{dt}} + m\mathbf{x} \times \mathbf{g} - \boldsymbol{\Omega} \times \mathbf{L} \tag{17}$$

where we have mechanical quantities of rotation axis $\boldsymbol{\Omega}$ and angular momentum **L**, and **x** is the distance vector from the center of rotation to the rotating mass point. The term $\boldsymbol{\Omega} \times \mathbf{L}$ is the Euler contribution due to rotation which is not contained in Newton's equations of motion. The latter are valid only for non-accelerated frames. Eq. (17) has to be translated into magnetic quantities to compute the behaviour of the spinning top. A magnetic dipole can be substituted by a ring current giving the same magnetic field as the dipole. As derived in [3], there is the equivalence

$$\boldsymbol{\mu} = \frac{q}{2m}\mathbf{L} \tag{18}$$

with *q* being the total charge of the substitutional current and *m* the mass of the dipole. The dipole moment is equivalent to a spin angular momentum. Thus the rotation speed of the top enters the calculation. With spinning frequency ω and moment of inertia θ the modulus of the angular momentum of the top is

$$L = \theta\omega \tag{19}$$

where the direction is governed by Eq. (17). The axis of precession $\boldsymbol{\Omega}$ is defined by the external magnetic field. By comparison of (16), (17) and (18) we obtain formally

$$\frac{q}{2m}\mathbf{L} \times \mathbf{B} = -\boldsymbol{\Omega} \times \mathbf{L} \tag{20}$$

or

$$\boldsymbol{\Omega} = \frac{q}{2m}\mathbf{B}\,. \tag{21}$$

This can be interpreted of $\boldsymbol{\Omega}$ of being the gravitomagnetic field. This interpretation is beyond classical mechanics as well as the standard model and can only be understood within the context of ECE theory, see for example [7].

Now we have enough information to solve the mechanical equation of motion derived from (17) for equilibrium of torque ($\mathbf{N}_{tot} = 0$):

$$\frac{\mathbf{dL}}{\mathbf{dt}} = -m\mathbf{x} \times \mathbf{g} - \frac{\mu_1}{\theta\omega}\mathbf{L} \times \mathbf{B}. \tag{22}$$

Most realistic calculation

Finally we sketch the theory in cases where we have to consider an extended rigid body instead of a point-like spinning top. The total torque acting on the body is

$$\mathbf{N} = \frac{1}{\mathrm{S}}\int_{\mathrm{S}} \mathbf{x}' \times \mathbf{f}_{\mathrm{S}}(\mathbf{x}')\, \mathrm{dS}' \tag{23}$$

where the integral extends over the sum of surfaces S and **x'** is the distance vector from the center. **N** determines the momentary axis of rotation. The top is rotated by an angle φ according to

$$m\frac{\partial^2\varphi}{\partial t^2} = \mathrm{N(t)}\ . \tag{24}$$

The position in space is approximately defined by

$$m\frac{\partial^2\mathbf{x}}{\partial t^2} = \boldsymbol{\nabla}\left(\boldsymbol{\mu}_1 \cdot \mathbf{B}\right) + m\mathbf{g} \tag{25}$$

where we have to calculate $\boldsymbol{\mu}_1$ numerically from the magnetization which is dependent on the external field (including directional and hysteresis effects):

$$\boldsymbol{\mu}_1 = \int \mathbf{M}(\mathbf{x})d^3x. \tag{26}$$

Taking the full coordinate dependence of $\boldsymbol{\mu}_1$ and $\mathbf{B}$ into account, we obtain for (25):

$$m\frac{\partial^2\mathbf{x}}{\partial t^2} = \int_V \boldsymbol{\nabla}'\left(\mathbf{M}(\mathbf{x}') \cdot \mathbf{B}(\mathbf{x}')\right) d^3x' + m\mathbf{g} \tag{27}$$

with V being the volume of the spinning top. The integral depends on the position **x** because **M** is a function of the magnetic field **B**(x). In total, a Finite Element Analysis has to evaluate the equation set

$$\left.\begin{aligned} &\mathbf{N} = \frac{1}{\mathrm{S}}\int_{\mathrm{S}} \mathbf{x}' \times \mathbf{f}_{\mathrm{S}}(\mathbf{x}')\, \mathrm{dS}' \\ &m\frac{\partial^2\varphi}{\partial t^2} = \mathrm{N(t)} \\ &m\frac{\partial^2\mathbf{x}}{\partial t^2} = \int_V \boldsymbol{\nabla}'\left(\mathbf{M}(\mathbf{B},\mathbf{x}') \cdot \mathbf{B}(\mathbf{x}')\right) d^3x' + m\mathbf{g} \end{aligned}\right\} \tag{28}$$

for each time step. Then the top has to be moved to the new position **x** and rotated by φ around the axis **N**.

A correct description of the problem by ECE theory requires solving the full field equations of dynamics where the transformation between Force **F** and Torsion **T** has to be made by

$$\mathbf{F} = \mathrm{E}_0\mathbf{T} \tag{29}$$

with E_0 being the Einstein rest energy mc^2 [6]. However the differences are expected to be very small except in cases where so-called spin connection resonance is at work.

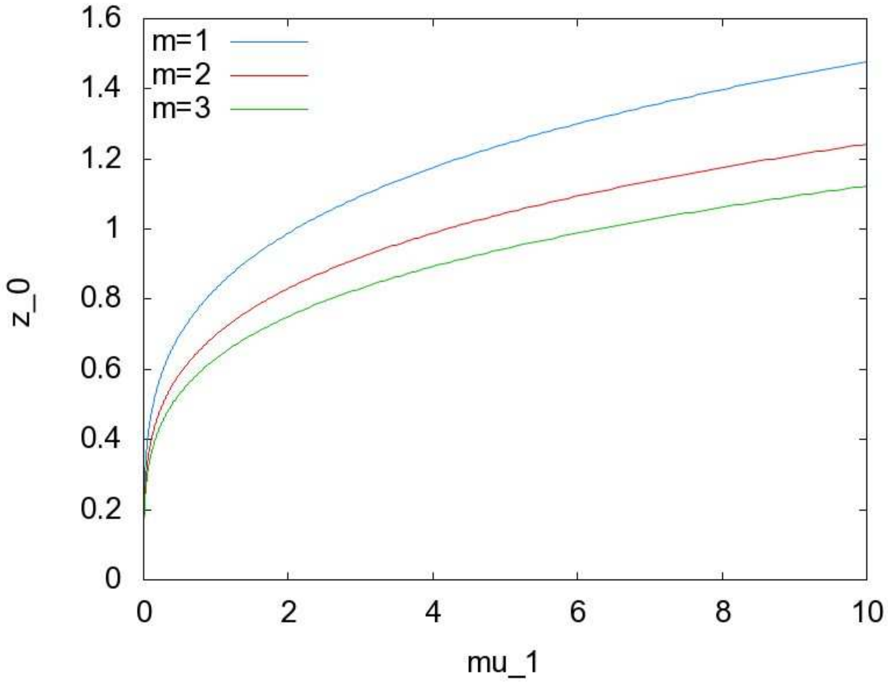

Fig. 3. Parameter study for equilibrium height z_0 in dependence of top dipole moment μ_1.

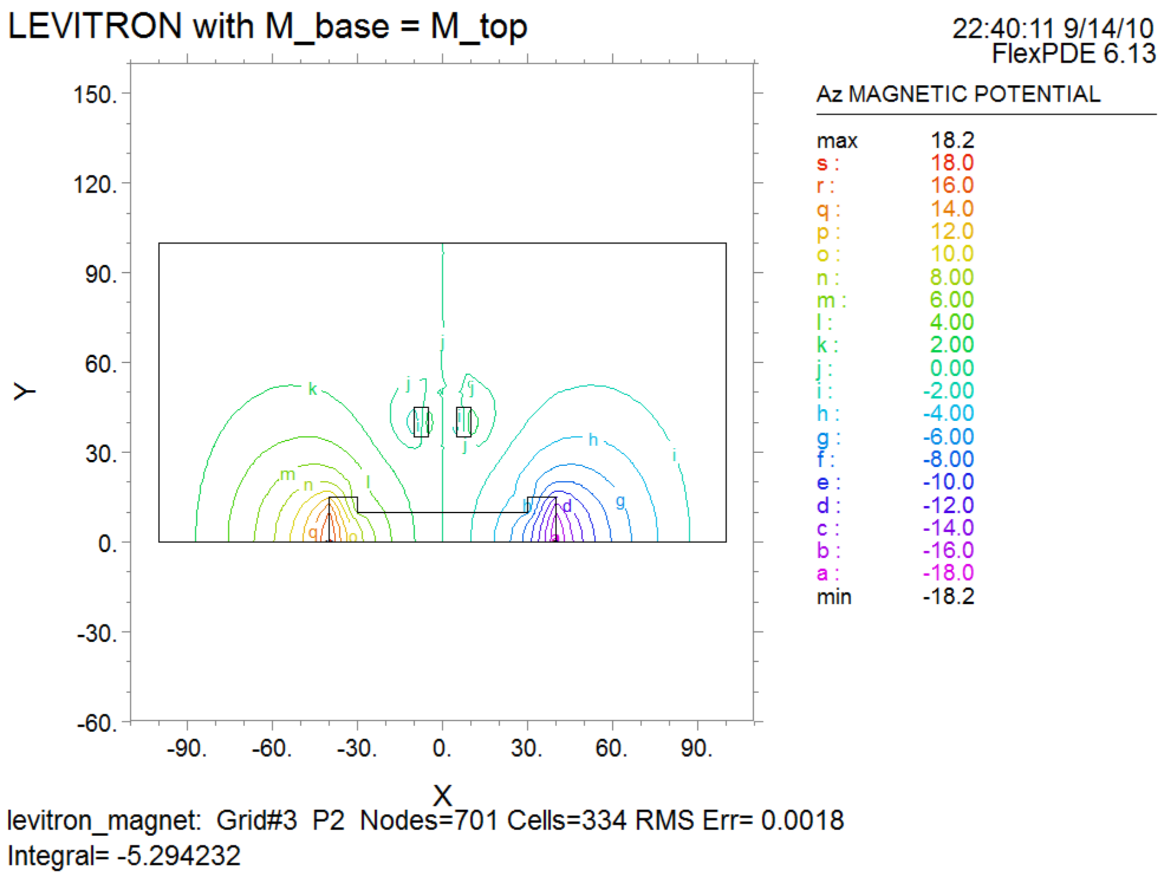

Fig. 4. Magnetic vector potential for a Levitron configuration with ring-like top, 2D model.

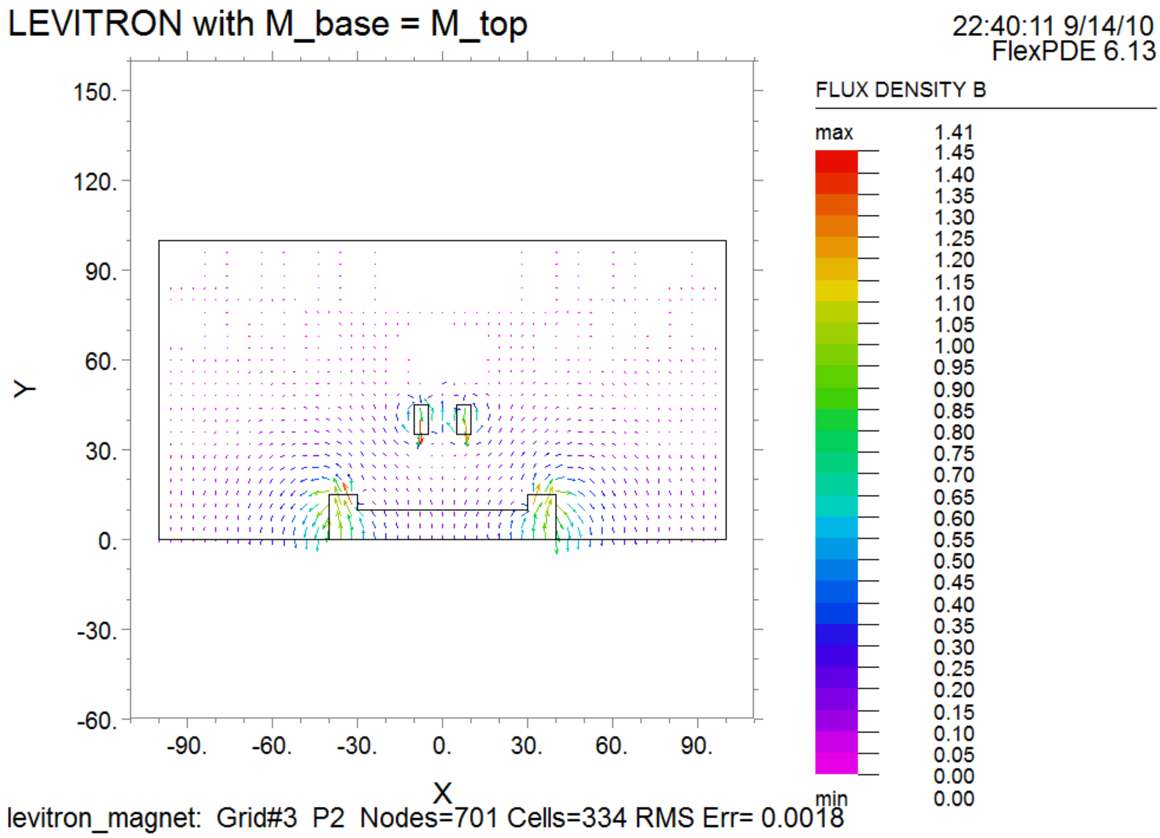

Fig. 5. Magnetic flux density for a Levitron configuration with ring-like top, 2D model.

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