——— THE ———
PHOTON'S MAGNETIC
FIELD

OPTICAL NMR SPECTROSCOPY

Myron W. Evans

Department of Physics, University of North Carolina
Charlotte, USA and Cornell Theory Center
Ithaca, NY 14853, USA

THE PHOTON'S MAGNETIC FIELD: OPTICAL NMR SPECTROSCOPY

Contents

Chapter 2

THE MAGNETOSTATIC FLUX DENSITY B_Π OF THE ELECTROMAGNETIC FIELD:

DEVELOPMENT AND CLASSICAL INTERPRETATION

by

M. W. Evans[1],

Department of Physics,

University of North Carolina,

Charlotte, NC 28223.

ABSTRACT

The classical electromagnetic field generates a static, uniform, magnetic flux density B_Π in its axis (Z) of propagation, proportional to the antisymmetric part of the tensor $E_i E_j^*$, (the vector quantity $E \times E^*$) where E is the electric field strength of the plane wave. The properties of B_Π are developed and interpreted in classical field theory. It is shown that B_Π obeys the novel continuity equation

$$\nabla \cdot B_\Pi = -\frac{\partial U_\Pi}{\partial t} = 0$$

where $U_\Pi = -B_0/c$ is a quantity interpreted as a magnetostatic density of the electromagnetic plane wave. Here B_0 is the magnetic flux density amplitude of the wave and c the speed of light. The vector potential, A_Π, of B_Π is defined in terms of $E \times E^*$, which is shown to be proportional to the antisymmetric component of Maxwell's electromagnetic stress tensor.

---

[1]Senior Visiting Research Associate, Materials Research Laboratory, Pennsylvania State University, University Park, PA 16802-4801. Visiting Scientist and Leverhulme Fellow, Cornell Theory Center, Ithaca, NY 14853.

Finally, the motion of an electron in the field B_{Π} is shown to be a helix by solving the novel

equation of motion $p + 2eA_{\Pi} = $ constant, where p is the momentum of the electron and e its

charge. This trajectory is shown to be equivalent to the motion of an electron in a circularly

polarised plane wave.

1. INTRODUCTION

It has been demonstrated {1-7} recently that the electromagnetic plane wave

generates the novel quantity B_{Π} which has the units of tesla, and the symmetry of uniform,

divergentless, magnetostatic flux density, i.e. positive to parity inversion $\hat{P}$ and negative to

motion reversal $\hat{T}$. In quantum field theory B_{Π} becomes {2} the novel magnetostatic flux

density quantum, $\hat{B}_{\Pi}$, a boson operator defined by $$\hat{B}_{\Pi} = \frac{B_0 \hat{J}}{\hbar},$$ where $\hat{J}$ is the angular

momentum boson operator of the photon, B_0 is the magnetic flux density amplitude associated

with a single photon, and $\hbar$ is the reduced Planck constant. The boson operator $\hat{B}_{\Pi}$ is

generated by each photon of the beam as it propagates linearly at the speed of light, c. For

this reason, and from considerations of symmetry {8}, there can be no electrostatic field E_{Π}

in X and Y of the plane wave, and E_{Π} cannot be generated from B_{Π} by Faraday induction in

free space. However, B_{II} interacts with matter through an interaction hamiltonian operator $-\hat{m} \cdot B_{II}$, where $\hat{m}$ is a magnetic dipole moment operator, and B_{II} can be used either in its classical or quantum forms to describe observable phenomena such as the inverse Faraday effect {9}; an optical Faraday effect {3}; optical Zeeman effect {4,5}; optical Cotton Mouton effect {6}; and other related phenomena {10,11} in which light magnetizes matter. Well known phenomena of optics can also be reinterpreted in terms of B_{II}, for example antisymmetric light scattering {12}. The classical vector B_{II} is proportional {1} to the vector cross product $E \times E^*$, where E is the usual electric field strength of an electromagnetic plane wave, and E^* is its complex conjugate {13-15}. Accordingly, in free space:

$$B_{II} = \frac{E \times E^*}{2E_0 ci} = B_0 k = (\frac{I_0}{\epsilon_0 c^3})^{\frac{1}{2}} k = (\frac{|N|}{2\epsilon_0 c^3}) k \qquad (1)$$

in which k is a unit <u>axial</u> vector in Z, the propagation axis, and where E_0 is the scalar amplitude of E; $B_0 = E_0 /c$ is the scalar magnetic flux density amplitude, and ϵ_0 is the free space permittivity. Here I_0 is the scalar intensity in watts m^{-2} of the plane wave and $|N|$ denotes the scalar magnitude of the well known Poynting vector

$$N = \frac{1}{\mu_0} E \times B^* . \qquad (2)$$

This paper is concerned with the development and classical interpretation of the novel vector B_{II} in analogy with its well known relative, N. The latter is a flux of energy density of the plane wave, and takes meaning {15} only when the wave interacts with matter, at the simplest level the electronic charge. It is well known that $N = 2I_0 n$, where n in free space is the unit propagation vector in the axis, Z, of propagation of the plane wave. The

vectors **N** and **n** are negative to both $\hat{P}$ *and* $\hat{T}$, and are polar vectors, whereas $\mathbf{B}_{\Pi}$ is $\hat{P}$

positive, $\hat{T}$ negative, and is an axial vector. **N** is proportional to the scalar part of the free

space intensity tensor

$$I_{ij} = \epsilon_0 c E_i E_j^*\qquad\qquad(3)$$

and $\mathbf{B}_{\Pi}$ is proportional to its antisymmetric part, which in vector notation is $\epsilon_0 c \mathbf{E}\mathbf{x}\mathbf{E}^*$.

Therefore **N** and $\mathbf{B}_{\Pi}$ are different parts of the same tensor property. It follows that as for **N**

{15}, its relative $\mathbf{B}_{\Pi}$ takes meaning only when the wave interacts with matter.

Section 2 develops a novel continuity equation for $\mathbf{B}_{\Pi}$ which links it to a novel

magnetic density $U_{\Pi} = -B_0/c$ in the same way as **N** is linked by a continuity equation of

electromagnetic field theory to the energy density U {15}. Thus, **N** and $\mathbf{B}_{\Pi}$ are vector fields

and U and U_{Π} are scalar fields. The scalar field U can also be interpreted as electromagnetic

power per unit volume, and **N** as power per unit area. In the same way $\mathbf{B}_{\Pi}$ becomes

interpretable in Section 2 as magnetostatic power per unit area, and U_{Π} as magnetostatic

power per unit volume generated by a completely circularly polarised electromagnetic plane

wave.

Section 3 defines the vector potential $\mathbf{A}_{\Pi}$ associated with $\mathbf{B}_{\Pi}$ starting directly from a

consideration of the cross product **E** x **E***, suitably scaled by the denominator $2E_0 c i$. It is

shown that a vector triple product of the type **r** x (**E** x **E***), where **r** is a position vector in (X,

Y, Z), has all the characteristics of the vector potential normally associated with a uniform,

divergentless, magnetostatic field. The latter is identified therefore as $\mathbf{B}_\Pi$, and is related to $\mathbf{A}_\Pi$ through

$$\boldsymbol{B}_\Pi = \nabla \times \boldsymbol{A}_\Pi . \tag{4}$$

Section 4 interprets the cross product $\mathbf{E} \times \mathbf{E}^*$ with the antisymmetric part of Maxwell's electromagnetic stress tensor {14}, which is part of the electromagnetic energy/momentum four tensor. Thus, $\mathbf{B}_\Pi$ is proportional to a vorticity in the classical electromagnetic field, and this is illustrated by determining the equations of motion of an electron in $\mathbf{B}_\Pi$ by solving the novel Lorentz equation

$$\boldsymbol{p} + 2e\boldsymbol{A}_\Pi = constant \tag{5}$$

where p is the electron's momentum and e its charge. The field $\mathbf{B}_\Pi$ drives the electron forward in a helical trajectory, with constant linear velocity in Z. It is shown finally that this is the same trajectory as that of the electron in a circularly polarised plane wave, obtained by solving the Lorentz equation for this case. In other words, the solutions of the Lorentz equation (5) are also solutions of the Lorentz equation of motion of the electron in a circularly polarised plane wave, showing that a plane wave can generate the characteristics of a magnetostatic flux density, $\mathbf{B}_\Pi$, which takes meaning as it interacts with the electron, driving the latter in a helical trajectory.

The paper ends with a short discussion of the interpretation of $\mathbf{B}_\Pi$ in the required relativistic context.

2. THE CONTINUITY EQUATION FOR B_x

Maxwell's phenomenological equations lead to the following well known $\{15\}$ continuity equation when electromagnetic radiation interacts with matter:

$$\nabla \cdot \boldsymbol{N} = -\frac{\partial U}{\partial t} - \boldsymbol{E} \cdot \boldsymbol{J}^* = 0 \tag{6}$$

where $\boldsymbol{J}^*$, the current density, is zero in free space. The energy density, U, is defined in free space as the time averaged quantity:

$$U = \frac{1}{2} (\boldsymbol{H}^* \cdot \boldsymbol{B} + \boldsymbol{E}^* \cdot \boldsymbol{D}) \tag{7}$$

with $\boldsymbol{B} = \mu_0 \boldsymbol{H}$ and $\boldsymbol{D} = \epsilon_0 \boldsymbol{E}$. Here μ_0 is the permeability of free space.

Clearly, in free space, the wave does no work on matter, such as electronic charge, because the density $\boldsymbol{E} \bullet \boldsymbol{J}^*$ of power lost from the fields $\boldsymbol{E}$ and $\boldsymbol{B}^*$ is zero. U is therefore a field energy density which takes meaning only if there is field / matter interaction.

By considering the product $\boldsymbol{E} \times \boldsymbol{E}^*$, which is proportional, as we have seen, to the antisymmetric vector part of light intensity, it can be demonstrated as follows that there exists a novel continuity equation linking B_x and U_n. Using the vector relation:

$$\nabla \cdot (\boldsymbol{E} \times \boldsymbol{E}^*) = \boldsymbol{E}^* \cdot (\nabla \times \boldsymbol{E}) - \boldsymbol{E} \cdot (\nabla \times \boldsymbol{E}^*) \tag{8}$$

and the Maxwell equations in free space:

$$\nabla \times \boldsymbol{E} = -\frac{\partial \boldsymbol{B}}{\partial t} \; ; \quad \nabla \times \boldsymbol{E}^* = -\frac{\partial \boldsymbol{B}^*}{\partial t} \tag{9}$$

implies

$$\mathbf{V} \cdot (\boldsymbol{E} \times \boldsymbol{E}^*) = \mathbf{V} \cdot (\boldsymbol{B} \times \boldsymbol{B}^*) = 0 \tag{10}$$

i.e.

$$\mathbf{V} \cdot \boldsymbol{B}_{II} = 0 \tag{11}$$

which shows that $\mathbf{B}_{II}$ is uniform and divergentless. From eqns (9) and (10), we can write:

$$\mathbf{V} \cdot (\boldsymbol{E} \times \boldsymbol{E}^*) = 2E_0 ci \mathbf{V} \cdot \boldsymbol{B}_{II} = -\boldsymbol{E}^* \cdot \frac{\partial \boldsymbol{B}}{\partial t} + \boldsymbol{E} \cdot \frac{\partial \boldsymbol{B}^*}{\partial t} \tag{12}$$

Integration by parts of the right hand side of eqn. (12) gives the result:

$$\int \boldsymbol{E}^* \cdot \frac{\partial \boldsymbol{B}}{\partial t} dt - \int \boldsymbol{E} \cdot \frac{\partial \boldsymbol{B}^*}{\partial t} dt =$$
$$\frac{1}{2} (\boldsymbol{E}^* \cdot \boldsymbol{B} - \boldsymbol{E} \cdot \boldsymbol{B}^*) = -2iE_0 B_0 \tag{13}$$

Defining the quantity

$$U_{II} = \frac{1}{2E_0 ci} (\int \boldsymbol{E}^* \cdot \frac{\partial \boldsymbol{B}}{\partial t} dt - \int \boldsymbol{E} \cdot \frac{\partial \boldsymbol{B}^*}{\partial t} dt) = -\frac{B_0}{c} \tag{14}$$

implies the result

$$\mathbf{V} \cdot \boldsymbol{B}_{II} = -\frac{\partial U_{II}}{\partial t} = 0 \tag{15}$$

which is a free space continuity equation for $\mathbf{B}_{II}$ • Eqn. (15) is the precise counterpart of the

free space continuity equation (6) for **N**:

$$\nabla \cdot \boldsymbol{N} = -\frac{\partial U}{\partial t} = 0 \qquad\qquad (16)$$

Eqn. (15) is novel to this work, whereas eqn. (16) is standard in classical electrodynamics {15}.

The continuity equations (15) and (16) have the same structure and must be interpreted in the same way. Thus, **N**, the Poynting vector, is, in classical electrodynamics, the flux of the density (U) of the electromagnetic energy. Similarly, the novel B_π is the flux of the magnetostatic density of the electromagnetic plane wave, a flux which is uniform, and does not vary with time. U is the electromagnetic field energy density in free space (i.e. electromagnetic power per unit volume {15}), and therefore U_π is the magnetostatic density in free space generated by an electromagnetic plane wave, or the magnetostatic power of the wave per unit volume occupied by that wave in free space.

Both eqns. (15) and (16) are continuity equations relating {15} the time rate of change of a density (the scalar fields U or U_π) to the divergence of a flux (the vector fields **N** or B_π). It is well established {15} that the notion of electromagnetic field energy density, U, takes meaning only when there is interaction between the field and matter. Similarly, U_π takes meaning only when there is wave/matter interaction. Eqns. (15) and (16) are both statements based on the existence in free space of the light intensity tensor I_{ij}, which is hermitian {16}. Eqn. (16) is concerned with the scalar part of I_{ij}, and eqn (15) with its vector part, which, as we have seen, is the quantity $\epsilon_0 c E \times E^*$ proportional to B_π. The existence of B_π, and of eqn. (15), is an inevitable consequence of the fact that I_{ij} is a tensor {16}, with a

vector (i.e. antisymmetric) component which is purely imaginary as a consequence of the hermitian nature of I_{ij} {16}.

It follows that the classical free space electromagnetic plane wave generates B_{II}, and its associated scalar field, U_{II}.

3. THE VECTOR POTENTIAL A_{II} IN FREE SPACE

Since B_{II} is a magnetostatic flux density it is assumed that there exists a vector potential A_{II} such that

$$B_{II} = \nabla \times A_{II}. \tag{17}$$

Since B_{II} and U_{II} are well defined in free space, it follows from the assumption (17) that A_{II} would also be well defined in free space. If eqn. (17) were true, it follows that A_{II} would be a function {17} of the type:

$$A_{II} = -\frac{1}{2} r \times B_{II} \tag{18}$$

where r is a positive coordinate vector in frame (X, Y, Z). From eqn. (1):

$$A_{II} = -\frac{1}{2} r \times \frac{(E \times E^{*})}{(2E_0 c i)} \tag{19}$$

Defining

$$r \equiv X i + Y j + Z k \tag{20}$$

and using the vector relation

$$\mathbf{r} \times (\mathbf{B} \times \mathbf{B}^*) = \mathbf{B}(\mathbf{r} \cdot \mathbf{B}^*) - \mathbf{B}^*(\mathbf{r} \cdot \mathbf{B}) \tag{21}$$

with {17-19}

$$\mathbf{B} = E_0 (\mathbf{1} - i\mathbf{j}) e^{i\phi}$$
$$\mathbf{B}^* = E_0 (\mathbf{1} + i\mathbf{j}) e^{-i\phi} \tag{22}$$

where ϕ is the phase {17} of the plane wave, we have

$$\mathbf{A}_{\mathrm{II}} = \frac{B_0}{2} (X\mathbf{j} - Y\mathbf{1}) . \tag{23}$$

Since

$$|\mathbf{B}_{\mathrm{II}}| = B_0 \tag{24}$$

it is clear that $\mathbf{B}_{\mathrm{II}}$ and $\mathbf{A}_{\mathrm{II}}$ of eqn. (24) are related by eqn. (17).

In other words there exists a vector potential $\mathbf{A}_{\mathrm{II}}$ in free space whose curl is $\mathbf{B}_{\mathrm{II}}$, and

which is defined by the vector field $\dfrac{B_0}{2}(X\mathbf{j} - Y\mathbf{i})$; a field that is generated in free space by

the classical electromagnetic plane waves (22), both solutions of Maxwell's equations. Note

that $\mathbf{B}_{\mathrm{II}}$ and $\mathbf{A}_{\mathrm{II}}$ **themselves** are not plane wave solutions of Maxwell's equations. In the

same way, **N** is not in itself a solution of Maxwell's equations, but is generated therefrom

through a cross product **E** x **B*** / μ_0.

It has therefore been demonstrated that $\mathbf{B}_{\mathrm{II}}$ is related to a well defined $\mathbf{A}_{\mathrm{II}}$ in free space,

as well as to a well defined scalar field $\mathbf{U}_{\mathrm{II}}$.

The next section considers the interaction of B_{II} with an electron, i.e. wave/matter interaction.

4. INTERACTION OF B_{II} WITH AN ELECTRON

From eqn. (18) of the preceding section it is clear that

$$\mathbf{A}_{II} = -\frac{1}{2}\mathbf{v} \times \mathbf{B}_{II} \qquad (25)$$

where $\mathbf{v} = \partial\mathbf{r}/\partial t$ is a velocity in frame (X, Y, Z). Consider the interaction of B_{II} with an electron. Since action and reaction are equal and opposite the action of B_{II} on the electronic charge e is balanced by a reaction of e upon B_{II}. However, as usual in classical electrodynamics, we assume that B_{II} is not changed greatly by the action of e upon it {13-15} so that the Lorentz equation of motion applies. Since there is no E_{II} present,

$$\dot{\mathbf{p}} = e\mathbf{v} \times \mathbf{B}_{II} \qquad (26)$$

where $\dot{\mathbf{p}}$ is the momentum of the electron, and v its velocity (p/m) in the field B_{II}. With eqn. (25) the Lorentz equation can be rewritten directly in terms of the novel vector potential $\mathbf{A}_{II}$:

$$\dot{\mathbf{p}} + 2\mathbf{A}_{II}e = 0 . \qquad (27)$$

The equations of motion of e in B_{II} thus become {15} those of Coriolis acceleration:

$$\dot{v}_X = \Omega v_Y; \quad \dot{V}_Z = 0; \quad \dot{v}_Y = -\Omega v_X; \qquad (28)$$

where the parameter Ω is defined as

$$\Omega = (1 - \frac{v^2}{c^2})^{\frac{1}{2}} (\frac{e}{m}) B_{\Pi z} . \qquad (29)$$

The equations (28) can be rewritten {15} as

$$\frac{d}{dt} (v_X + i v_Y) = -i\Omega (v_X + i v_Y) ;$$
$$v_X + i v_Y = a \exp(-iot) . \qquad (30)$$

where a is complex, and defined by

$$a = v_{ot} \exp(-i\alpha) \qquad (31)$$

where v_α and α are real. Here

$$v_{ot} = (v_X^2 + v_Y^2)^{\frac{1}{2}} \qquad (32)$$

is the velocity of the electron in the XY plane. The trajectory of the electron in the field B_Π

is therefore that of a helix

$$X = X_0 + r \sin (\Omega t + \alpha)$$
$$Y = Y_0 + r \cos (\Omega t + \alpha) \qquad (33)$$
$$Z = Z_0 + v_{0z} t$$

where the radius of the helix r is defined by

$$r = \frac{p_t}{(eB_{\Pi z})} = \frac{m}{eB_{\Pi z}} v_{ot} (1 - \frac{v^2}{c^2})^{\frac{1}{2}} \qquad (34)$$

where p_t is the projection of the momentum on to the XY plane.

Note that the same result can be obtained directly from the equation

$$\dot{p} = -2A_{II}e \qquad (35)$$

to give

$$\dot{v}_X = -\Omega v_Y \ etc. \qquad (36)$$

We now demonstrate that the motion of an electron in a circularly polarised plane wave is precisely the same as the motion of an electron in the novel magnetic field B_{II}.

Assume that the reaction of the electron upon the circularly polarised electromagnetic field is negligible, so that the Lorentz equation again applies:

$$\dot{p} = e(E + v \times B) \qquad (37)$$

Here E and B are plane wave solutions of Maxwell's equations in free space:

$$E = E_0(i + ij)e^{-i\phi};$$
$$B = B_0(j - ii)e^{-i\phi} \qquad (38)$$

i.e. they are the usual oscillating, phase dependent, electric and magnetic field vectors of the wave. Note carefully that B is quite different in nature from B_{II}. The velocity of the electron in frame (X, Y, Z) is

$$v = v_X i + v_Y j + v_Z k. \qquad (39)$$

Substituting eqns. (38) to (39) into the Lorentz equation (37) and using $|B_n| = B_0 = E_0 /c$

gives the equation of motion

$$\dot{v}_x = \Omega (c - v_z) e^{-i\phi} \tag{40}$$

$$\dot{v}_y = i\Omega (c + v_z) e^{-i\phi} \tag{41}$$

$$\dot{v}_z = \Omega (v_x + i v_y) e^{-i\phi} . \tag{42}$$

Multiplying eqn (41) by i and adding to eqn. (40) gives

$$\frac{d}{dt} (v_x + i v_y) = -2 v_z \Omega e^{-i\phi} \tag{43}$$

If we assume a solution of the type

$$v_z = \frac{i}{2} (v_x + i v_y) e^{i\phi} \tag{44}$$

separation of variables occurs

$$\frac{d}{dt} (v_x + i v_y) = -i\Omega (v_x + i v_y) \tag{45}$$

$$\frac{d}{dt} v_z = -2 i \Omega v_z e^{-2i\phi} \tag{46}$$

Eqn. (44) is a consistent solution of eqns. (40) to (42), a solution which implies that

the time average of v_z must be zero

$$\langle v_z \rangle_t = 0 \tag{47}$$

because the time average of the oscillating function

$$\langle e^{i\phi}\rangle_t = \langle\cos\,\phi\rangle_t + i\langle\sin\,\phi\rangle_t \tag{48}$$

is zero {15}. This is consistent with the fact that the unaveraged v_z itself is in general non-

zero and complex. Eqns (42) and (46) both imply that $\langle\dot{v_z}\rangle_t$ is also zero. Eqn. (44) is

therefore a solution of eqns. (40) to (42) when both $\langle v_z\rangle_t$ and $\langle\dot{v}_z\rangle_t$ vanish.

Note that eqn. (45) is the same precisely as eqn. (30), derived when considering the

motion of one electron in B_Π and can be solved to give the trajectory of the electron in a

circularly polarised plane wave. This is, from eqn. (45), a circle

$$
\begin{aligned}
X &= X_0 + r\sin(\Omega t + \alpha)\\
Y &= Y_0 + r\cos(\Omega t + \alpha)\\
Z &= Z_0
\end{aligned}
\tag{49}
$$

a conclusion which is consistent with that of Landau and Lifshitz {15} from the relativistic

Hamilton Jacobi equations of the electromagnetic field, but derived in a different way by

solving the Lorentz equation assuming eqns. (38).

For an initially stationary electron, i.e. for $v_{oz} = 0$, eqns (49) are identical with

eqns. (33a) to (33c), describing the trajectory of the electron in the novel field B_Π. The

trajectories (49) are consistent with the assumption (44), an assumption which implies that the

time averages of v_z and $\dot{v}_z$ both vanish.

In summary, the trajectory of an initially stationary electron in the field B_Π is the

same as that in the fields **E** and **B**, an initially stationary electron moves in a circle under the

influence either of B_{π} or of E and B. In the former, the velocity in Z vanishes identically, in the latter this component is zero on the average.

An observer noting this trajectory would not be able to define unambiguously the influence that causes the electron to move as it does in a circle, be this the wave or the field B_{π}. The influence upon an initially stationary electron of a circularly polarised electromagnetic plane wave is identical in all respects in the plane perpendicular to the propagation axis with the influence of B_{π} upon that electron. Therefore the motion in this plane of an electron in a circularly polarised electromagnetic plane wave can be represented exactly by a magnetostatic field B_{π}, which influences an electron to move in the same trajectory. B_{π} is therefore a real, physically meaningful, influence.

Furthermore, we have shown that B_{π} has the units of tesla, is $\hat{T}$ negative, $\hat{P}$ positive, is accompanied by a well defined scalar field U_{π}, and a well defined vector potential A_{π}. It follows that B_{π} has several characteristics of a magnetostatic field. However, B_{π}, is clearly not identical with an ordinary magnetostatic field, because it is a property of light. It is important to note that apparently there are no X and Y components of E_{π}, and E_{π} cannot be generated from B_{π} by Faraday induction. If there were an E_{π} in X and Y an electron's trajectory in that E_{π} would be a catenary {15} and clearly, from eqns. (33) and (49), this is not the case. An electric field E_{π} cannot be generated from products such as E x E*, B x B., E x B*, or B x E* on the grounds of fundamental $\hat{P}$ *and* $\hat{T}$ symmetry.

Finally, further physical interpretation can be placed upon E x E* by considering the well known Maxwell stress tensor {15}

$$\sigma_{\alpha\beta} = [-\epsilon_0 E_\alpha E_\beta^* - \mu_0 H_\alpha H_\beta^* + \frac{1}{2}\delta_{\alpha\beta}(\epsilon_0 E_\alpha E_\beta^* + \mu_0 H_\alpha H_\beta^*)] \tag{50}$$

which is the momentum flux density of electromagnetic radiation, and part of its energy momentum four tensor {15}. It is clear that the antisymmetric component of $\sigma_{\alpha\beta}$ is proportional to E x E* in vector notation, so that the field B_{Π} is also proportional to the antisymmetric part of $\sigma_{\alpha\beta}$. It is significant to note in this context that the antisymmetric part of stress in mechanics is a vorticity, with the same symmetry as angular momentum, so we deduce that B_{Π} is proportional to the angular momentum of classical radiation, as in our

operator equation $\quad \hat{B}_\Pi = \dfrac{B_0 \hat{J}}{\hbar}$ {2} of quantum field theory. Clearly, angular momentum takes

meaning in the energy momentum four tensor of the electromagnetic field through the antisymmetric part of the Maxwell stress tensor, a vorticity, i.e. the antisymmetric part of the momentum tensor per unit volume.

DISCUSSION

It has been argued that the notion of B_{Π} is consistent with several properties of a magnetostatic field, but it must be borne in mind that B_{Π} is generated by a photon which travels at the speed of light. The classical theory with which we have been concerned in this

paper must come to terms with the fact that B_π is relativistic in nature. In so doing {15,8} it

becomes clear that the only relativistically invariant component of B_π is that in the

propagation axis, which is, of course, consistent with our eqn. (1). Furthermore, it can be

shown {8} that there is no Faraday induction by a time modulated B_π, i.e. the hypothetical

E_π cannot be generated from $\partial B_\pi / \partial t$ through Faraday's Law. This is a consequence of the

Lorentz transformations. Furthermore, the existence of a B_π is consistent with the fact that a

valid solution of the Maxwell equations in free space is

$$B = B_0 (J - iI) e^{-i\phi} + B_z k \qquad (51)$$

where $B_z k$ is in general a magnetostatic field such as B_π. It has been shown that there exists

a vector potential A_π such that B_π

$= \nabla \times A_\pi$, and there exists a scalar field U_π linked to B_π by a continuity equation. Since

A_π is defined in terms of X and Y coordinates it is relativistically invariant, i.e. does not

change under Lorentz transformation. This is consistent with the fact that B_π is defined in Z,

and is a magnetic flux density, and so is also invariant to Lorentz transformation.

Finally, it is straightforward to deduce that

$$N = \pm cU\mathbf{n}; \quad B_{II} = \pm cU_{II}k \qquad (52)$$

where n is a propagation vector whose magnitude is refractive index, and k is a unit axial

vector. The first of these equations shows that the relation between energy and momentum in

an electromagnetic field is the same as that in a particle moving at the speed of light {15},

i.e. the photon of quantum field theory. The second equation shows that there exists a similar

proportionality between the magnetic flux density B_π and U_π.

CONCLUSION

The classical theory of fields has been used to develop and interpret the concepts $E \times E^*$ and B_{II}, and it has been shown that these concepts are self-consistent and physically meaningful, for example, the motion of an electron in B_{II} is the same as that in E and B of a plane wave. The latter can therefore be thought of as generating a magnetostatic field.

ACKNOWLEDGEMENTS

REFERENCES

{1}   M. W. Evans, Physica B, in press, (1992) (classical theory).

{2}   ibid., in press, 1992 (quantum theory).

{3}   ibid., submitted, 1992 (optical Faraday effects).

{4}   ibid., submitted, 1992 (anomalous optical Zeeman effect).

{5}   ibid., in press, 1992 (optical Zeeman effect).

{6}   ibid., submitted, 1992 (optical Cotton Mouton effect).

{7}   ibid., submitted, 1992 (symmetry duality).

{8}   ibid., submitted, 1992 (on Faraday induction).

{9}   P. W. Atkins, "Molecular Quantum Mechanics." (Oxford Univ. Press, 1983, 2nd. ed.).

{10} R. Tanaś and S. Kielich, J. Mod. Opt., 37 (1990) 1935.

{11} W. S. Warren, S. Mayr, D. Goswami, and A. P. West, Jr., Science, 255 (1992) 1683.

{12} L. D. Barron, "Molecular Light Scattering and Optical Activity." (Cambridge Univ. Press, Cambridge, 1982).

{13} S. Kielich, in M. Davies (Senior Reporter), "Dielectric and Related Molecular Processes." (Chem. Soc., London, 1972), vol. 1.

{14} S. Kielich, "Nonlinear Molecular Optics.", (Nauka, Moscow, 1981).

{15} L. D. Landau and E.M. Lifshitz, "The Classical Theory of Fields." (Pergamon, Oxford, 1975, 4th ed.).