EXPERIMENTAL DETECTION OF THE PHOTON'S FUNDAMENTAL STATIC MAGNETIC FIELD OPERATOR: THE ANOMALOUS OPTICAL ZEEMAN AND OPTICAL PASCHEN-BACK EFFECTS

I. INTRODUCTION

In this paper we continue a systematic theoretical search for a method of detecting and measuring unequivocally the photon's fundamental static magnetic field operator[1-5]

$$\hat{B}_{\Pi} = B_0 \frac{\hat{J}_{\Pi}}{\hbar} \tag{1}$$

where B_0 is the scalar magnetic flux density amplitude in tesla of a circularly polarized laser beam, made up of N photons, $\hat{J}_{\Pi}$ is the photon's angular momentum operator, and $\hbar$ is the reduced Planck constant $h/2\pi$.

Fragmentary experimental evidence for the existence of $\hat{B}_{\Pi}$ is available through the inverse Faraday effect (IFE),[6-9] and the recent emergence of optical NMR (ONMR), or laser-enhanced nuclear magnetic resonance spectroscopy (LENS).[10,11] Both techniques measure the ability of circularly polarized laser radiation to magnetize. The IFE measures bulk magnetization and ONMR measures light-induced resonance shifts, which are different for each resonating site and are therefore useful for sample identification and spectral analysis. However, the theoretical existence of $\hat{B}_{\Pi}$ also implies other effects, such as an optical Zeeman effect, in which the magnetic effect of a circularly polarized laser splits electric dipole transitions in atoms occurring in the visible frequency range. This paper provides a fairly rigorous quantum theory of the anomalous optical Zeeman effect and the optical Paschen-Back effect, in which both spin and orbital electronic angular momenta are considered in various coupling schemes.

The existence of an optical Zeeman effect in atoms appears to have been implicit in the theory of the inverse Faraday effect proposed by Pershan et al.,[6-9] Kielich et al.,[12] and Atkins and Miller.[13] The present author independently arrived at the existence of an optical Zeeman effect in a series of papers[14-17] based on symmetry considerations and semiclassical theory, considerations that also led to ONMR.[18-21] Recently, he has proposed theoretically the existence of the photon's $\hat{B}_{\Pi}$ operator, a fundamental property of the photon itself, whose classical equivalent is a static magnetic field, $\hat{B}_{\Pi}$, produced by circularly polarized electromagnetic radiation at all frequencies.[1-4] It is important to realize that the operator $\hat{B}_{\Pi}$ (or the classical $\hat{B}_{\Pi}$) is different fundamentally from the usual **B** vector of electromagnetic plane waves.[22] The **B** vector is frequency dependent, whereas the **B** vector is not. The **B** vector has components in X and Y directions mutually perpendicular to the propagation axis Z of the laser, and no component in the Z axis, whereas $\mathbf{B}_{\Pi}$ is directed in the Z axis only. The **B** vector depends on the photon's linear momentum vector $\boldsymbol{\kappa}$ (i.e., the propagation vector), whereas $\mathbf{B}_{\Pi}$ does not. Again, $\mathbf{B}_{\Pi}$ is positive to the parity inversion operator $\hat{P}$ and negative to the motion reversal operator $\hat{T}$, and is therefore fundamentally different in symmetry from the Poynting vector[23] and the propagation vector.[24] It appears that $\mathbf{B}_{\Pi}$ (and its quantum field equivalent $\hat{B}_{\Pi}$) is a fundamentally new concept in electromagnetic field theory.

The optical Zeeman effect appears to be a promising method of detecting the effects of $\hat{B}_{\Pi}$ experimentally. Electric dipole transitions in atoms are readily measured and identified spectrally.[24] The key to the optical Zeeman effect is to replace the magnet of the conventional Zeeman effect[24] by a circularly polarized laser. In the ordinary Zeeman effect, where there is no consideration given to the role of net electronic spin angular momentum,[22] a singlet $^1S \rightarrow {}^1P$ transition is split by a magnet into three lines. Its optical equivalent has recently been proposed theoretically[5] using the concept summarized in Eq. (1), and produces a splitting of the original $^1S \rightarrow {}^1P$ electric dipole transition, a splitting pattern whose details are different in quantum-field theory (in which the operator $\hat{B}_{\Pi}$ forms a Hamiltonian with the electronic magnetic dipole moment operator $\hat{m}$ of the atom) and in semiclassical theory (in which the Hamiltonian is formed from a product of $\hat{m}$ and the classical vector $\mathbf{B}_{\Pi}$). The pattern also depends on the type of angular momentum interaction used in the quantum-field theory, i.e., whether a coupled or uncoupled representation of $\hat{B}_{\Pi}$ and $\hat{m}$ is used.[22] The semiclassical result is recovered[5] only in the uncoupled representation. In the coupled representation, the quantum field theory produces three lines, but the central line is displaced in frequency. In the uncoupled representation of the quantum field theory of the optical Zeeman effect, and in the semiclassical representation of the same problem, the splitting pattern obtained[5] is the same as that in the conventional Zeeman effect, i.e., two lines each side of a central line situated at the original frequency of the electric dipole transition $^1S \rightarrow {}^1P$.

In Section II, these findings are augmented by the consideration of electronic spin angular momentum in quantum field and semiclassical approaches using in the former different coupling models for the angular momenta involved in the interaction Hamiltonians. In Section III, the details are given of the spectral splitting due to the circularly polarized laser for each theoretical approach of Section II. Finally, some experimental details are discussed and estimates of the splittings in hertz are given for each theoretical approach, namely the quantum field theory in the fully coupled, semicoupled, and uncoupled representations, and the semiclassical approach where $\mathbf{B}_\Pi$ is considered as a classical field vector. Conditions are discussed under which the anomalous optical Zeeman effect gives way to the optical Paschen-Back effect in the quantum field and semiclassical representations of the same phenomena.

II. QUANTUM FIELD AND SEMICLASSICAL INTERACTION HAMILTONIANS

The core of the description of the anomalous optical Zeeman effect and the optical Paschen-Back effect in atoms is the construction of the interaction Hamiltonians between the novel photon property $\hat{B}_\Pi$ (quantum field theory) or $\mathbf{B}_\Pi$ (semiclassical theory) and the atom's net electronic magnetic dipole moment operator $\hat{m}$. In this section, first-order interaction Hamiltonians are constructed in the framework of quantum field and semiclassical descriptions of the same problem.

A. Quantum Field Theory

The interaction Hamiltonian is the operator product

$$\Delta \hat{H}_1 = -\hat{m} \cdot \hat{B}_\Pi \tag{2}$$

from which the energy of interaction is calculated from an expectation value such as

$$\Delta E_1 = -\frac{\gamma_e B_0}{\hbar} \langle SLJJ_\Pi F M_F | \hat{L} + 2.002\hat{S} | S'L'J'J'_\Pi F'M'_F \rangle \tag{3}$$

In this expression, the magnetic dipole moment of the atom is developed as

$$\hat{m} = \gamma_e(\hat{L} + 2.002\hat{S}) \tag{4}$$

where γ_e is the electronic gyromagnetic ratio, $\hat{L}$ is the operator describing the net electronic orbital angular momentum, and $2.002\hat{S}$ is the operator description of the net electronic spin angular momentum.[22] The $\hat{B}_\Pi$ operator is developed in terms of the photon's angular momentum operator in Eq. (1). The angular momentum quantum numbers S, L, and J_Π are associated with the operators $\hat{S}$, $\hat{L}$, and $\hat{J}_\Pi$, respectively. The interaction

energy (3) is one in which a coupled representation[22] is considered for the three angular momenta just introduced. In this case the coupling scheme is

$$\mathbf{J} = \mathbf{L} + \mathbf{S} \qquad \mathbf{F} = \mathbf{J} + \mathbf{J}_\Pi \tag{5}$$

so that the values of the quantum number J, associated with the operator $\hat{J}$ are given as usual by the Clebsch-Gordan series

$$J = L + S, \ldots, |L - S| \tag{6}$$

Similarly, the overall quantum number F is defined by

$$F = J + J_\Pi, \ldots, |J - J_\Pi| \tag{7}$$

i.e., from a coupled representation of the $\hat{J}$ operator of the atom and the novel[1-5] $\hat{J}_\Pi$ operator of the photon whose effects we are attempting to describe.

There are other ways of writing the interaction energy for the given Hamiltonian (2), these being the semicoupled and uncoupled representations. In the former, the operators $\hat{L}$ and $2.002\hat{S}$ of the atom are combined in a coupled angular momentum representation[22] to give the $\hat{J}$ operator, but the interaction energy is formed as follows:

$$\Delta E_2 = -\frac{\gamma_e B_0}{\hbar} \langle SLJM_J; J_\Pi M_{J_\Pi} | \hat{L} + 2.002\hat{S} | S'L'J'M'_J; J'_\Pi M'_{J_\Pi} \rangle \tag{8}$$

i.e., with $\hat{J}$ and $\hat{J}_\Pi$ considered in an uncoupled representation, in which the projections $\hat{M}_J$ and $\hat{M}_{J_\Pi}$ onto the azimuthal axis (Z, the laser's propagation axis) are well defined, but in which the net angular momentum operator $\hat{F}$ is not. In the latter, the interaction energy is written as

$$\Delta E_3 = -\frac{\gamma_e B_0}{\hbar} \langle SM_S; LM_L; J_\Pi M_{J_\Pi} | \hat{L} + 2.002\hat{S} | S'M'_S; L'M'_L; J'_\Pi M'_{J_\Pi} \rangle \tag{9}$$

in which all three angular momentum operators—$\hat{L}$ and $2.002\hat{S}$ of the atom, and $\hat{J}_\Pi$ of the photon—are considered in a fully uncoupled representation. All three representations are possible theoretically, and which is the most appropriate can be determined only by independent consideration of the physics of the problem.

1. *The Coupled Representation*

The first stage is the usual one. The Wigner-Eckart theorem is used to separate out the M_F dependence[22, 25]:

$$\Delta E_1 = -(-1)^{F-M_F}\begin{pmatrix} F & 0 & F \\ -M_F & 0 & M_F \end{pmatrix}\langle SLJJ_\Pi F\|\hat{m}\cdot\hat{B}_\Pi\|SLJJ_\Pi F\rangle \qquad (10)$$

We have restricted our consideration to diagonal elements of the interaction energy, and in this case, the 3-j symbol is[25]

$$(-1)^{F-M_F}\begin{pmatrix} F & 0 & F \\ -M_F & 0 & M_F \end{pmatrix} = (2F+1)^{-1/2} \qquad (11)$$

The quantum nature of the interaction energy is therefore contained within the reduced matrix element in Eq. (10). This is a problem of the type first considered by Curl and Kinsey[26] and which is summarized in Eq. (13.8) of Silver,[25] one in which there are three types of commuting (independent) angular momenta, described by operators $2.002\hat{S}$, $\hat{L}$, and $\hat{J}_\Pi$ in spaces 1, 2, and 3, in the fully coupled representation of angular momentum quantum theory.[22] It is helpful to write the interaction energy (10) as

$$\Delta E_1 = -\frac{\gamma_e B_0}{\hbar}(2F+1)^{-1/2}\Big(\langle SLJJ_\Pi F\Big\|\Big[\big[\hat{1}^0\otimes\hat{L}^1\big]^1_0\otimes\hat{J}^1_{\Pi 0}\Big]^0_0\Big\|SLJJ_\Pi F\rangle$$
$$+2.002\langle SLJJ_\Pi F\Big\|\Big[\big[\hat{S}^1\otimes\hat{1}^0\big]^1_0\otimes\hat{J}^1_{\Pi 0}\Big]^0_0\Big\|SLJJ_\Pi F\rangle\Big) \qquad (12)$$

in terms of tensor products of the type illustrated in Eq. (13.7) of Silver,[25] to which we refer the reader for background and details of irreducible tensorial methods. These methods allow the reduced matrix element to be written in terms of the 9-j symbols of atomic quantum mechanics,[25] allowing the interaction energy to be expressed simply as

$$\Delta E_1 = -\gamma_e B_0 \hbar g_1 \qquad (13)$$

where the g_1 factor is a complicated combination of terms defined

through the individual angular momentum quantum numbers:

$$g_1 = (2J+1)[3(2F+1)J_\Pi(J_\Pi+1)(2J_\Pi+1)]^{1/2}\begin{bmatrix} J & J & 1 \\ J_\Pi & J_\Pi & 1 \\ F & F & 0 \end{bmatrix}$$
$$\times[(2S+1)L(L+1)(2L+1)]^{1/2}\begin{bmatrix} S & S & 0 \\ L & L & 1 \\ J & J & 1 \end{bmatrix} \qquad (14)$$
$$+\,2.002[(2L+1)S(S+1)(2S+1)]^{1/2}\begin{bmatrix} S & S & 1 \\ L & L & 0 \\ J & J & 1 \end{bmatrix}$$

Note at this stage that there are several energy levels, because there are several allowed combinations of quantum numbers through the appropriate Clebsch-Gordan series. For each energy level there will be an individual g_1 factor. The physical meaning of this coupled representation is discussed later, and in Section III the result (13) is used in the context of electric dipole transitions in atomic states split by the photon property $\hat{B}_\Pi$ generated by a circularly polarized laser.

2. *The Semicoupled Representation*

This is, perhaps, the most realistic representation of the problem in quantum field theory, because of the nature of the photon operator $\hat{B}_\Pi$. The photon propagates at the speed of light and is massless, so that the azimuthal components of the angular momentum operator $\hat{J}_\Pi$ are always well defined (i.e., specified)[22] in terms of the azimuthal quantum numbers

$$M_{J_\Pi} = \pm 1$$

(A sign change in this context denotes switching from left to right circular polarization.) Relativity theory forbids any component of the photon angular momentum perpendicular to the azimuthal (propagation) axis Z. It appears natural, therefore, to combine the angular momentum operators $\hat{J}$ and $\hat{J}_\Pi$ in the uncoupled representation of the quantum theory of angular momentum coupling,[22, 25] a representation in which the azimuthal components of the angular momenta are specified, but in which the resultant angular momentum is not.[22]

In the semicoupled representation, the interaction energy (8) can therefore be written

$$\Delta E_2 = -\frac{\gamma_e B_0}{\hbar}\Big(\langle SLJM_J|\big[\hat{1}^0 \otimes \hat{L}^1\big]_0^1|S'L'J'M_J'\rangle\Big) + 2.002 \times \langle SLJM_J|\big[\hat{S}^1 \otimes \hat{1}^0\big]_0^1|S'L'J'M_J'\rangle\Big)\langle J_\Pi M_{J_\Pi}|\hat{J}^1_{\Pi 0}|J'_\Pi M'_{J_\Pi}\rangle \tag{15}$$

an expression that can be reduced using tensorial methods (using Eqs. (14.10) ff. of Silver[25]) to the form

$$\Delta E_2 = -\left(\frac{\gamma_e B_0}{\hbar}\right)M_J\left[\frac{3J(J+1) - L(L+1) + S(S+1)}{2J(J+1)}\right](-1)^{J_\Pi - M_{J_\Pi}} \times \begin{pmatrix} J_\Pi & 1 & J'_\Pi \\ -M_{J_\Pi} & 0 & M'_{J_\Pi}\end{pmatrix}\langle J_\Pi\|\hat{J}_\Pi\|J'_\Pi\rangle \tag{16}$$

This can be reduced further to the simple result

$$\Delta E_2 = -\gamma_e B_0 g_L M_J M_{J_\Pi}\hbar \tag{17}$$

using the following results[25] and notation:

$$g_L = \frac{3J(J+1) - L(L+1) + S(S+1)}{2J(J+1)} \tag{18}$$

$$(-1)^{J_\Pi - M_{J_\Pi}}\begin{pmatrix} J_\Pi & 1 & J_\Pi \\ -M_{J_\Pi} & 0 & M_{J_\Pi}\end{pmatrix} = M_{J_\Pi}[J_\Pi(J_\Pi+1)(2J_\Pi+1)]^{-1/2} \tag{19}$$

$$\langle J_\Pi\|\hat{J}_\Pi\|J_\Pi\rangle = \hbar[J_\Pi(J_\Pi+1)(2J_\Pi+1)]^{1/2} \tag{20}$$

In this semicoupled representation, therefore, the interaction energy in quantum field theory becomes the product of a g_L factor which depends only on the quantum numbers J, L, and S, with the azimuthal quantum number product $M_J M_{J_\Pi}$. The g_L factor in this case is recognizable as the Landé factor of atomic theory.[22, 25]

3. *The Uncoupled Representation*

This is a possible representation of the same problem, in which the three operators $2.002\hat{S}$, $\hat{L}$, and $\hat{J}_\Pi$ are decoupled operators acting independently on decoupled states, each operator acting independently on states built from independent sets of coordinates in spaces 1, 2, and 3 (Ref. 22). The interaction energy (9) is therefore written as

$$\Delta E_3 = -\frac{\gamma_e B_0}{\hbar}\Big(\langle SM_S; LM_L; J_\Pi M_{J_\Pi}|\hat{1}^0_0\hat{L}^1_0\hat{J}^1_{\Pi 0}|S'M'_S; L'M'_L; J'_\Pi M'_{J_\Pi}\rangle + 2.002\langle SM_S; LM_L; J_\Pi M_{J_\Pi}|\hat{S}^1_0\hat{1}^0_0\hat{J}^1_{\Pi 0}|S'M'_S; L'M'_L; J'_\Pi M'_{J_\Pi}\rangle\Big) \tag{21}$$

and the Wigner-Eckart theorem applied three times to give a superficially complicated result:

$$\begin{aligned}\Delta E_3 = {} & -\gamma_e B_0\hbar(-1)^{S-M_S+L-M_L+J_\Pi-M_{J_\Pi}} \\ & \times\Bigg[\begin{pmatrix} S & 0 & S' \\ -M_S & 0 & M'_S\end{pmatrix}\begin{pmatrix} L & 1 & L' \\ -M_L & 0 & M'_L\end{pmatrix}\begin{pmatrix} J_\Pi & 1 & J'_\Pi \\ -M_{J_\Pi} & 0 & M'_{J_\Pi}\end{pmatrix} \\ & \times\langle S\|\hat{1}\|S'\rangle\langle L\|\hat{L}\|L'\rangle\langle J_\Pi\|\hat{J}_\Pi\|J'_\Pi\rangle \\ & + 2.002\begin{pmatrix} S & 1 & S' \\ -M_S & 0 & M'_S\end{pmatrix}\begin{pmatrix} L & 0 & L' \\ -M_L & 0 & M'_L\end{pmatrix}\begin{pmatrix} J_\Pi & 1 & J'_\Pi \\ -M_{J_\Pi} & 0 & M_{J_\Pi}\end{pmatrix} \\ & \times\langle S\|\hat{S}\|S'\rangle\langle L\|1'\|L'\rangle\langle J_\Pi\|J_\Pi\|J'_\Pi\rangle\Bigg]\end{aligned} \tag{22}$$

However, with standard results,[22, 25] such as

$$\langle S\|\hat{S}\|S\rangle = [S(S+1)(2S+1)]^{1/2}\hbar \tag{23}$$

$$\langle S\|\hat{1}\|S\rangle = (2S+1)^{1/2} \tag{24}$$

$$\begin{pmatrix} S & 1 & S \\ -M_S & 0 & M_S\end{pmatrix} = (-1)^{S-M_S}M_S[S(S+1)(2S+1)]^{-1/2} \tag{25}$$

$$\begin{pmatrix} S & 0 & S \\ -M_S & 0 & M_S\end{pmatrix} = (-1)^{S-M_S}(2S+1)^{-1/2} \tag{26}$$

the interaction energy in the decoupled representation of quantum field theory collapses to

$$\Delta E_3 = -\gamma_e B_0 \hbar M_{J_{\Pi}} (M_L + 2.002 M_S) \tag{27}$$

in which there is no g factor at all, and which is a simple product of azimuthal quantum numbers of the atom and the photon's novel $\hat{B}_{\Pi}$ operator in which we are interested.

B. Semiclassical Theory

The semiclassical representation of the anomalous optical Zeeman and Paschen-Back effects depends on the interaction Hamiltonian

$$\Delta \hat{H}_2 = -\hat{m} \cdot \mathbf{B}_{\Pi} \tag{28}$$

where $\mathbf{B}_{\Pi}$ is now a classical field vector,[1-5] not a quantum-mechanical operator. The interaction energy in this case is

$$\Delta E_4 = -\gamma_e |\mathbf{B}_{\Pi}| \langle SLJM_J | \hat{L} + 2.002 \hat{S} | S'L'J'M_J' \rangle \tag{29}$$

which can be reduced to

$$\Delta E_4 = -\gamma_e |\mathbf{B}_{\Pi}| g_L \hbar M_J \tag{30}$$

where g_L is the same Landé factor as in Eq. (17).

III. APPLICATION TO ELECTRIC DIPOLE TRANSITIONS IN ATOMS

In this section the results of Section II are applied to predict the splitting of a visible frequency electric dipole transition in an atom by a circularly polarized laser generating the flux quantum $\hat{B}_{\Pi}$ of Eq. (1). The selection rules governing such a transition in the conventional theory of the anomalous Zeeman effect in atoms are well known.[22, 25, 26] They are determined by rules on the existence of the 3-j symbol in the Wigner-Eckart expansion of the matrix elements of the transition electric dipole moment operator $\hat{\mu}$. For the Z component

$$\langle SLJM_J | \hat{\mu}_0^1 | S'L'J'M_J' \rangle = (-1)^{J-M_J} \begin{pmatrix} J & 1 & J' \\ -M_J & 0 & M_J' \end{pmatrix} \langle SLJ \| \hat{\mu}_0^1 \| S'L'J' \rangle \tag{31}$$

and the selection rules are

$$\Delta J = 0, \pm 1 \qquad \Delta M_J = 0 \tag{32}$$

Similarly, for the X and Y components of $\hat{\mu}$,

$$\Delta J = 0, \pm 1 \qquad \Delta M_J = \pm 1 \tag{33}$$

However, in the anomalous optical Zeeman effect, the atomic terms between which the electric dipole transition takes place are each being considered in the presence of the operator $\hat{B}_{\Pi}$, in the various coupling schemes of Section II. Therefore, the electric dipole transition selection rules must also be derived in the appropriate coupling scheme.

We shall consider an atomic transition[22] between the atomic terms ${}^2P_{1/2}$ and ${}^2D_{3/2}$. In the former, $L = 1$, $S = \frac{1}{2}$, and $J = \frac{1}{2}$; and in the latter, $L = 2$, $S = \frac{1}{2}$, and $J = \frac{3}{2}$. The Laporte (or parity) selection rule is also obeyed in such a transition, i.e., $\Delta L = 1$ in this case. The transition occurs at a frequency that is determined from the appropriate electric dipole selection rules,[22] and the spectrum in the absence of $\hat{B}_{\Pi}$ consists of a single line which can be measured at visible frequencies in a spectrometer.

We are specifically interested in how this line is affected by the presence of an additional, circularly polarized laser, generating the flux quantum $\hat{B}_{\Pi}$ of Eq. (1), and substituting for the usual magnet of the Zeeman effect.[22] In examining the effect of $\hat{B}_{\Pi}$ we use the four results, Eqs. (13), (17), (27), and (30), in turn. In each case the effect of $\hat{B}_{\Pi}$ is first determined on the ${}^2P_{1/2}$ atomic term, and then on the ${}^2D_{3/2}$ term. Each of these two terms is split into nondegenerate energy levels by the addition of quantized energy such as ΔE_1, described by Eq. (13), for example. Various electric dipole transitions can then occur between the split ${}^2P_{1/2}$ term and the split ${}^2D_{3/2}$ term according to the transition electric dipole selection rules appropriate for the coupling scheme. Overall, therefore, we expect that the novel flux quantum $\hat{B}_{\Pi}$ splits the original line corresponding to the transition ${}^2P_{1/2} \rightarrow {}^2D_{3/2}$ in an atom. The details of the splitting pattern depend on which of the various schemes of Section II are chosen. This procedure is similar to the standard theory of the conventional Zeeman effect,[22-24] but in the optical Zeeman effect, $\hat{B}_{\Pi}$ is a quantum-mechanical operator. In the conventional Zeeman effect, the applied magnetic field $\mathbf{B}_0$ is always a classical, magnetostatic field vector, whose origin is not electromagnetic.

A. Quantum Field Theory, Coupled Representation, Eq. (13)

The atomic $^2P_{1/2}$ term is split from Eq. (13) into two levels by the novel operator $\hat{B}_{\Pi}$ of the photon:

$$g_1(F = \tfrac{1}{2}, J_{\Pi} = 1, J = \tfrac{1}{2}, L = 1, S = \tfrac{1}{2})$$
$$g_2(F = \tfrac{3}{2}, J_{\Pi} = 1, J = \tfrac{1}{2}, L = 1, S = \tfrac{1}{2})$$

and there is a different g factor for each level. The atomic $^2D_{3/2}$ term is split into the three levels:

$$g_3(F = \tfrac{1}{2}, J_{\Pi} = 1, J = \tfrac{3}{2}, L = 2, S = \tfrac{1}{2})$$
$$g_4(F = \tfrac{3}{2}, J_{\Pi} = 1, J = \tfrac{3}{2}, L = 2, S = \tfrac{1}{2})$$
$$g_5(F = \tfrac{5}{2}, J_{\Pi} = 1, J = \tfrac{3}{2}, L = 2, S = \tfrac{1}{2})$$

each with a different g factor. In general the five g factors (two in the lower term and three in the upper) are all different. Electric dipole transitions within the atom can now occur between the two lower levels and three upper levels with selection rules determined as follows.

The transition electric dipole moment operator is developed using the Wigner-Eckart theorem between coupled states to give

$$\langle SLJJ_{\Pi}FM_F|\hat{\mu}_0^1|S'L'J'J'_{\Pi}F'M'_F\rangle = (-1)^{F-M_F}\begin{pmatrix} F & 1 & F' \\ -M_F & 0 & M'_F \end{pmatrix}\langle SLJJ_{\Pi}F\|\hat{\mu}_0^1\|S'L'J'J'_{\Pi}F'\rangle \tag{34}$$

This procedure yields immediately the selection rules on the Z component of $\hat{\mu}$:

$$\Delta F = 0, \pm 1 \qquad \Delta M_F = 0 \tag{35}$$

Similarly, for X and Y components of $\hat{\mu}$,

$$\Delta F = 0, \pm 1 \qquad \Delta M_F = \pm 1 \tag{36}$$

All selection rules now refer to the net quantum number F.

There are six possible spectral lines generated by electric dipole transitions between the two $^2P_{1/2}$ levels and the three $^2D_{3/2}$ levels, but one of these, from the $F = \frac{1}{2}$ level of the split $^2P_{1/2}$ term to the $F = \frac{5}{2}$ level of the split $^2D_{3/2}$ term, is forbidden by the selection rule (35, 36) just derived, i.e., by the fact that the maximum change in F must be $+1$.

Discussion of the physical meaning of this result is given later. The quantum field theory of the anomalous optical Zeeman effect in the coupled representation splits the original visible frequency spectral line into five, each displaced from the original frequency.

B. Quantum Field Theory, Semicoupled Representation, Eq. (17)

In this case the selection rules on the electric dipole transitions are obtained by the development (for the Z component):

$$\begin{aligned}\langle SLJM_J; J_{\Pi}M_{J_{\Pi}}|\hat{\mu}_0^1\hat{1}_0^0|S'L'J'M'_J; J'_{\Pi}M'_{J_{\Pi}}\rangle &= \langle SLJM_J|\hat{\mu}_0^1|S'L'J'M'_J\rangle\langle J_{\Pi}M_{J_{\Pi}}|\hat{1}_0^0|J'_{\Pi}M'_{J_{\Pi}}\rangle \\ &= (-1)^{J-M_J}\begin{pmatrix} J & 1 & J' \\ -M_J & 0 & M'_J \end{pmatrix}\langle SLJ\|\hat{\mu}_0^1\|S'L'J'\rangle\end{aligned} \tag{37}$$

so that the 3-j symbol is nonzero if and only if

$$\Delta J = 0, \pm 1 \qquad \Delta M_J = 0 \tag{38}$$

Similarly, for X and Y components of $\hat{\mu}$,

$$\Delta J = 0, \pm 1 \qquad \Delta M_J = \pm 1 \tag{39}$$

The Landé g_L factor of Eq. (17) is the same for each level of the $^2P_{1/2}$ term. For each level of the split $^2D_{3/2}$ term the Landé factor is again the same, g_{L1}. Transitions between the levels are controlled by the selection rule $\Delta M_J = 0, \pm 1$. The resulting spectral pattern is three groups of doublets, i.e., six lines. This is recognizable as the same pattern observed in the conventional semiclassical theory of the anomalous Zeeman effect, as illustrated, for example, in Fig. 9.27 of Ref. 22.

C. Quantum Field Theory, Uncoupled Representation, Eq. (27)

In this case the electric dipole (Z component) transition selection rules are determined from the development

$$\langle SM_S; LM_L; J_{\Pi}M_{J_{\Pi}}|\hat{1}_0^0\hat{\mu}_0^1\hat{1}_0^0|S'M'_S; L'M'_L; J'_{\Pi}M'_{J_{\Pi}}\rangle = (-1)^{L-M_L}\begin{pmatrix} L & 1 & L' \\ -M_L & 0 & M'_L \end{pmatrix}\langle L\|\hat{\mu}_0^1\|L'\rangle \tag{40}$$

Assuming, as usual,[22–24] that the spin selection rule

$$\Delta S = 0 \tag{41}$$

is obeyed, we obtain

$$\Delta L = 0, \pm 1 \qquad \Delta M_L = 0 \tag{42}$$

Similarly, for the X and Y components of $\hat{\mu}$,

$$\Delta L = 0, \pm 1 \qquad \Delta M_L = \pm 1 \tag{43}$$

In this case there are no g factors in either term, and both $^2P_{1/2}$ and $^2D_{3/2}$ terms are split to the same extent.[22] The result is a spectral pattern of three lines, which can be thought of as three coincidental doublets. This is recognizable as the same pattern obtained in the conventional theory of the Paschen-Back effect.[22] In the uncoupled representation of quantum field theory, therefore, the novel $\hat{B}_\Pi$ operator is expected to produce the optical Paschen-Back effect.

D. Semiclassical Theory, Eq. (30)

It is straightforward to see that in this case the transition electric dipole moment selection rules are those given by Eqs. (38) and (39), and that the splitting pattern is the same as that in the conventional theory of the anomalous Zeeman effect, consisting of three doublets.

IV. DISCUSSION

In four different schemes we have deduced that the novel property $\hat{B}_\Pi$ of the photon[1–5] splits electric dipole transitions occurring at visible frequencies in atoms. It is appropriate to ask which scheme is likely to be the most realistic. It is well known[22] that in the quantum theory of angular momentum coupling, the uncoupled representation leaves the magnitude of the total angular momentum undefined and says nothing about the relative orientation of the contributing individual angular momenta, but defines individual components. The coupled representation defines the total angular momentum but leaves individual components undefined. Either scheme is equally valid and acceptable mathematically. In Section II we found that there is also a third scheme, which we have called the semicoupled representation. All three are valid in the quantum-field theoretical description of the effect of $\hat{B}_\Pi$ on atomic transitions.

However, it is independently known that the photon propagates at the speed of light, which implies that the component of $\hat{B}_\Pi$ in the azimuthal axis be well defined, because there cannot be any perpendicular components from the theory of relativity. Therefore, it appears that in our coupled representation of Section II, there is a conflict of reasoning, in that the total angular momentum is defined as well as the azimuthal component of the novel field operator of the photon $\hat{B}_\Pi$. Therefore, the commutator $[F^2, J_{\Pi_Z}]$ is not zero. However, it is well known[22] that this type of "paradox" can be resolved by remembering that the commutator is an operator, which acts on a wave function, ψ, and if the result

$$\left[F^2, J_{\pi_Z}\right]\psi = 0 \tag{44}$$

is true, then F^2 and M_{J_Π} can be simultaneously well defined in the quantum theory of angular momentum coupling.[22]

This is the mathematical basis for our coupled representation of the problem in Section II. In physical terms, the coupled representation leads to five lines, instead of six as in the semicoupled representation, and this can be tested experimentally to reveal which is the truer representation.

The semicoupled representation treats $\hat{J}$ and $\hat{J}_\Pi$ in an uncoupled scheme, so that azimuthal components of both are well defined, but their resultant $\hat{F}$ is not. Therefore, F does not appear in Eq. (17) and there is clear definition of directionality, in that the azimuthal quantum number M_J does appear in Eq. (17) and controls the selection rules as described in Section III. The directionality comes from the presence of the circularly polarized laser, generating the quantity $\hat{B}_\Pi$, a laser that propagates in the azimuthal axis Z. In the coupled representation that gives Eq. (13) no azimuthal quantum number appears, but F is well defined and selection rules on F now govern the effect of $\hat{B}_\Pi$ on atomic transitions.

In the uncoupled representation of Section II, the only difference from the semicoupled representation is that the operator $2.002\hat{S}$ has been decoupled from $\hat{L}$, leading to the optical Paschen-Back effect. This is therefore a type of strong field limit, in which $2.002\hat{S}$, $\hat{L}$, and $\hat{J}_\Pi$ precess independently[22] about the propagation or azimuthal axis Z.

In the semiclassical representation, $\mathbf{B}_\Pi$ is a classical field vector, and the treatment of both the anomalous optical Zeeman effect and of the optical Paschen-Back effect becomes the same as conventional theory, leading to the same physical considerations.[22] This is because in the semiclassical representation, $\mathbf{B}_\Pi$ is akin to a magnetostatic field, albeit generated by a laser.[1]

Finally, it is straightforward to derive order of magnitude estimates of the splitting from any of the equations (13), (17), (27), and (30), given the relation[1-5]

$$|\mathbf{B}_{\Pi}| = B_0 = \left(\frac{2I_0}{\varepsilon_0 c^3}\right)^{1/2} \sim 10^{-7} I_0^{1/2} \quad (45)$$

between $|\mathbf{B}_{\Pi}|$ and the intensity of the laser in watts per square meter. Here, ε_0 is the permittivity in vacuo in S.I. units:

$$\varepsilon_0 = 8.854 \times 10^{-12} J^{-1} C^2 m^{-1} \quad (46)$$

For an intensity of 100 W cm^{-2} (10^6 W m^{-2}) we expect that the novel property $\hat{B}_{\Pi}$ will shift the original $^2P_{1/2} \rightarrow {}^2D_{3/2}$ transition typically by of the order 1.5×10^6 Hz. This is 5×10^{-5} cm^{-1} (inverse centimeters), and the splitting is expected to be proportional to the square root of the laser's intensity. There should be no splitting if the laser has no degree of circular polarity. These features should help in identifying the effect of the new fundamental photon property $\hat{B}_{\Pi}$ in which we are interested, and which has recently been proposed theoretically.[1-5]

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THE OPTICAL FARADAY EFFECT AND OPTICAL MCD

I. INTRODUCTION

This paper continues a series of articles in which the consequences are developed of the recent deduction[1-6] that the photon generates a magnetic flux quantum, an operator

$$\hat{B}_{\Pi} = B_0 \frac{\hat{J}}{\hbar} \quad (1)$$

Here B_0 is the scalar magnetic flux density amplitude of a beam of N photons (a circularly polarized generator laser), $\hat{J}$ is the photon's angular momentum operator[7] whose eigenvalues are $\pm M_J \hbar$, where M_J is plus or minus one, and where $\hbar$ is the reduced Planck constant. The operator $\hat{B}_{\Pi}$ changes sign with the circular polarity of the generator laser, is unlocalized in space, and has eigenvalues $\pm M_J B_0$, where B_0 is the laser's scalar flux density amplitude. Its classical equivalent is the axial vector $\mathbf{B}_{\Pi}$, a novel magnetostatic flux density generated by circularly polarized electromagnetic plane waves.[1-6] The theoretical existence of $\hat{B}_{\Pi}$ is supported by the experimental evidence for light-induced magnetic effects. The first to be described (in the 1960s) was the inverse Faraday effect,[8-14] and recently it has been shown[15, 16] that NMR resonances are shifted in new and useful ways by the magnetizing effect of circularly polarized argon ion radiation