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THE PHOTON'S MAGNETOSTATIC FLUX DENSITY $\hat{B}_{\Pi}$: THE INVERSE FARADAY EFFECT REVISITED*

I. INTRODUCTION

Intense, circularly polarized laser pulses produce a net magnetization M_Z ($\mathrm{A\ m^{-1}}$) in atomic, molecular, and other condensed material such as dilute magnetic semiconductors.[1,2] This magneto-optic property was first proposed by Piekara and Kielich,[3–6] and was demonstrated experimentally in the early 1960s by Pershan et al.[7,9,10] and Shen.[8] Since then, no further experimental work appears to have been reported on the effect. The theory of the inverse Faraday effect rests on the foundations built by Piekara and Kielich,[3–6] and was developed by Pershan et al.[7,9,10] in terms of the antisymmetric part of the tensor $E_i E_j^*$, where E_i is the electric field strength of a circularly polarized laser pulse in $\mathrm{V\ m^{-1}}$ and E_i^* is its own complex conjugate. This antisymmetric intensity is conveniently expressed in vector notation as the cross product $\mathbf{E} \times \mathbf{E}^*$, which is negative[11,12] to motion reversal ($\hat{T}$) and positive to parity inversion ($\hat{P}$). It therefore has the necessary $\hat{P}$ and $\hat{T}$ symmetries of magnetic flux density, which is the qualitative explanation for the ability of a circularly polarized laser to magnetize.

Further development of the theory is due to Kielich,[13–15] Atkins and Miller,[16] Wagnière,[17] Woźniak et al.,[18,19] and Evans et al.[20–22] with computer simulation of the magnetization. These theories all rely on the property $\mathbf{E} \times \mathbf{E}^*$ of the laser pulse. However, it has been shown recently[23–27] that this property, $\mathbf{E} \times \mathbf{E}^*$, is directly proportional to a novel, fundamental, magnetic flux density vector, $\mathbf{B}_{\Pi}$, of the classical electromagnetic field. In quantum-field theory[24] this becomes the novel, fundamental, and ubiquitous magnetic flux density operator, $\hat{B}_{\Pi}$, of the photon itself. In this paper it is argued that the inverse Faraday effect must be described semiclassically as a combination of terms in all positive integral powers of the classical $\mathbf{B}_{\Pi}$. The original theory, which relies on $\mathbf{E} \times \mathbf{E}^*$, is shown to be equivalent to considering only the term in $|\mathbf{B}_{\Pi}|^2$.

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II. DESCRIPTION OF $\mathbf{B}_{\Pi}$

It is straightforward to show that[11,12,17,23–27]

$$\mathbf{E} \times \mathbf{E}^* = 2E_0^2\,\mathrm{i}\mathbf{k} \tag{1}$$

where $\mathbf{k}$ is a unit axial vector, negative to $\hat{T}$ and positive to $\hat{P}$. This is purely imaginary and proportional to the square of E_0, the scalar electric field strength amplitude of a circularly polarized laser. In free space $E_0 = cB_0$ and

$$\mathbf{E} \times \mathbf{E}^* = 2E_0c\,\mathrm{i}B_0\mathbf{k} \equiv 2E_0c\,\mathrm{i}\mathbf{B}_{\Pi} \tag{2}$$

where B_0 is the scalar magnetic flux density amplitude (tesla) and c is the speed of light. The vector $\mathbf{B}_{\Pi}$ is the product $B_0\mathbf{k}$, which is in units of tesla. From these simple considerations,

$$\begin{aligned}\mathbf{B}_{\Pi} &= \frac{\mathbf{E} \times \mathbf{E}^*}{2E_0c\,\mathrm{i}} = B_0\mathbf{k} = \frac{E_0}{c}\mathbf{k} = \left(\frac{I_0}{\varepsilon_0c^3}\right)^{1/2}\mathbf{k} \sim 10^{-7}I_0^{1/2}\mathbf{k}\\ &= \left(\frac{|\mathbf{N}|}{2\varepsilon_0c^3}\right)^{1/2}\mathbf{k}\end{aligned} \tag{3}$$

Here I_0 is the scalar intensity in $\mathrm{W\ m^{-2}}$, which in free space is

$$I_0 = \varepsilon_0cE_0^2 \tag{4}$$

where ε_0 is the free space permittivity. In Eq. (3), $|\mathbf{N}|$ is the scalar magnitude of Poynting's vector

$$\mathbf{N} = \frac{1}{\mu_0}\mathbf{E} \times \mathbf{B}^* \tag{5}$$

where μ_0 is the free space permeability. The vector $\mathbf{N}$ (Ref. 28) is a flux of energy density, and the novel vector $\mathbf{B}_{\Pi}$ is a flux of magnetic density. Although $\mathbf{N}$ and $\mathbf{B}_{\Pi}$ have the same negative $\hat{T}$ symmetry, the former is negative to $\hat{P}$ and the latter, as we have seen, is positive to $\hat{P}$. We note that $\hat{N}$ is nonzero in linear polarization, but $\mathbf{B}_{\Pi}$ vanishes, because the latter reverses sign with circular polarity, whereas the former does not. Furthermore, $\mathbf{N}$ can be expressed as

$$\mathbf{N} = 2I_0\mathbf{n} \tag{6}$$

where $\mathbf{n}$ is the vector whose scalar magnitude is the real refractive index in the direction of propagation, and which is well known[28, 29] to be proportional to the $\hat{P}$- and $\hat{T}$-negative polar wave vector κ. This must be carefully distinguished from the $\hat{P}$-positive, $\hat{T}$-negative unit axial vector $\mathbf{k}$, which is multiplied by B_0 to form the novel $\mathbf{B}_{\Pi}$. Note that both $\mathbf{N}$ and $\mathbf{B}_{\Pi}$ are independent of the phase of the laser, and therefore of its angular frequency ω. In other words, the time averages over many cycles of both $\mathbf{N}$ and $\mathbf{B}_{\Pi}$ are nonzero, and it follows that $\mathbf{B}_{\Pi}$ is quite different from the usual oscillating $\mathbf{B}$ field of the electromagnetic plane wave, which vanishes when averaged over time. $\mathbf{B}$ is a complex quantity, with components mutually orthogonal (i.e., in X and Y) to the propagation direction (Z) of the wave, but none in that direction itself. In contrast, $\mathbf{B}_{\Pi}$ is a purely real quantity,[23–27] and is directed exclusively in Z, with no components in X and Y. Remarkably, its existence appears to have gone unrecognized in the long and illustrious history of the theory of electromagnetic fields.

III. THE ROLE OF $\mathbf{B}_{\Pi}$ IN THE INVERSE FARADAY EFFECT: SEMICLASSICAL TREATMENT

Using the vector $\mathbf{B}_{\Pi}$ it becomes straightforward to develop any magnetic effect of the circularly polarized electromagnetic plane wave, because we can now say that such a wave can magnetize material with which it interacts. There exists in nature an optical magnet, which delivers a magnetic flux density in tesla of

$$|\mathbf{B}_{\Pi}| \sim 10^{-7} I_0^{1/2} \tag{7}$$

Thus, for a circularly polarized laser of intensity $I_0 = 10\,000$ W m^{-2} (1.0 W cm^{-2}) the $\mathbf{B}_{\Pi}$ field is 10^{-5} T, or 0.1 G, about a tenth of the earth's mean magnetic field.

When $\mathbf{B}_{\Pi}$ is used, the theory of magnetization by circularly polarized light becomes standard and straightforward, because we have only to

adapt the existing semiclassical theory[28–30] of magnetization by an ordinary magnetostatic field, $\mathbf{B}_S$, and replace $\mathbf{B}_S$ everywhere by $\mathbf{B}_{\Pi}$. Thus, the magnetization is given by

$$M_Z = \frac{1}{\mu}\left(\frac{\kappa}{1+\kappa}\right)B_{\Pi Z} = N\langle m_Z\rangle_U \tag{8}$$

where κ is the volume susceptibility, μ_0 is the vacuum permeability, N is the number density, and $\langle m_Z\rangle$ is the mean magnetic dipole moment. It is assumed here that the total magnetic dipole moment is a sum of permanent and induced components:

$$\begin{aligned} m_Z &= m_Z^{(0)} + m_Z^{(\text{ind})} \\ m_Z^{(\text{ind})} &= \tfrac{1}{2}\xi_{ZZ}B_{\Pi Z}/\mu_0 \end{aligned} \tag{9}$$

where ξ_{ZZ} is the molecular magnetizability.[28–30] The ensemble average appearing in Eq. (8) is assumed to be the usual thermodynamic average[28–30]

$$\langle m_Z\rangle_U = \frac{\int\left(m_Z^{(0)} + m_Z^{(\text{ind})}\right)e^{-U/kT}\,d\Omega}{\int e^{-U/kT}\,d\Omega} \tag{10}$$

where the energy of interaction U is defined by

$$U = -m_Z^{(0)}B_{\Pi Z} - \frac{1}{2}\frac{\xi_{ZZ}}{\mu_0}B_{\Pi Z}^2 \tag{11}$$

The approximation[28–30]

$$\langle m_Z\rangle_U = \langle m_Z\rangle_0 - \frac{1}{kT}\left(\langle m_Z U\rangle_0 - \langle m_Z\rangle_0\langle U\rangle_0\right) + \cdots \tag{12}$$

is used for the thermodynamic average. Here $\langle\ \rangle_0$ denotes ensemble averaging in the absence of U, and $\langle\ \rangle_U$ denotes ensemble in averaging the presence of U. Since $\langle m_Z\rangle_0$ is zero in an initially isotropic material such as a gas or liquid, we have

$$\langle m_Z\rangle_U = -\frac{1}{kT}\langle m_Z U\rangle_0 + \cdots \tag{13}$$

This entirely standard semiclassical approach gives the following result for the temperature-dependent part of the effect we seek to describe:

$$\langle m_Z \rangle_U = \frac{1}{kT}\left(\langle m_Z^{(0)2} \rangle_0 B_{\Pi Z} + \frac{1}{\mu_0} \langle \xi_{ZZ} m_Z^{(0)} \rangle_0 B_{\Pi Z}^2 + \frac{1}{4\mu_0^2} \langle \xi_{ZZ}^2 \rangle_0 B_{\Pi Z}^3 \right) + \cdots \quad (14)$$

which shows that the magnetization (A/m) is described by terms in the first three powers of $B_{\Pi Z}$ within the first approximation (12) of the thermodynamic average (10). The term in $B_{\Pi Z}^2$ in Eq. (14) vanishes, however, because the theory of tensor invariants[28–30] shows that the ensemble average $\langle \xi_{ZZ} m_Z^{(0)} \rangle_0$ must vanish in isotropic media (but not in certain crystals). So we are left with terms in $B_{\Pi Z}$ and $B_{\Pi Z}^3$ for which the premultiplying ensemble averages do not vanish in liquids or gases.

In addition to these temperature-dependent terms, there exists the temperature-independent term considered in the usual theory of the inverse Faraday effect,[3–22] which has recently been put in the following simple form by Woźniak et al.[18]:

$$\langle m_Z \rangle_U = \frac{E_0^2}{3} \left({}^m\gamma_{123}^{ee} + {}^m\gamma_{231}^{ee} + {}^m\gamma_{312}^{ee} \right) \quad (15)$$

Here ${}^m\gamma_{ijk}^{ee}$ are molecule fixed-frame components of the appropriate[18, 19] molecular hyperpolarizability tensor. Using

$$E_0 = c|\mathbf{B}_\Pi| \quad (16)$$

it is immediately clear that this term is proportional to $|\mathbf{B}_\Pi|^2$. The complete expression for the inverse Faraday effect within the approximations we have made here is therefore

$$M_Z \doteqdot \frac{Nc^2}{3} \left({}^m\gamma_{123}^{ee} + {}^m\gamma_{231}^{ee} + {}^m\gamma_{312}^{ee} \right) B_{\Pi Z}^2 + \frac{N}{kT} \left(\langle m_Z^{(0)2} \rangle_0 B_{\Pi Z} + \frac{\langle \xi_{ZZ}^2 \rangle_0}{4\mu_0^2} B_{\Pi Z}^3 \right) \quad (17)$$

IV. ORDER OF MAGNITUDE ESTIMATES

To estimate the various orders of magnitude of the contributing terms in Eq. (17), the magnetic dipole moment is estimated roughly as a tenth of the electronic Bohr magneton, i.e., as 10^{-24} J T^{-1}. A rough order of magnitude approximation to the volume magnetic susceptibility κ is obtained from a model calculation given by Atkins,[28] which gives κ of about 10^{-5}. From this, the magnetizability can be obtained using $\kappa = N\xi_{ZZ}$ where N is the number density.[28] The order of magnitude of the hyperpolarizability ${}^m\gamma_{ijk}^{ee}$ is obtained from the Faraday effect theory of Woźniak et al.[18, 31] as about 10^{-45} A m^4V^{-2} for a typical diamagnetic. For a paramagnetic with a permanent magnetic dipole moment it is assumed that the hyperpolarizability is roughly 100 times bigger, i.e., 10^{-43} A m^4V^{-2}.

In Eq. (17) N is set at 10^{28} molecules for the molar volume in meters cubed (Ref. 18) and kT at 4×10^{-21} J molecule^{-1}, equivalent to 300 K. The order of magnitude of $B_{\Pi Z}$ is set at 1.0 T, corresponding to a pulse of intensity about 3×10^{15} W m^{-2}, available from a contemporary mode-locked laser, which must be accurately circularly polarized.

These rough estimates give an order of magnitude of magnetization (A m^{-1}) of about 2.5 A m^{-1} for the term in $B_{\Pi Z}$, about 2.0 A m^{-1} for the term in $B_{\Pi Z}^3$, and about 30 A m^{-1} for the temperature-independent term proportional to $B_{\Pi Z}^2$. Clearly these figures depend on the estimates we have used for $m_Z^{(0)}$, ξ_{ZZ}, and ${}^m\gamma_{ijk}^{ee}$ but all three terms contribute to the total magnetization. In our estimate, the term in $B_{\Pi Z}^2$ happens to be dominant, but at very low T and with less intense laser pulses, the term in $B_{\Pi Z}$ dominates, provided there is a permanent magnetic dipole moment. (If the latter is zero, there are terms in $B_{\Pi Z}^2$ and $B_{\Pi Z}^3$, but not in $B_{\Pi Z}$.)

Note that the magnetization changes sign with the circular polarity of the laser. The term in $B_{\Pi Z}$ changes sign because the vector $\mathbf{B}_\Pi$ is switched from positive (left) to negative (right). The conjugate product $\mathbf{E} \times \mathbf{E}^*$ changes sign with circular polarization,[18] and the product $\mathbf{E} \times \mathbf{E}^*$ is proportional to $|\mathbf{B}_\Pi|^2\mathbf{k}$, where $\mathbf{k}$ is a unit axial vector.

V. CONCLUSION

The inverse Faraday effect is characterized by a laser-induced magnetization that is proportional to all positive integral powers of $B_{\Pi Z}$. The conventional theory[18] is based solely on a consideration of $\mathbf{E} \times \mathbf{E}^*$, and produces a magnetization proportional to $B_{\Pi Z}^2$ only, from $\mathbf{E} \times \mathbf{E}^*$ multiplied by the sample's hyperpolarizability. We have argued that there is also a magnetization produced by a product of $B_{\Pi Z}$ and the sample's magnetizability.

Acknowledgments

The Leverhulme Trust is thanked for a Fellowship, and the Cornell Theory Center and Penn State Materials Research Laboratory are thanked for additional support.

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THE PHOTON'S MAGNETOSTATIC FLUX QUANTUM: THE OPTICAL COTTON-MOUTON EFFECT

I. INTRODUCTION

The ability of circularly polarized electromagnetic radiation to produce anisotropy in magnetic permeability was first proposed by Piekara and Kielich,[1,2] who systematically described light-induced anisotropy in material electric permittivity ($\Delta\varepsilon$), magnetic permeability ($\Delta\mu$), and refractive index (Δn). In Ref. 1 for example, formulated in the pre-laser era, it was proposed that "On observe alors des changements de ε, μ, ou n, dus à l'action du champ polarisant." We are concerned in this paper with the formulation of an optical Cotton-Mouton effect, a relative of the optical Kerr effect first proposed by Buckingham[3] and classified by Piekara and Kielich in their references. We define the novel optical Cotton-Mouton effect as a change in refractive index (linear dichroism) due to the novel, recently proposed, static magnetic field ($\mathbf{B}_\Pi$) of a circularly polarized electromagnetic plane wave.[4–8] Piekara and Kielich[1,2] described "saturation optique dans un champ optique." This effect later became known as the optical Kerr effect, or Buckingham effect.[3]

This paper is developed from the recent deduction[4–8] that the photon carries a magnetostatic flux quantum, $\hat{B}_\Pi$, whose classical equivalent is a phase-independent magnetic field $\mathbf{B}_\Pi$ generated in a circularly polarized light beam, an axial vector with the symmetry characteristics of a static magnetic flux density (tesla). The latter must be an axial vector positive to the parity inversion operator $\hat{P}$, and negative to the motion reversal operator $\hat{T}$ (Ref. 9). The classical field $\mathbf{B}_\Pi$ of the circularly polarized electromagnetic plane wave is a purely real quantity that is proportional to the antisymmetric (purely imaginary) part of the tensor $E_iE_j^*$, where E_i is the electric field strength of the wave in volts per meter. The scalar part of the tensor $E_iE_j^*$ is proportional to the phase-independent intensity of the plane wave in watts per meter squared, and we have shown elsewhere[4–8] that the vector part of $E_iE_j^*$ (i.e., its antisymmetric part) is proportional to the phase-independent magnetic flux density $\mathbf{B}_\Pi$ and vanishes if there is no degree of circular polarity. Furthermore, we have shown[8] that $\mathbf{B}_\Pi$ can be expressed in terms of the ubiquitous third Stokes parameter S_3 (Ref. 10) and therefore that phenomena such as circular dichroism and ellipticity are fundamentally magnetic.

The definition[4–8] of the $\hat{B}_\Pi$ operator per photon allows a wide range of novel optical/photonic phenomena to be forecasted straightforwardly, on the grounds that circularly polarized electromagnetic radiation can magne-