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The magnetostatic and electrostatic fields generated by light in free space: delta function description

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Abstract

In general, the Maxwell equations can be solved with electric and magnetic fields which are sums of oscillatory and static components, the latter being directed in the propagation axis of the electromagnetic plane wave. The electrostatic and magnetostatic fields are described classically in terms of travelling delta functions, which in quantum field theory becomes a description of the magnetostatic and electrostatic fields generated by a photon travelling at the speed of light in free space. The interaction of these fields with matter is investigated and the basic equations of electrodynamics derived in terms of the travelling delta function formalism of the classical field.

Introduction

The Maxwell equations of classical field theory can be solved to give electric and magnetic fields in the form [1]

$$\boldsymbol{E}^{G} = \frac{E_0}{\sqrt{2}}(\boldsymbol{i} \pm \mathrm{i}\boldsymbol{j})\mathrm{e}^{\mathrm{i}\phi} \pm E_\pi \boldsymbol{k} \quad (1)$$

$$\boldsymbol{B}^{G} = \frac{B_0}{\sqrt{2}}(\boldsymbol{j} \mp \mathrm{i}\boldsymbol{i})\mathrm{e}^{\mathrm{i}\phi} \pm B_\pi \boldsymbol{k} \quad (2)$$

showing that in general, Maxwell's equations in free space support components in $\boldsymbol{i}$, $\boldsymbol{j}$ and $\boldsymbol{k}$, unit vectors in directions x, y, and the propagation axis z. The components

$$\boldsymbol{B}_\pi = \pm\frac{B_0}{\sqrt{2}}(\mathrm{i}+1)\boldsymbol{k} \quad (3)$$

and

$$\boldsymbol{E}_\pi = \pm\frac{E_0}{\sqrt{2}}(\mathrm{i}-1)\boldsymbol{k} \quad (4)$$

are independent of the phase ϕ of the electromagnetic plane wave and are responsible for several observable effects. Among those due to $\boldsymbol{B}_\pi$ are optical NMR spectroscopy [2–8], the optical Zeeman effect [9, 10], optical forward backward birefringence [11], and several other related effects due to the magnetizing properties of light [12], if $\boldsymbol{B}_\pi$ is real and $\boldsymbol{E}_\pi$ is imaginary.

In the first part of this paper we investigate the interaction of the novel fields $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ with matter (in the first instance an electron) using a representation in terms of moving delta functions. The interaction of $\boldsymbol{B}_\pi$ with an atom is then explored in terms of quantum field theory, in which the interaction hamiltonian is used in a Heisenberg equation to test for the presence of squeezing effects due to $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$. Finally, the delta function description is developed based on the Fourier transformation of the $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ fields.

The delta function description

The delta function description of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ comes from the fact that the fields are generated in free space by a travelling wave, which in the

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quantized description becomes a photon propagating at the speed of light. Therefore, $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are not conventional electrostatic and magnetostatic fields but are present in free space as parts of the complete solution of Maxwell's equations as described in Eqs. (1) and (2). It can be shown that the phase-independent solution $\boldsymbol{B}_\pi$ can be related to the vector product

$$\boldsymbol{B}_\pi = \pm\frac{(\mathrm{i}+1)}{\sqrt{2}}\frac{\boldsymbol{E}\times\boldsymbol{E}^*}{E_0 c} \quad (5)$$

$$\boldsymbol{E} = \frac{E_0}{\sqrt{2}}(\boldsymbol{i}\pm \mathrm{i}\boldsymbol{j})\mathrm{e}^{\mathrm{i}\phi} \quad (6)$$

$$\boldsymbol{E}^* = \frac{E_0}{\sqrt{2}}(\boldsymbol{i}\mp \mathrm{i}\boldsymbol{j})\mathrm{e}^{-\mathrm{i}\phi} \quad (7)$$

and c is the speed of light. Therefore the general solution given in Eq. (2) can be thought of as a nonlinear solution of Maxwell's equation in free space

$$\boldsymbol{B}^{\mathrm{G}} = \frac{B_0}{\sqrt{2}}(\boldsymbol{j}\mp \mathrm{i}\boldsymbol{i})\mathrm{e}^{\mathrm{i}\phi} \pm \frac{(\mathrm{i}+1)}{\sqrt{2}}\frac{\boldsymbol{E}\times\boldsymbol{E}^*}{E_0 c} \quad (8)$$

$$\boldsymbol{B}^{\mathrm{G}} = \boldsymbol{B}(\boldsymbol{r},t) \pm \boldsymbol{B}_\pi \quad (9)$$

In quantum field theory both $\boldsymbol{B}_\pi$ and $\boldsymbol{E}_\pi$ can be expressed in terms of the standard creation and annihilation operators [1]

$$\mathrm{Re}(\hat{\boldsymbol{B}}_\pi) = \pm\frac{B_0}{2\sqrt{2}}(\hat{a}_x\hat{a}_y^\dagger - \hat{a}_y\hat{a}_x^\dagger)\boldsymbol{k} \quad (10)$$

$$\mathrm{Re}(\hat{\boldsymbol{E}}_\pi) = \mp\frac{E_0}{2\sqrt{2}}(\hat{a}_x\hat{a}_y^\dagger - \hat{a}_y\hat{a}_x^\dagger)\boldsymbol{k} \quad (11)$$

a description which shows that the fields are generated by the simultaneous creation and annihilation of a quantum of energy $h\omega/2\pi$. They are therefore produced with no net change in energy in the quantized field. This argument is developed in Ref. 1, where it is shown that $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are consistent with the law of conservation of electromagnetic energy.

From Eqs. (5) and (9) it is clear that the novel fields $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are free space solutions of Maxwell's equations, and must be considered as being present in a vacuum in the same way as the usual oscillating solutions $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$. The source of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ is the same as that of $\boldsymbol{E}(\boldsymbol{r},t)$ and $\boldsymbol{B}(\boldsymbol{r},t)$. Therefore, $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are uniform, divergentless, and time-independent [1] but cannot be thought of in terms of conventional electrostatic and magnetostatic fields. This is clear from a description such as that of Landau and Lifshitz [13] of the classical conventional electrostatic field. We quote. "Electromagnetic fields occurring in vacuum in the absence of charges are called electromagnetic waves ... First of all we note that such fields must necessarily be time varying". Landau and Lifshitz then go on to assert that an electrostatic solution of Maxwell's equations which is not time varying is zero because of the absence of charges and currents in free space. Their argument is based on the relation

$$\boldsymbol{E}_\mathrm{s} = -\nabla\phi \quad (12)$$

between an electrostatic field and a scalar potential ϕ and the relation

$$\phi = \int\frac{\rho \mathrm{d}v}{R} \quad (13)$$

between the scalar potential and charge density ρ (the Poisson equation). Thus, if charge is zero then ϕ is zero and the conventional electrostatic field $\boldsymbol{E}_\mathrm{s}$ is zero. The argument is based on Coulomb's relation between $\boldsymbol{E}_\mathrm{s}$ and charge e

$$\boldsymbol{E}_\mathrm{s} = \frac{e\boldsymbol{R}}{R^3} \quad (14)$$

where $\boldsymbol{R}$ is the position vector of the point of interest relative to charge e.

Clearly, therefore, the fields $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ introduced and developed in Ref. 1 cannot be interpreted as electrostatic and magnetostatic fields in the conventional electrodynamical theory. Yet $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are, at the same time, mathematically valid solutions of Maxwell's phenomenological equations, and can, furthermore, be related to the oscillating solutions of those equations through Eqs. (5) and (9) of this paper. It is therefore necessary to provide a fundamentally new interpretation of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$, one which distinguishes these from the conventional interpretation of, for example, Landau and Lifshitz. It is emphasized that both $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ are physically meaningful electric and magnetic fields which propagate through free space, and which interact with matter to give observable phenomena such as optical NMR [2], in which $\boldsymbol{B}_\pi$ induces shifts in NMR resonances, shifts which are too large to be explained by hyperpolarizability induction mechanisms such as those used in the conventional theory of the inverse Faraday effect [1].

In this section we describe $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ classically in terms of Dirac delta functions which are moving in free space at the speed of light. Thus, more rigorously, we define the electric field density

$$\boldsymbol{E}_\pi^{(\mathrm{d})} = \frac{E_0}{\sqrt{2}}(\mathrm{i}-1)\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{k} \tag{15}$$

and the magnetic field density

$$\boldsymbol{B}_\pi^{(\mathrm{d})} = \frac{B_0}{\sqrt{2}}(\mathrm{i}+1)\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{k} \tag{16}$$

where

$$\delta(\boldsymbol{r}-\boldsymbol{r}_0) = \delta(x-x_0)\delta(y-y_0)\delta(z-ct) \tag{17a}$$

(Note that the magnetic field density $\boldsymbol{B}_\pi^{(\mathrm{d})}$ should not be confused with the magnetic flux density $\boldsymbol{B}_\pi$.) By definition of the Dirac delta function, the fields $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ vanish unless $x = x_0$, $y = y_0$, and $z = ct$, where c is the speed of light at an instant t. Here $\boldsymbol{k}$ for $\boldsymbol{E}_\pi$ is a polar vector in z, and for $\boldsymbol{B}_\pi$ an axial unit vector in z, the propagation axis of the laboratory frame (x, y, z). The scalar quantities E_0 and B_0 are amplitudes, as for the oscillating components of Eqs. (1) and (2).

By definition, the Dirac delta function obeys the relation

$$\int_{-\infty}^{\infty} f(x)\delta(x-a)\mathrm{d}x = f(a) \tag{17b}$$

so that the electric field $\boldsymbol{E}_\pi$ is an integral over the corresponding electric field density

$$\boldsymbol{E}_\pi(\boldsymbol{r}_0) = \int_{-\infty}^{\infty} \boldsymbol{E}_\pi(\boldsymbol{r})\delta(\boldsymbol{r}-\boldsymbol{r}_0)\mathrm{d}^3\boldsymbol{r} \tag{17c}$$

The limit of integration need not be $\pm\infty$: the range of integration can be arbitrary, provided it includes the point at which the δ function does not vanish.

It is seen that Eq. (17c) defines a field $\boldsymbol{E}_\pi(\boldsymbol{r}_0)$ which is differentiable, but which can be expressed as an integral over the corresponding field density function

$$\boldsymbol{E}_\pi(\boldsymbol{r}_0) = \int_{-\infty}^{\infty} \boldsymbol{E}_\pi^{(\mathrm{d})}\mathrm{d}^3\boldsymbol{r} \tag{17d}$$

Similarly

$$\boldsymbol{B}_\pi(\boldsymbol{r}_0) = \int_{-\infty}^{\infty} \boldsymbol{B}_\pi^{(\mathrm{d})}\mathrm{d}^3\boldsymbol{r} \tag{17e}$$

In this section we show that these definitions are consistent with the fundamental equations of electrodynamics and are interpretable as electro- and magnetostatic fields travelling at the velocity of light, so that z is always defined by ct and both fields vanish unless $z = ct$ at the point (x_0, y_0) in vacuo. The interaction of these fields with matter is therefore fundamentally different from the equivalent interactions of conventionally interpreted electrostatic and magnetostatic fields.

First, there exists in free space a scalar potential density

$$\begin{aligned}\phi_\pi^{(\mathrm{d})} &= \mp\frac{E_0}{\sqrt{2}}(\mathrm{i}-1)\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{k} \Leftrightarrow z = ct \\ \phi_\pi^{(\mathrm{d})} &= 0 \Leftrightarrow \text{otherwise}\end{aligned} \tag{18}$$

The potential ϕ_π is the integral over $\phi_\pi^{(\mathrm{d})}$ (cf. Eqs. (17d) and (17e)) such that

$$\boldsymbol{E}_\pi = -\nabla\phi_\pi \tag{19}$$

and this scalar potential exists in the absence of charges and currents. Similarly there exists a vector potential defined by

$$\boldsymbol{B}_\pi = \nabla\times\boldsymbol{A}_\pi \tag{20}$$

so that

$$\int_{\mathrm{S}} \nabla\times\boldsymbol{A}_\pi\mathrm{d}s = \oint_{\mathrm{C}} \boldsymbol{A}_\pi\cdot\mathrm{d}\boldsymbol{l} \tag{21}$$

with the vector potential density

$$A_{\pi}^{(d)} = \frac{B_0}{\sqrt{2}}(i+1)\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{k} \times \boldsymbol{R} \tag{22}$$

It is seen that the vector potential density is an annulus in free space travelling at the speed of light. Both $\phi_{\pi}^{(d)}$ and $A_{\pi}^{(d)}$ are defined without reference to a charge, and can therefore be nonzero in free space.

Similarly, Gauss's law in a source free region

$$\nabla \times \cdot \boldsymbol{E}_{\pi} = 0 \tag{23}$$

Faraday's law

$$\nabla \times \boldsymbol{E}_{\pi} = -\frac{\partial \boldsymbol{B}_{\pi}}{\partial t} \tag{24}$$

and Ampere's law

$$\nabla \times \boldsymbol{B}_{\pi} = \frac{1}{c^2}\frac{\partial \boldsymbol{E}_{\pi}}{\partial t} \tag{25}$$

are satisfied by $\boldsymbol{E}_{\pi}$ and $\boldsymbol{B}_{\pi}$ as defined in Eqs. (17d) and (17e).

The interaction of $\boldsymbol{E}_{\pi}^{(d)}$ and $\boldsymbol{B}_{\pi}^{(d)}$ with an electron is described through the use of their respective real parts in the Lorentz equation, so that the force density is split into two parts

$$\boldsymbol{F}^{(d)}(t) = \boldsymbol{F}_{\boldsymbol{E}}^{(d)}(t) + \boldsymbol{F}_{\boldsymbol{B}}^{(d)}(t) \tag{26}$$

the force between $\boldsymbol{E}_{\pi}$ and e and $\boldsymbol{B}_{\pi}$ and e. The two force density components are described by

$$\boldsymbol{F}_{\boldsymbol{E}}^{(d)}(t) = \frac{eE_0}{\sqrt{2}}\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{k} \tag{27}$$

and

$$\boldsymbol{F}_{\boldsymbol{B}}^{(d)}(t) = \frac{eB_0}{\sqrt{2}}\delta(\boldsymbol{r}-\boldsymbol{r}_0)\boldsymbol{V}_0 \times \boldsymbol{k} \tag{28}$$

Therefore, the electron is accelerated and reaches a velocity density

$$v^{(d)} = v_0^{(d)} + \frac{e}{m_e}\delta(x-x_0)\delta(y-y_0) \times \left[E_0\int \mathrm{d}t\delta(z-ct_0)\boldsymbol{k} + B_0\int \mathrm{d}t\delta(z-ct_0)\boldsymbol{V}_0 \times \boldsymbol{k}\right] \tag{29}$$

where $v_0^{(d)}$ is its initial velocity density. The trajectory of the electron is given by an equation in position density

$$r^{(d)} = r_0^{(d)} + v_0^{(d)}t + \frac{e}{m_e}\delta(x-x_0)\delta(y-y_0) \times E_0\left[\int\int_0^t \mathrm{d}t'\mathrm{d}t\delta(z-ct_0)\boldsymbol{k} + B_0\int\int_0^t \mathrm{d}t'\mathrm{d}t\delta(z-ct_0)\boldsymbol{V}_0 \times \boldsymbol{k}\right] \tag{30}$$

which can be expressed as

$$\boldsymbol{r}^{(d)} = \boldsymbol{r}_0^{(d)} + \boldsymbol{V}_0^{(d)}t + \frac{e}{m_e c}\delta(x-x_0)\delta(y-y_0) \times [E_0\boldsymbol{k} + B_0\boldsymbol{V}_0 \times \boldsymbol{k}] \tag{31}$$

Therefore, there is no effect on either $\boldsymbol{E}_{\pi}^{(d)}$ or $\boldsymbol{B}_{\pi}^{(d)}$ unless the conditions $x = x_0$, $y = y_0$, and $z = ct$ are satisfied simultaneously. This can be interpreted to mean that unless the travelling wave is defined by these conditions, there is no effect on the electron. Therefore, the photon and electron must both be at this position for interaction to occur. Thereafter the trajectory of the electron is described by Eq. (31). Therefore, if $\boldsymbol{E}_{\pi}^{(d)}$ and $\boldsymbol{B}_{\pi}^{(d)}$ are nonzero there should be an effect of a circularly polarized laser beam or X-ray beam on an electron beam. In particular an electron beam coaxial in z with a circularly polarized laser beam should be accelerated in z.

The quantum statistical nature of $\boldsymbol{E}_{\pi}$ and $\boldsymbol{B}_{\pi}$

It has been shown in Ref. 1 that both $\boldsymbol{E}_{\pi}$ and $\boldsymbol{B}_{\pi}$ can be interpreted rigorously in terms of creation and annihilation operators [4], and $\boldsymbol{B}_{\pi}$ can be related to the third Stokes operator [13] of the quantized field. Through this relation, $\boldsymbol{B}_{\pi}$ can be expressed directly in terms of the angular momentum of a single photon, defined as the expectation value of an angular momentum operator between the photon number states $\langle 1|$ and $|1\rangle$. In this formulation of quantum field theory, the interaction between the field operator $\boldsymbol{B}_{\pi}$ and an atom can be described in terms of an interaction hamiltonian operator

$$\hat{H}_I = \frac{B_0}{2}\hat{m}_0^1(\hat{a}_-\hat{a}_+^\dagger - \hat{a}_+\hat{a}_-^\dagger) \tag{32}$$

where $\hat{m}_0^1$ is the atom's magnetic dipole moment operator.

The Heisenberg equation of motion can be used to describe the interaction of $\boldsymbol{B}_\pi$ and the atom

$$\frac{\mathrm{d}}{\mathrm{d}z}\hat{a}_\pm(z) = -\frac{\mathrm{i}n}{\hbar c}[\hat{a}_\pm, \hat{H}_I] \tag{33}$$

Here the replacement of time by $-n_\omega Z/c$ has occurred, as described in Ref. 15, so that the Heisenberg equation becomes a description of the trajectory of the annihilation or creation operator in the coordinate z. Using the commutator relations

$$\begin{aligned} &[\hat{a}_+, \hat{a}_-\hat{a}_+^\dagger - \hat{a}_+\hat{a}_-^\dagger] = \hat{a}_- \\ &[\hat{a}_-, \hat{a}_-\hat{a}_+^\dagger - \hat{a}_+\hat{a}_-^\dagger] = -\hat{a}_+ \\ &[\hat{a}_+^\dagger, \hat{a}_-\hat{a}_+^\dagger - \hat{a}_+\hat{a}_-^\dagger] = \hat{a}_-^\dagger \\ &[\hat{a}_-^\dagger, \hat{a}_-\hat{a}_+^\dagger - \hat{a}_+\hat{a}_-^\dagger] = -\hat{a}_+^\dagger \end{aligned} \tag{34}$$

leads to the equations of motion

$$\begin{aligned} &\frac{\mathrm{d}}{\mathrm{d}z}\hat{a}_+(z) = \mathrm{i}A\hat{a}_-(z) \\ &\frac{\mathrm{d}}{\mathrm{d}z}\hat{a}_-(z) = -\mathrm{i}A\hat{a}_+(z) \\ &A = -\frac{n_\omega B_0 m_0^1}{2\hbar c} \end{aligned} \tag{35}$$

which have the solution

$$\hat{a}_+(Z) + \mathrm{i}\hat{a}_-(Z) = \mathrm{e}^{Az}[\hat{a}_+(0) + \mathrm{i}\hat{a}_-(0)] \tag{36}$$

Similarly

$$\hat{a}_+^\dagger(z) + \mathrm{i}\hat{a}_-^\dagger(z) = \mathrm{e}^{Az}[\hat{a}_+^\dagger(0) + \mathrm{i}\hat{a}_-^\dagger(0)] \tag{37}$$

for the creation operators. Separating real and imaginary parts gives

$$\hat{a}_\pm(z) = \mathrm{e}^{Az}\hat{a}_\pm(0) \qquad \hat{a}_\pm^\dagger(z) = \mathrm{e}^{Az}\hat{a}_\pm^\dagger(0) \tag{38}$$

The existence of squeezing effects in the interaction of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ with an atom with a magnetic dipole moment operator $\hat{m}_0^1$ can now be investigated with the usual time ordered and normalized quadrature parameters [14, 15]

$$\begin{aligned} \langle(\Delta\hat{Q})^2\rangle = {} & \langle(\Delta\hat{a})^2\rangle + \langle[(\Delta\hat{a})^\dagger]^2\rangle \\ & + 2(\langle\hat{a}^\dagger\hat{a}\rangle - \langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle) \end{aligned} \tag{39}$$

and

$$\begin{aligned} \langle(\Delta\hat{P})^2\rangle = {} & (-\langle(\Delta\hat{a})^2\rangle) - \langle[(\Delta\hat{a})^\dagger]^2\rangle \\ & + 2(\langle\hat{a}^\dagger\hat{a}\rangle - \langle\hat{a}^\dagger\rangle\langle\hat{a}\rangle) \end{aligned} \tag{40}$$

squeezing being defined when one of these is less than zero.

Fourier analysis of the delta function representation

By definition, the Dirac delta function is a Fourier integral of the type

$$\delta(\boldsymbol{r} - \boldsymbol{r}_0) = \frac{1}{(2\pi)^3}\int \mathrm{e}^{\mathrm{i}\kappa\cdot\boldsymbol{r}}\mathrm{e}^{-\mathrm{i}\kappa\cdot\boldsymbol{r}_0}\mathrm{d}\kappa \tag{41}$$

which, applied to an electromagnetic plane wave implies that $\boldsymbol{\kappa}$ is the wave vector. At the point $\boldsymbol{r} = \boldsymbol{r}_0$, the Dirac delta function is therefore an integral over all wave vectors, or modes, of the electromagnetic plane wave. For a mode u_κ, Eq. (41) can be rewritten as

$$\delta(\boldsymbol{r} - \boldsymbol{r}_0) = \int u_\kappa(\boldsymbol{r})u_\kappa^*(\boldsymbol{r}_0)\mathrm{d}\boldsymbol{\kappa} \tag{42}$$

The field densities $\boldsymbol{E}_\pi^{(d)}$ and $\boldsymbol{B}_\pi^{(\mathrm{d})}$ can therefore be rewritten in terms of scalar mode expansions of the plane wave

$$\begin{aligned} &B_0\boldsymbol{k}\delta(x - x_0) = (B_0\boldsymbol{k})\int u_{\boldsymbol{\kappa}}(x)u_{\boldsymbol{\kappa}}^*(x_0)\mathrm{d}\boldsymbol{\kappa} \\ &B_0\boldsymbol{k}\delta(y - y_0) = (B_0\boldsymbol{k})\int u_{\boldsymbol{\kappa}}(y)u_\kappa^*(y_0)\mathrm{d}\boldsymbol{\kappa} \\ &B_0\boldsymbol{k}\delta(z - ct) = (B_0\boldsymbol{k})\int u_{\boldsymbol{\kappa}}(z)u_{\boldsymbol{\kappa}}^*(ct_0)\mathrm{d}\boldsymbol{\kappa} \end{aligned} \tag{43}$$

Therefore it becomes clear that the representation of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$ in terms of products of delta functions is equivalent to an integration over $\boldsymbol{\kappa}$ of the scalar basis field product $u_{\boldsymbol{\kappa}}(\boldsymbol{r})u_{\boldsymbol{\kappa}}^*(\boldsymbol{r}_0)$.

In the quantum field [14] the integrals are replaced by sums

$$B_0 k\delta(r - r_0) = B_0 k \sum_{\kappa} u_\kappa(r) u^*_\kappa(r_0) \quad (44)$$

over modes of the type

$$u_\kappa(r) = \frac{e^{i\kappa\cdot r}}{V} \qquad u^*_\kappa(_0) = \frac{e_0^{-i\kappa\cdot r}}{V} \quad (45)$$

where V is the quantum volume, and this provides an opportunity of linking conceptually the delta representations given in Eqs. (15) and (16) of the classical field with representations in terms of creation and annihilation operators (Eqs. (10) and (11)) of the quantum field theory. This is accomplished through a mode expansion of the quantum field in free space, using the representation [14]

$$\hat{E}(r,t) = \hat{E}^{(+)}(r,t) + \hat{E}^{(-)}(r,t)$$
$$\hat{E}^{(+)}(r,t) = \sum_\lambda C_\lambda \underline{u}_\lambda(r)\hat{a}_\lambda(t) \quad (46)$$
$$\hat{E}^{(-)}(r,t) = \sum_\lambda C^*_\lambda \underline{u}^*_\lambda(r)\hat{a}^\dagger_\lambda(t) = [\hat{E}^{(+)}(r,t)]^*$$

where C_λ and C^*_λ are proportionality constants. It is assumed in these expansions that the spatial dependence occurs in the mode fields $\underline{u}_\lambda(r)$ while the time dependence and quantum mechanical operator properties occur in $\hat{a}_\lambda$ and $\hat{a}^\dagger_\lambda$

$$\hat{a}_\lambda(t) = \exp(-ick_\lambda t)\hat{a}_\lambda(0)$$
$$\hat{a}^\dagger_\lambda(t) = \exp(ick_\lambda t)\hat{a}^\dagger_\lambda(0) \qquad \omega_\lambda = ck_\lambda \quad (47)$$

These are oscillator variables with frequencies ω_λ fixed by the eigenvalues of the Helmholtz equation, i.e. by the spatial properties of the chosen basis fields. Classically, they are generalized spatial Fourier components; quantum mechanically, they become operators in the Heisenberg picture, Eq. (33) satisfying the equal time commutator

$$[\hat{a}_\lambda(t), \hat{a}^\dagger_\lambda(t)] = \delta(\lambda, \lambda') \quad (48)$$

Thus $\hat{a}_\lambda$ and $\hat{a}^\dagger_\lambda$ are annihilation and creation operators for photons in the mode λ. Using the concept of mean number (n_λ) of photons in mode λ, defined in the quantum field by

$$\langle \hat{a}_\lambda \hat{a}^\dagger_\lambda \rangle = \bar{n}_\lambda \delta(\lambda, \lambda') \quad (49)$$

where δ is a Kronecker delta function, we arrive at a definition of $\langle \hat{\underset{\sim}{B}}_\pi \rangle$ based on the quantum equivalent of Eq. (3), i.e.

$$\langle \hat{\underset{\sim}{B}}_\pi \rangle = \frac{1}{\epsilon_0 E_0 c} \sum_\lambda \hbar\omega_\lambda \check{}_\lambda(r) \times \underline{u}^*_\lambda(r)\bar{n}_\lambda$$
$$= \frac{\langle \hat{\underset{\sim}{E}}^{(+)} \times \hat{\underset{\sim}{E}}^{(-)} \rangle}{E_0 c} \quad (50)$$

where $\underline{u}_\lambda$ and $\underline{u}^*_\lambda$ are vector quantities in mode λ, and where the summation is over all modes λ. Replacing this sum by an integral, and reverting to the classical field, it becomes apparent that the description given in Eq. (16) is the equivalent of the field $\boldsymbol{B}_\pi$, in classical terms, through the delta function defined by Fourier transformation in Eq. (41).

The conclusion of this section is that the description of $\boldsymbol{B}_\pi$ and $\boldsymbol{E}_\pi$ in terms of creation and annihilation operators has its classical equivalent in terms of the delta function description of Eqs. (15) and (16).

Discussion

The $\boldsymbol{B}_\pi$ field is the expectation value between photon eigenstates of the field operator $\hat{\boldsymbol{B}}_\pi$, which can be expressed [1] in terms of creation and annihilation operators as

$$\hat{\boldsymbol{B}}_\pi = \frac{B_0}{2}(\hat{a}_x\hat{a}^\dagger_y - \hat{a}_y\hat{a}^\dagger_x)\boldsymbol{k} \quad (51)$$

For one electromagnetic mode, this is equivalent to the delta function definition introduced in the third section (The quantum statistical nature of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$). This can be shown by separating the classical $\boldsymbol{B}_\pi$ as

$$\boldsymbol{B}_\pi(z_0) = \frac{B_0}{2\pi}\boldsymbol{k}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty} e^{ikz}e^{-ikz_0}\,d\kappa\,dz \quad (52)$$

where

$$u_\kappa \equiv u_\kappa(0)\frac{e^{i\kappa z}}{\sqrt{2\pi}} \qquad u^*_\kappa \equiv u^*_\kappa(0)\frac{e^{-i\kappa z_0}}{\sqrt{2\pi}} \quad (53)$$

are modes (see the fourth section (Fourier analysis of the delta function representation)) of the plane wave. Using the methods of the fourth section, the transition from the classical to the quantum field is accomplished by replacing the integral in Eq. (52) with sums over photon modes, giving

$$\hat{B}_\pi(z_0) = \left(\sum_z \sum_\kappa \hat{\boldsymbol{u}}_\kappa(z_0)\hat{\boldsymbol{u}}^*_\kappa(z_0)\right) B_0 \boldsymbol{k} \tag{54}$$

Therefore the quantum operator $\hat{\boldsymbol{B}}_\pi$ is the sum over all modes labelled $\boldsymbol{\kappa}$, and all points labelled z, of the photon beam, multiplied by the amplitude B_0 and the axial unit vector $\boldsymbol{k}$ (not to be confused with the wave number κ).

For an electromagnetic wave in free space, it is possible to make the identities

$$\kappa = \frac{\omega}{c} \qquad z = ct \tag{55}$$

where ω is the angular frequency and c is the velocity of light. The modes $\hat{\boldsymbol{u}}_\kappa(z_0)$ and $\hat{\boldsymbol{u}}^*_\kappa(z_0)$ can therefore be written as

$$u_\omega \equiv u_\omega(0)\frac{e^{i\omega z}}{\sqrt{2\pi}} \qquad u^*_\omega \equiv u^*_\omega(0)\frac{e^{-i\omega z_0}}{\sqrt{2\pi}} \tag{56}$$

Equation (56) can be compared with the definitions of creation and annihilation operators in quantum field theory

$$\hat{a}^\dagger(t) = \hat{a}^\dagger(0)e^{i\omega t} \qquad \hat{a}(t_0) = \hat{a}(0)e^{-i\omega t_0} \tag{57}$$

We can compare Eqs. (56) and (57) to give

$$\begin{aligned} &\hat{u}_\kappa = \hat{a}^\dagger && \hat{u}_\kappa = \hat{a}^\dagger \\ &\frac{u_\kappa(0)}{\sqrt{2\pi}} \equiv \hat{a}^\dagger(0) && \frac{u^*_\kappa(0)}{\sqrt{2\pi}} \equiv \hat{a}(0) \end{aligned} \tag{58}$$

so that

$$\sum_z \sum_\kappa \hat{\boldsymbol{u}}_\kappa(z_0)\hat{\boldsymbol{u}}^*_\kappa(z_0) \equiv \sum_t \sum_\omega \hat{\boldsymbol{a}}^\dagger(t)\hat{\boldsymbol{a}}(t_0) \tag{59}$$

If the wave is monochromatic, ω is a fixed angular frequency, and the sum Σ_ω becomes superfluous. At this instant $t = t_0$ the sum over t becomes superfluous because we are concerned with one instant only, defined at $t = t_0$.

This implies

$$\hat{\boldsymbol{B}}_\pi(t = t_0) = B_0\hat{\boldsymbol{a}}^\dagger(t)\hat{\boldsymbol{a}}(t_0)\boldsymbol{k} \tag{60}$$

Equation (51) is a particular case of Eq. (60) when the combination of components is defined by

$$\hat{\boldsymbol{a}}^\dagger(t)\hat{\boldsymbol{a}}(t_0) \equiv \tfrac{1}{2}(\hat{\boldsymbol{a}}_x\hat{\boldsymbol{a}}^\dagger_y - \hat{\boldsymbol{a}}_y\hat{\boldsymbol{a}}^\dagger_x) \equiv \hat{\boldsymbol{a}}_x\hat{\boldsymbol{a}}^\dagger_y \tag{61}$$

where we have used [1] the result

$$\hat{a}_x\hat{a}^\dagger_y = -\hat{a}_y\hat{a}^\dagger_x \tag{62}$$

which comes from the result [1] that the classical $\boldsymbol{B}_\pi$ is defined through a vector cross product (see Eq. (5)).

It is concluded that the quantum field theoretical description (Eq. (51)), which is a consequence of Eqs. (5) and (57), is equivalent to the classical description in terms of the delta function obtained in the third section (The quantum statistical nature of $\boldsymbol{E}_\pi$ and $\boldsymbol{B}_\pi$) provided that the wave is monochromatic and provided that it is considered at the instant $t = t_0$. The latter condition is equivalent to removing the phase dependence of the wave in the cross product (Eq. (5)) of $\boldsymbol{E}$ and its complex conjugate $\boldsymbol{E}^*$.

When discussing the physical meaning of $\boldsymbol{B}_\pi$ and $\boldsymbol{E}_\pi$ it is significant to recall that in the relativistic quantum theory of fields [16] the photon has four polarizations in general, but only two helicities. It can be shown [16] that $\boldsymbol{B}_\pi$ and $\boldsymbol{E}_\pi$ can be related to physically meaningful expectation values over timelike (0) and longitudinal, spacelike (1) polarization of the photon, in such a way that

$$\begin{aligned} &\boldsymbol{B}_\pi = B_0\hat{\boldsymbol{k}} && \boldsymbol{E}_\pi = E_0\hat{\boldsymbol{k}} \\ &\boldsymbol{B}_\pi \cdot \boldsymbol{B}_\pi = B_0^2 && \boldsymbol{E}_\pi \cdot \boldsymbol{E}_\pi = E_0^2 \end{aligned} \tag{63}$$

We have therefore shown that $\boldsymbol{B}_\pi$ and $\boldsymbol{E}_\pi$ are physically meaningful magnetic and electric fields and have self-consistently defined these fields in the classical and quantum theories.

Application to the structure of atoms and molecules

Having shown that $\hat{\boldsymbol{B}}_\pi$ is a physically meaningful field in free space it is possible to use it to investigate spectroscopically the structure of atoms and molecules using several new techniques based on the interaction of $\hat{\boldsymbol{B}}_\pi$ with the atomic and molecular magnetic dipole moment $\hat{\boldsymbol{m}}$. The analytical spectroscopic techniques using $\hat{\boldsymbol{B}}_\pi$, for example, parallel those using a permanent magnetostatic field $\boldsymbol{B}$. Examples include optical NMR spectroscopy [2, 6], the optical Zeeman and anomalous optical Zeeman effects, optical ESR, the inverse Faraday effect (magnetization due to $\hat{\boldsymbol{B}}_\pi$), the optical Faraday effect (azimuth rotation due to $\hat{\boldsymbol{B}}_\pi$), the optical Cotton–Mouton effect and Majorana effects due to $\hat{\boldsymbol{B}}_\pi$, and optical forward backward birefringence due to $\hat{\boldsymbol{B}}_\pi$. All of these techniques rely on molecular property tensors, and therefore provide novel information on atomic and molecular structure. The source of this novelty is that $\hat{\boldsymbol{B}}_\pi$, unlike a conventional magnetostatic field, is in quantum theory an operator, so that the hamiltonian formed between $\hat{\boldsymbol{B}}_\pi$ and $\hat{\boldsymbol{m}}$, for example is a product of operators. The use of $\hat{\boldsymbol{B}}_\pi$ for investigating atomic structure is illustrated in this section with the optical Zeeman and anomalous Zeeman effects (spectral splitting due to $\hat{\boldsymbol{B}}_\pi$ of a circularly polarized laser beam) and with the optical Faraday effect (azimuth rotation due to a circularly polarized laser beam).

The optical Zeeman effect due to $\hat{\boldsymbol{B}}_\pi$

In the context of atomic structure and spectral absorption lines, we consider in this first example the splitting due to the operator $\hat{\boldsymbol{B}}_\pi$ of an atomic $^1S \rightarrow {}^1P$ optical frequency absorption line by a circularly polarized pump laser. The interaction energy between the pump laser and the atom is the expectation value

$$\Delta\mathscr{E} = -\langle LJFM_F|\hat{\boldsymbol{m}}\cdot\hat{\boldsymbol{B}}_\pi|L'J'F'M'_F\rangle \tag{64}$$

where $\hat{\boldsymbol{m}}$ is the magnetic dipole moment operator of the atom, proportional to an orbital angular momentum operator $\hat{\boldsymbol{L}}$

$$\hat{\boldsymbol{m}} = \gamma_e\hat{\boldsymbol{L}} \tag{65}$$

through the gyromagnetic ratio γ_e as usual. The quantum number F in Eq. (64) is defined by the usual Clebsch–Gordan condition

$$F = L + J, \ldots, |L - J| \qquad M_F = M_L + M_J \tag{66}$$

Using standard procedures it can be shown that Eq. (64) can be written in the form

$$\Delta\mathscr{E}_\pi = -g_L\gamma_e|\hat{\beta}_\pi|\hbar \tag{67}$$

where the Landé factor

$$g_L = \tfrac{1}{2}[F(F+1) - J(J+1) - L(L+1)] \tag{68}$$

is recognizable as that given by a simple vector coupling model of the theory of atomic structure in which the L and J quantum numbers of the atom's $\hat{\boldsymbol{m}}$ operator and the photon's $\hat{\boldsymbol{B}}_\pi$ operator have been considered. A parallel development for molecules is necessarily more complicated but essentially follows the same principles.

Novel information about atomic structure is therefore provided by the splitting pattern generated by $\hat{\boldsymbol{B}}_\pi$ of a circularly polarized pump laser. For example, in the ^{1}S ground state we have $L = 0$ and we assume that $J = 1$ for the photon. Therefore $F = 1$ in the ^{1}S state. In the ^{1}P state, $L = 1$, $J = 1$, and $F = 2, 1, 0$. Transitions occur between $F = 1$ ground state (^{1}S) and the three F states of ^{1}P. Therefore, we observe three lines in this simple example of an optical Zeeman effect in atoms. In this type of spectrum, the circularly polarized laser acts as a light magnet, which replaces the permanent magnet of the well known conventional Zeeman effect. Furthermore, the splitting pattern is different from that for the same atomic transition in the conventional Zeeman effect. In the optical Zeeman effect the pattern is

$$\begin{aligned}\Delta\mathscr{E}_\pi(F=0) &= 2\gamma_e|\hat{\beta}_\pi|\hbar \\ \Delta\mathscr{E}_\pi(F=1) &= \gamma_e|\hat{\beta}_\pi|\hbar \\ \Delta\mathscr{E}_\pi(F=2) &= -\gamma_e|\hat{\beta}_\pi|\hbar\end{aligned} \tag{69}$$

and in the conventional Zeeman effect it is

$$\Delta\mathscr{E}(M_L = 1) = -\gamma_e|\boldsymbol{B}|\hbar$$
$$\Delta\mathscr{E}(M_L = 0) = 0 \tag{70}$$
$$\Delta\mathscr{E}(M_L = -1) = \gamma_e|\boldsymbol{B}|\hbar$$

We arrive at the important conclusion that $\hat{\boldsymbol{B}}_\pi$ generates a different pattern of Zeeman lines from $\boldsymbol{B}$ of a conventional magnet.

This means that the optical Zeeman effect is a new source of information on atomic and molecular structure as studied by spectroscopic methods in the gaseous phase of matter.

The anomalous optical Zeeman and optical Paschen–Back effects

In the anomalous optical Zeeman and Paschen–Back effects, the structure of an atom or molecule is further elucidated by the involvement of electronic spin angular momentum, and the spin quantum number S. The interaction energy (Eq. (64)) is therefore modified to

$$\Delta\mathscr{E}^{(A)} = -\frac{\gamma_e}{\hbar}\langle SLJJ_1FM_F|\hat{\boldsymbol{m}}\cdot\hat{\boldsymbol{B}}_\pi|S'L'J'J'_1F'M'_F\rangle \tag{71}$$

where the magnetic dipole moment of the atom is developed as

$$\hat{\boldsymbol{m}} = \gamma_e(\hat{\boldsymbol{L}} + 2.002\hat{\boldsymbol{S}}) \tag{72}$$

The theoretical prediction of the splitting pattern expected in the anomalous optical Zeeman effect depends on the coupling scheme chosen for the various electronic angular momenta of the atom, and therefore depends on the model chosen for the atomic structure. In other words, the splitting pattern of the anomalous optical Zeeman effect depends on the nature of the Landé factors emerging from the particular angular momentum coupling scheme chosen to evaluate Eq. (71). The most appropriate scheme can be elucidated only by reference to experimental data, generated by splitting an atomic absorption line with a circularly polarized pump laser generating $\hat{\boldsymbol{B}}_\pi$.

This shows clearly that $\hat{\boldsymbol{B}}_\pi$ provides novel information on atomic structure, in particular the way in which the atomic angular momenta L, S, and J interact in a coupling scheme. It is emphasized that the splitting pattern is different from that of the conventional Zeeman effect because $\hat{\boldsymbol{B}}_\pi$ is an operator of quantum field theory, and not a simple vectorial magnetostatic field.

The optical Faraday effect due to $\hat{\boldsymbol{B}}_\pi$; optical MCD

In the optical Faraday effect the operator $\hat{\boldsymbol{B}}_\pi$ from a circularly polarized pump laser rotates the plane of polarization of a probe laser which is linearly polarized. In this context information is provided about the internal structure of atoms and molecules through the molecular property tensors controlling the optical Faraday effect. The optical Faraday effect due to $\hat{\boldsymbol{B}}_\pi$ is ubiquitous, i.e. it occurs in all atoms and molecules in the gaseous, liquid and solid states of matter. The generator of the $\hat{\boldsymbol{B}}_\pi$ operator is the circularly polarized pump laser, which can be pulsed to high intensity. The effect is, furthermore, dependent on the frequency of the linearly polarized probe laser, so that there is generated, in principle, an optical magnetic circular dichroism across a spectrum of frequencies. This provides a new analytical technique of general interest, which can be referred to as "optical MCD". The origin of optical MCD in essence can be traced to the semi-classical time-dependent perturbation theory, which produces expressions for the polarizability components as given in the conventional theory of reduced matrix elements of electric and magnetic transition dipole moment operators. For a given circularly polarized pump laser frequency the optical MCD spectrum is a plot of the $\hat{\boldsymbol{B}}_\pi$ induced angle of rotation against the frequency of the linearly polarized probe laser. In other words optical MCD is produced in essence by replacing the magnet of the conventional MCD technique by a circularly polarized pump laser.

Again, it is important to note that the optical MCD spectrum is expected to be different in general from the MCD spectrum under the same conditions because the former is generated by the operator $\hat{\boldsymbol{B}}_\pi$ (described in this paper) and the latter by a magnetostatic field (a vector).

Optical NMR spectroscopy

This has been confirmed experimentally, in essence, and has been reported in the literature [2]. It is the site-specific shift of NMR resonances by a circularly polarized laser. The effect is small but significant, because it immediately introduces the possibility of a new analytical technique in which the permanent magnetic field of the NMR instrument is supplemented by the $\hat{\boldsymbol{B}}_\pi$ operator of a circularly polarized laser. Optical NMR provides, of course, novel and useful information on the structure of atoms and molecules, including complex molecules such as proteins in solution.

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