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FORMS OF THE EVANS-VIGIER FIELD $B^{(3)}$

M. W. Evans

Department of Physics
University of North Carolina
Charlotte, North Carolina 28223

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The newly inferred longitudinal magnetic field of vacuum electromagnetism is given in a number of equivalent forms derived in several different ways. It is therefore overwhelmingly likely that the Evans-Vigier field $B^{(3)}$ will be isolated experimentally through its characteristic square root power density dependence. It is the first classical field of vacuum electromagnetism to be inferred since Maxwell and as such fundamentally extends our understanding of the nature of electromagnetism and field-particle theory.

Key words: $B^{(3)}$ field, vacuum electromagnetism.

1. INTRODUCTION

Our understanding of vacuum electromagnetism has been augmented recently by the inference [1-11] that there exists, in addition to the ordinary plane waves [13-15], a spin field $B^{(3)}$ which is a phase-free magnetic flux density, an axial vector oriented in the axis of propagation of the electromagnetic beam. It can therefore be referred to as a longitudinal magnetic field, meaning that the quantum of energy, the photon, carries mass [12]. The Evans-Vigier field $B^{(3)}$ can be isolated experimentally [7] in principle through observation of its characteristic dependence on the square root of the beam power density in magneto-optic phenomena. An example is the magnetization of an electron plasma [16] with microwave pulses of high power density under conditions which can be defined precisely from the relativistic Hamilton-Jacobi (HJ) equation [7] of one electron e in the electromagnetic field, represented by the four potential A_μ. The demonstration of $\boldsymbol{B}^{(3)}$ is also a demonstration of finite photon mass and is therefore of great interest.

In this Letter, several forms for $\boldsymbol{B}^{(3)}$ are summarized (Table 1) using different methods of derivation. In total, ten forms of the Evans-Vigier field are deduced and tabulated. Its origins are therefore understood with precision in the classical and quantum theories. Section 2 initiates the analysis with the form for $\boldsymbol{B}^{(3)}$ as originally inferred [1] from the conjugate product of non-linear optics [8], i.e., the cross product of two electric fields, plane waves in the circular basis. Section 3 develops these classical, *double field*, forms for use with the quantum theory using the fundamental theory of non-Abelian, O(3), gauge geometry [17], which leads to the charge quantization condition. The latter, in turn, is used to express $\boldsymbol{B}^{(3)}$ in a Biot-Savart-Ampère (BSA) form, and other forms such as that derivable from the Dirac equation of one electron in the electromagnetic field [7]. These forms are discussed in Sec. 4, and the overall inference made that the source of $\boldsymbol{B}^{(3)}$ in the vacuum are the plane waves themselves. This statement can be expressed in BSA form, or equivalently in a *curl A* form, both being well known classical methods of expressing the origin of a magnetic field. The $\boldsymbol{B}^{(3)}$ field propagates through the vacuum because the plane waves propagate, the source of $\boldsymbol{B}^{(3)}$ being the plane waves themselves.

2. CONJUGATE PRODUCTS AS SOURCES OF $\boldsymbol{B}^{(3)}$ IN THE VACUUM

The $\boldsymbol{B}^{(3)}$ field was inferred originally through the equation [1]

$$\boldsymbol{B}^{(3)*} = -\frac{i}{cE^{(0)}}\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} \tag{1}$$

where $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$ is an ordinary vacuum plane wave, in this case of the electric field strength, with scalar amplitude $E^{(0)}$. Here c is the speed of light in vacuo for the hypothetically massless field. Equation (1) is expressed in a circular basis [7] defined by the unit vectors

$$\mathbf{e}^{(1)} = \mathbf{e}^{(2)*} = \frac{1}{\sqrt{2}}(\mathbf{i} - i\mathbf{j}), \qquad \mathbf{e}^{(3)} = \mathbf{k}, \tag{2}$$

where $\mathbf{i}$, $\mathbf{j}$ and $\mathbf{k}$ are the usual Cartesian unit vectors, $\mathbf{k}$ being in the propagation axis. The vector product $\mathbf{E}^{(1)} \times \mathbf{E}^{(2)}$ is the ordinary conjugate product of non-linear optics [8], through which magneto-optic phenomena such as the inverse Faraday effect are routinely explained [18]. The form (1) is listed as *double E* in Table 1, and was the first to be discovered chronologically [1]. However, it does not seem to resemble, at first, the classical equations for the origin of a magnetic field, such as the Biot-Savart-Ampère (BSA) law [13]. In Sec. 3, it is shown that Eq. (1) is indeed a BSA law, given in Table 1 as the *BSA* form for $\mathbf{B}^{(3)}$. It is straightforward to deduce [2] from Eq. (1) that

$$\mathbf{B}^{(3)*} = -\frac{i}{B^{(0)}}\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} \tag{3}$$

where $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$ is the ordinary vacuum plane wave of magnetic flux density, with amplitude $B^{(0)}$, the *double B* form of Table 1; and

$$\mathbf{B}^{(3)*} = -i\frac{\kappa}{A^{(0)}}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{4}$$

which is the *double A* form. Here the curl of $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$ is defined as $\mathbf{B}^{(1)} = \mathbf{B}^{(2)*}$ and $\mathbf{A}^{(1)}$ is therefore another plane wave, a vector potential [7]. In Eq. (4) κ is the wave vector magnitude and $A^{(0)}$ the amplitude associated with $\mathbf{A}^{(1)} = \mathbf{A}^{(2)*}$.

3. FORMS OF $\mathbf{B}^{(3)}$ IN THE QUANTIZED FIELD

The forms discussed in Sec. 2 are purely classical, and could have been deduced at the time of Maxwell. The fundamental and widely known axioms of the quantum theory assert, however, that

$$\nabla = \frac{i}{\hbar}\mathbf{p}, \qquad \frac{\partial}{\partial t} = -\frac{i}{\hbar}En, \tag{5}$$
$$En = \hbar\omega, \qquad \mathbf{p} = \hbar\boldsymbol{\kappa},$$

so it is possible to write transverse momenta in the circular basis as wave vectors and del operators. For example

$$\boldsymbol{p}^{(1)} = \hbar\boldsymbol{\kappa}^{(1)} = -i\hbar\nabla^{(1)}, \qquad \boldsymbol{p}^{(2)} = \hbar\boldsymbol{\kappa}^{(2)} = -i\hbar\nabla^{(2)}. \tag{6}$$

The origin of these transverse momenta is the spinning motion of the classical field in the vacuum as it propagates. These axioms link together undulatory and particulate concepts of light and matter [7], as inferred originally by de Broglie in the famous concept of wave-particle duality. The classical forms for the Evans-Vigier field in Sec. 2 show that $\boldsymbol{B}^{(3)}$ is a physical observable in an axis *orthogonal* to the plane of definition of the waves, and so the gauge group [17] of electromagnetism becomes the non-Abelian group of rotations in three dimensions, O(3). This replaces the conventional O(2) = U(1), an inference which is one of the fundamental consequences of $\boldsymbol{B}^{(3)}$. The fundamental theory of O(3) gauge geometry [17] leads to the result [7]

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{7}$$

in which appears the Dirac constant $\hbar$. Therefore Eq. (7) is an expression of the quantum field theory, and is a form for $\boldsymbol{B}^{(3)}$ which is a direct consequence [7] of the basic theory of O(3) gauge geometry itself. In this form, labelled as *O(3) gauge* in Table 1, e can be interpreted in two ways: a) the coupling constant of the O(3) gauge; b) the charge on the electron. Comparison of Eqs. (4) and (7) leads to the charge quantization condition

$$\hbar\kappa = eA^{(0)}, \tag{8}$$

which means that the classical equivalent of the magnitude of the photon momentum $\hbar\kappa$ is $eA^{(0)}$. The physical meaning of Eq. (8) is discussed further elsewhere [19], and it has been derived independently [20] using the HJ equation. Using Eq. (8), the de Broglie matter-wave equation emerges consistently [21] from the relativistic factor of the HJ equation.

Equation (8) means that the transverse momentum $\boldsymbol{p}^{(1)}$ in the vacuum is proportional to the plane wave $\boldsymbol{A}^{(1)}$

$$\boldsymbol{p}^{(1)} = e\boldsymbol{A}^{(1)} = \boldsymbol{p}^{(2)*}. \tag{9}$$

Use of this in the O(3) gauge form (7) produces the *Dirac* form (Table 1) of the Evans Vigier field

$$B^{(3)*} = -\frac{i}{\hbar}p^{(1)} \times A^{(2)}. \tag{10}$$

This form is so called because it is identical with the expression for $B^{(3)}$ that emerges directly from the Dirac equation of e in A_μ [7], in which $p^{(1)}$ is an operator. This inference independently demonstrates the validity of the charge quantization condition (8), which links consistently the O(3) gauge equation (7) and quantum mechanical (10), *both of which are independently derivable from first principles.*

The first classical law for the origin of a magnetic field is that of Biot and Savart, greatly elaborated by Ampère [13]. So we refer to it as the BSA law. The latter can be derived directly from the double A form of $B^{(3)}$, Eq. (4), by using [22]

$$A^{(2)} = \frac{E^{(2)}}{i\omega}, \tag{11}$$

which gives the intermediate *AE* form listed in Table 1.

$$B^{(3)*} = -\frac{1}{cA^{(0)}}A^{(1)} \times E^{(2)}. \tag{12}$$

If the charge quantization condition in the form of Eq. (9) is used in Eq. (12), and if $p^{(0)} = eA^{(0)}$ is used by definition, the BSA form for $B^{(3)}$ is obtained straightforwardly,

$$B^{(3)*} = -\frac{1}{cp^{(0)}}p^{(1)} \times E^{(2)}, \tag{13}$$

and identifies the field $B^{(3)}$ of the vacuum in classical terms, as the cross product of a transverse momentum and a transverse electric field. This shows clearly that the source of $B^{(3)}$ exists within the propagating electromagnetic wave *itself*, the transverse momentum being directly proportional to a transverse vector potential $A^{(1)}$ through the charge quantization condition. Thus e is a proportionality constant between $p^{(1)}$ and $A^{(1)}$ as well as the charge on the electron. Therefore, the vector potential itself takes on physical meaning, as in the Aharonov-Bohm effect [7].

Another well known classical result is that any magnetic field is the curl of a vector potential. The *curl A* form of B (Table 1) is obtained from the double A form, Eq. (4), by using the charge quantization condition in the form

$$e\boldsymbol{A}^{(1)} = -i\hbar\boldsymbol{\nabla}^{(1)}, \tag{14}$$

where $\boldsymbol{\nabla}^{(1)}$ is a phase-dependent curl operator in the circular basis (2) rather than the usual Cartesian basis. Equation (14) results immediately in

$$\boldsymbol{B}^{(3)*} = -\boldsymbol{\nabla}^{(1)} \times \boldsymbol{A}^{(2)}, \tag{15}$$

which shows that $\boldsymbol{B}^{(3)}$ is the curl of $\boldsymbol{A}^{(2)}$ in the circular basis (2).

Finally, by using the charge quantization condition in wave-vector form

$$e\boldsymbol{A}^{(1)} = \hbar\boldsymbol{\kappa}^{(1)}, \tag{16}$$

where $\boldsymbol{\kappa}^{(1)} = \boldsymbol{\kappa}^{(2)*}$ is a transverse wave vector in the circular basis, not to be confused with the ordinary longitudinal wavevector, it is possible to obtain two more forms for $\boldsymbol{B}^{(3)}$:

$$\boldsymbol{B}^{(3)*} = -i\frac{B^{(0)}}{\kappa^2}\boldsymbol{\kappa}^{(1)} \times \boldsymbol{\kappa}^{(2)} \tag{17}$$

and

$$\boldsymbol{B}^{(3)*} = -i\frac{B^{(0)}}{p^{(0)2}}\boldsymbol{p}^{(1)} \times \boldsymbol{p}^{(2)}, \tag{18}$$

which are the *double p* and *double* κ forms in Table 1. In so doing, we note the following relation between magnitudes for the transverse and longitudinal wave-vectors in the electromagnetic wave:

$$|\boldsymbol{\kappa}^{(1)} \times \boldsymbol{\kappa}^{(2)}| = i\kappa|\boldsymbol{\kappa}^{(3)*}|. \tag{19}$$

DISCUSSION

The field $\boldsymbol{B}^{(3)}$ has been expressed in ten different forms, and these are summarized in Table 1. The only form in which the charge e appears explicitly is the O(3) gauge form, Eq. (7). However, since all ten forms are obviously for the same field, the Evans-Vigier field $\boldsymbol{B}^{(3)}$, charge is implicit in the nine accompanying variations. All ten balance $\hat{C}$ symmetry therefore, on both sides of the defining equation the $\hat{C}$ symmetry is negative. The BSA and curl A forms are

Table 1: Forms of the Evans-Vigier field $\boldsymbol{B}^{(3)}$

FORM	$\boldsymbol{B}^{(3)*}$
1. Double A	$\boldsymbol{B}^{(3)*} = -i\frac{\kappa}{A^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$
2. O(3) Gauge	$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$
3. Biot-Savart-Ampère	$\boldsymbol{B}^{(3)*} = -\frac{i}{cp^{(0)}}\boldsymbol{p}^{(1)} \times \boldsymbol{E}^{(2)}$
4. AE	$\boldsymbol{B}^{(3)*} = -\frac{1}{cA^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{E}^{(2)}$
5. Curl A	$\boldsymbol{B}^{(3)*} = -\boldsymbol{\nabla}^{(1)} \times \boldsymbol{A}^{(2)}$
6. Dirac	$\boldsymbol{B}^{(3)*} = -\frac{i}{\hbar}\boldsymbol{p}^{(1)} \times \boldsymbol{A}^{(2)}$
7. Double B	$\boldsymbol{B}^{(3)*} = -\frac{i}{B^{(0)}}\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$
8. Double E	$\boldsymbol{B}^{(3)*} = -\frac{i}{cE^{(0)}}\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$
9. Double p	$\boldsymbol{B}^{(3)*} = -i\frac{B^{(0)}}{p^{(0)2}}\boldsymbol{p}^{(1)} \times \boldsymbol{p}^{(2)}$
10. Double $\boldsymbol{\kappa}$	$\boldsymbol{B}^{(3)*} = -i\frac{B^{(0)}}{\kappa^2}\boldsymbol{\kappa}^{(1)} \times \boldsymbol{\kappa}^{(2)}$

classical expressions for a magnetic field, showing that $\boldsymbol{B}^{(3)}$ is a physical magnetic flux density in the vacuum, an experimental observable. It is the first classical field to be inferred in the vacuum since Maxwell, and as such is also defined in the quantum field, for example in the Dirac form. The latter can be derived independently from a first principles [7] solution of the Dirac equation of e in A_μ. The double A form, Eq. (4), is also derivable independently in the classical field. The charge quantization condition (8) produces the Dirac form directly from the O(3) gauge form, and arises independently from O(3) gauge theory [17] by a comparison of the O(3) gauge form and double A form. Therefore the charge quantization condition is shown to be a fundamental condition of field theory, and shows that the classical equivalent of the photon momentum magnitude $\hbar\kappa$ is $eA^{(0)}$. This result is obtainable in yet another independent manner [7,20] from the relativistic HJ equation of e in A_μ. The BSA form is obtainable from the double A form using it, and the BSA form is the classical expression for the source of a magnetic field. The source of $\boldsymbol{B}^{(3)}$ is therefore found in the propagating transverse waves themselves, and $\boldsymbol{B}^{(3)}$ propagates because the waves propagate.

The fact that $\boldsymbol{B}^{(3)}$ can be understood in several ways, which are consistent within field theory, means that it becomes overwhelmingly probable that it will be isolated experimentally. With the benefit of hindsight, the double B and double E forms have already shown that $iB^{(0)}\boldsymbol{B}^{(3)*}$ has been observed in magneto-optics at visible frequencies, for example in the inverse Faraday effect [7,18], optical Faraday effect [7,23], optical Zeeman effect [3], light shifts in a circularly polarized beam [24], and the optical Cotton Mouton effect [4]. Less clearly, but definitively, light has been shown to shift NMR resonances [7], another magneto-optic phenomenon. Use of the HJ equation for one electron in A_μ shows [7] that at visible frequencies the term in $iB^{(0)}\boldsymbol{B}^{(3)*}$ dominates, so that effects are proportional to I_0, the beam power density. The same equation shows however, that at microwave frequencies, the term in $\boldsymbol{B}^{(3)}$ itself dominates, with an expected $I_0^{1/2}$ dependence, which however, has yet to be isolated experimentally. Since $\boldsymbol{B}^{(3)}$ implies finite photon mass [25] the existing data are already sufficient to show that the electromagnetic field is massive. The only alternative is to assert that $\boldsymbol{B}^{(3)}$ is zero, and Table 1 shows that this is unreasonable, such an assertion invalidates the structure and internal consistency of field theory. Table 1 is in effect the result of a careful check for self-consistency, and produces the same $\boldsymbol{B}^{(3)}$ field using several independent methods in the classical and quantized theory.

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