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THE CHARGE QUANTIZATION CONDITION IN O(3) VACUUM ELECTRODYNAMICS

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The existence of the longitudinal field $\boldsymbol{B}^{(3)}$ in the vacuum implies that the gauge group of electrodynamics is O(3), and not U(1) [or O(2)]. This results directly in the charge quantization condition $e = \hbar\kappa/A^{(0)}$. This condition is derived independently in this Letter from the relativistic motion of one electron in the field, and is shown to be that in which the electron travels infinitesimally close to the speed of light.

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~~Keywords: Charge Quantization; O(3) Electrodynamics~~

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1. INTRODUCTION

By magnetizing an electron plasma with megawatt pulses of microwave radiation, it is possible in principle to demonstrate experimentally (1,2) ~~[1,2]~~ that under well-defined conditions the magnetization depends on the *square root* of the power density (I_0) of the beam (W m^{-2}). *This $I_0^{1/2}$ dependence constitutes unequivocal experimental proof of the existence in the vacuum of the longitudinal magnetic flux density* $\boldsymbol{B}^{(3)} = B^{(0)}\boldsymbol{e}^{(3)}$. We consider a circular basis (1), (2), (3) in which $\boldsymbol{e}^{(3)}$ is a unit vector in the propagation axis, 3, of the electromagnetic beam. It is not possible to obtain an $I_0^{1/2}$ dependence from the vacuum plane waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ because they time average to zero at first order in $B^{(0)}$, the magnetic flux density amplitude of the beam. Such a dependence can be obtained only from the phase-independent (3-8) ~~[3-8]~~ $\boldsymbol{B}^{(3)}$. The three physical fields $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$, and $\boldsymbol{B}^{(3)}$ are related in the vacuum by

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \text{ et cyclicum}, \tag{1}$$

in which $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is the observable (9,10) ~~[9, 10]~~ conjugate product of the theory of non-linear optics. The belated recognition (3-8) ~~[3-8]~~ of the existence of the observable field

$\mathbf{B}^{(3)}$ of vacuum electrodynamics has several fundamental consequences in field theory (5) ~~[5]~~. One of these is the extension of the gauge group of electromagnetism from $O(2)$ to $O(3)$. Self-consistent geometrical methods (5) ~~[5]~~ along these lines result in the vacuum *charge quantization condition*,

$$\hbar\boldsymbol{\kappa} = eA^{(0)}, \tag{2}$$

in which the usual photon momentum, $\hbar\boldsymbol{\kappa}$, is identified with the product $eA^{(0)}$ of the electronic charge with the scalar magnitude of the vector potential of the beam. Therefore, charge occurs in units of $\hbar$ and is quantized. In this letter, $eA^{(0)}$ is identified as the classical momentum of an electron travelling infinitesimally close to the speed of light in the vacuum, c.

In a self-consistent, three-dimensional gauge group $O(3)$, fundamental geometry (5, 11) ~~[5, 11]~~ shows that

$$\mathbf{B}^{(3)} = -i\frac{e}{\hbar}\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}, \tag{3}$$

where $\hbar$ is the Dirac constant and $\mathbf{A}^{(1)}$ and $\mathbf{A}^{(2)}$ are vector potentials of plane waves. In the ordinary $O(2)$ gauge group, on the other hand, Eqs. (1) mean that (3-8) ~~[3-8]~~

$$\mathbf{B}^{(3)} = -i \frac{\kappa}{A^{(0)}} \mathbf{A}^{(1)} \times \mathbf{A}^{(2)}, \tag{4}$$

and the $O(2)$ gauge group *self-indicates* that it is incomplete, because the physical $\mathbf{B}^{(3)}$ is orthogonal to its own plane of definition. Comparison of Eqs. (3) and (4) gives Eq. (2), the charge quantization condition (5) ~~[5]~~.

In this Letter we derive Eq. (2) from the relativistic, but classical, theory of one electron accelerated in an electromagnetic field infinitesimally close to *c*.

2. FUNDAMENTAL ENERGY CONSIDERATIONS

Talin *et al.* (12) ~~[12]~~ have shown that the interaction between circularly polarized electromagnetic radiation and a material system results both in energy and angular momentum being transferred from the field to the system, in the simplest case one electron. Excluding consideration of the purely quantum mechanical intrinsic spin of the electron, its orbital angular momentum before the field is switched on is zero. In an adiabatic process, the angular momentum acquired by the electron is equal to that lost by the field (12) ~~[12]~~. Thus, orbital angular momentum can be *completely* lost by the field, and given up *entirely* to the electron, total angular momentum being conserved. Similar considerations obtain for energy and linear momentum, and conservation laws make it possible for the electron to acquire completely the energy and momenta of the field. The electron is accelerated thereby infinitesimally close to c, and takes on all the characteristics of the field. In this condition, its radiation becomes indistinguishable from that of the vacuum electromagnetic field, as shown classically be Jackson ~~[13]~~ (13).

In the quantum field theory, the original incoming photon is annihilated and a new *photon* created from the electron. The energy and momenta of the created photon are the usual $\hbar\omega$, $\hbar\boldsymbol{\kappa}$ and $\hbar$, where ω is the angular frequency of the annihilated photon and $\boldsymbol{\kappa}$ its wave vector.

It is shown in this section that the condition under which this occurs is precisely that under which the $I_0^{1/2}$ dependence of $\boldsymbol{B}^{(3)}$ can be isolated clearly from the concomitant second-order I_0 dependence ~~[14]~~ (14) of the conventional theory of the inverse Faraday effect. This $I_0^{1/2}$ condition obtains when a high intensity, low-frequency, and circularly polarized electromagnetic field acts on an electron. This process is described classically by the *relativistic* version of the Hamilton-Jacobi equation of fundamental dynamics, an equation which is applied in the purely relativistic limit ~~[15]~~,

$$\omega < \frac{e}{m} B^{(0)} \tag{5}$$

where e/m is the charge-to-mass ratio of the electron.

3. CONSIDERATION OF THE HAMILTON-JACOBI EQUATION OF e IN A_μ

The classical, orbital, angular momentum of the electron in the electromagnetic field represented by its four-potential (13) ~~[13]~~ A_μ can be shown (5) ~~[5]~~ to be

$$J_{e1}^{(3)} = \frac{e^2 c^2}{\omega^2} \left(\frac{B^{(0)}}{(m^2\omega^2 + e^2 B^{(0)2})^{\frac{1}{2}}} \right) \boldsymbol{B}^{(3)} \tag{6}$$

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and is induced and governed completely by the field $\boldsymbol{B}^{(3)}$. In the condition (5) Eq. (5) reduces to

$$J_{e1}^{(3)} \rightarrow \frac{ec^2}{\omega^2} \boldsymbol{B}^{(3)} \tag{7}$$

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so that the total energy (12) ~~[12]~~ acquired by the electron becomes

$$En_{el} = \omega |\mathbf{J}_{el}^{(3)}| \rightarrow \frac{ec^2}{\omega} B^{(0)} \qquad (8)$$

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This is a classical, relativistic, result, but let us consider the limit in which this energy has been wholly acquired from *one photon* of energy $\hbar\omega$ of the quantized field. The photon has lost $\hbar\omega$ and has given it up entirely to the electron. Thus,

$$\frac{ec^2 B^{(0)}}{\omega} = \hbar\omega \qquad (9)$$

and in this limit, photon energy has been annihilated. Using (5) [5] $B^{(0)} = \kappa A^{(0)}$ and $\kappa = \omega/c$, Eq. (9) becomes Eq. (2).

The charge quantization condition is therefore that in which the electron is accelerated infinitesimally close to the speed of light, and becomes indistinguishable from the photon.

The electron acquires an angular momentum $\mathbf{J}_{el}^{(3)}$, but in roman the condition (9), *this is also the angular momentum of a photon*. Both angular momenta are governed *entirely* by the field $\mathbf{B}^{(3)}$, which is obviously non-zero and observable through the spinning motion of the electron. To emphasize this point, the radius of the electron's spinning trajectory in the classical field is, in S.I. units,

$$r = \frac{c}{\omega} \cdot \frac{eB^{(0)}}{(m^2\omega^2 + e^2B^{(0)2})^{\frac{1}{2}}} \tag{10}$$

which, in the limit (5), becomes

$$r \to \frac{c}{\omega} = \frac{1}{\kappa}, \tag{11}$$

where κ is the usual classical wave-vector in the vacuum. In the quantized field, r is the scalar magnitude of the photon radius.

In the same limit (6), finally, the magnetization produced by $B^{(3)}$ in an N electron plasma becomes

$$|M^{(3)}| \to -\frac{Ne^2}{2m\omega^2}\left(\frac{c}{\epsilon_0}\right)^{\frac{1}{2}} I_0^{1/2} \tag{12}$$

where ϵ_0 is the free space permittivity. This $I_0^{1/2}$ dependence of $|M^{(3)}|$ obviously signals the existence of $B^{(3)}$ in the vacuum and is experimentally measurable using Faraday induction (16) ~~[16]~~. In this eventually, a U(1) gauge group for electromagnetism can no longer be sustained, and must be replaced by O(3), whereupon vacuum electromagnetism becomes a non-Abelian theory (5) ~~[5]~~.

4, DISCUSSION

Equation (2) means that the photon momentum $\hbar\boldsymbol{\kappa}$ is the product of two $\hat{C}$ negative quantities, e and $A^{(0)}$. We have therefore shown why the photon momentum is $\hat{C}$ positive, while its concomitant fields are $\hat{C}$ negative. As in any non-Abelian field theory (5,11) ~~[5, 11]~~, the field becomes its own source, an electron travelling infinitesimally near c. Charge, e, now occurs in units of $\hbar$,

$$e = \hbar\left(\frac{\boldsymbol{\kappa}}{A^{(0)}}\right), \tag{13}$$

which supplements the usual

$$En = \hbar\omega, \quad \boldsymbol{p} = \hbar\boldsymbol{\kappa}. \tag{14}$$

In the conventional O(2) gauge group [often referred to (11) ~~[11]~~ as U(1)] the existence of $\boldsymbol{B}^{(3)}$ in the vacuum is inconsistent, because it is orthogonal to the plane of definition. Therefore U(1) is disallowed experimentally by the $I_0^{1/2}$ profile discussed already. Classical electrodynamics produces $\boldsymbol{B}^{(3)}$ through equations such as (1), (4), and (5) to (12), and is incompatible therefore with a two-dimensional group structure such as O(2) which allows no physical field in the vacuum perpendicular to the O(2) plane. Extension of the gauge group to O(3) produces Eq. (2), which is also the result of classical electrodynamics (the relativistic Hamilton-Jacobi equation). The O(2) group must therefore be replaced by the O(3) group, using methods akin to general relativity (5,11) ~~[5, 11]~~. As shown elsewhere (5) ~~[5]~~, these methods generalize the vacuum Maxwell equations, adding a third equation linking $\boldsymbol{B}^{(3)}$ to the formal (and unphysical) longitudinal electric field $i\boldsymbol{E}^{(3)}$. The latter is necessary for mathematical self-consistency and formal completeness in special relativity (5) ~~[5]~~, but produces no physical effects at first order. No such effects have been observed to date experimentally, whereas effects due to $\boldsymbol{B}^{(3)}$ are well known (3-8) ~~[3-8]~~.

A profoundly important consequence of $\boldsymbol{B}^{(3)}$ is that in the relativistic quantum field theory (5) ~~[5]~~, *the photon acquires mass* (m_0) and therefore Eq. (12) can be viewed as a crucially important indicator of non-zero m_0 (17) ~~[17]~~. In the conventional view, a massless particle with spin produces the E(2) little group of Wigner (18) ~~[18]~~. As discussed by Weinberg (19) ~~[19]~~, this is the unphysical group of rotations and translations in a plane. *Therefore the concept of a massless particle is unphysical*. It is uncritically accepted conventionally because it is convenient for the production of a zero $m_0 A_\mu A_\mu$ term, which is gauge invariant. The massless boson has two helicities, +1 and -1, whereas mass, however tiny in magnitude, means that the third helicity state, 0, appears due to longitudinal as well as transverse polarization. In other words, a spinning particle with mass is three dimensional in three-dimensional space. A *massless* particle is two-dimensional in the same space. Since $\boldsymbol{B}^{(3)}$ is obviously longitudinal, physical, and observable, it follows that the electromagnetic field carries mass and is described by the Proca equation (5) ~~[5]~~. The experimental detection of the $I_0^{1/2}$ profile of Eq. (12) therefore becomes central to field theory, and can be observed with a straightforward modification of the thirty-year-old technology used by Deschamps *et al.* (16) ~~[16]~~ in their original demonstration of the inverse Faraday effect in an electron plasma set up in helium gas by microwave pulses. More details of this crucial experiment are given elsewhere (20-23) ~~[20-23]~~.

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REFERENCES

[1] J. Deschamps, M. Fitaire, and M. Lagoutte, *Rev. Appl. Phys.* **7**, 155 (1972).

[2] S. Woźniak, M. W. Evans and G. Wagnière, *Mol. Phys.* **75**, 81, 99 (1992).

[3] M. W. Evans, *Physica B* **182**, 227, 237 (1992).

[4] M. W. Evans, *Physica B* **183**, 103 (1993); **190**, 310 (1993).

[5] M. W. Evans and J.-P. Vigier, *The Enigmatic Photon, Volume 1, The Field* $\mathbf{B}^{(3)}$ (Kluwer, Dordrecht, 1994); *The Enigmatic Photon, Volume 2, Non-Abelian Electrodynamic*, in preparation.

[6] M. W. Evans and A. A. Hasanein, *The Photomagneton in Quantum Field Theory* (World Scientific, Singapore, 1994).

[7] M. W. Evans, *The Photon's Magnetic Field* (World Scientific, Singapore, 1992).

[8] M. W. Evans and S. Kielich, eds., *Modern Nonlinear Optics*, Vol. 85(2) of *Advances in Chemical Physics*, I. Prigogine and S. A. Rice, eds. (Wiley-Interscience, New York, 1993/1994).

[9] reviewed by R. Zawodny in Vol. 85(1) of Ref. 8.

[10] S. Woźniak, G. Wagnière and R. Zawodny, *Phys. Lett. A* **154**, 259 (1991).