Paper 6

Unified Field Theory and $B^{(3)}$

The recent discovery of the vacuum spin field $B^{(3)}$ of the electromagnetic sector means that unified field theory is also affected at the most fundamental level. It is shown that $B^{(3)}$ changes the gauge symmetry of the electromagnetic sector from *U(1) = O(2) to O(3),* the rotation group symmetry. Accordingly, the massive bosons of *GWS* also acquire (3) components, but the ability of *GWS* to predict the correct masses is not affected.

Key words: $B^{(3)}$ Field, Unified Field Theory.

6.1 Introduction

Electromagnetism in unified field theory [1—4] is conventionally the *U(1)* sector of theories such as *GWS* or *SU(5)*. The term *U(1) sector* derives from the *U(1)* = *O(2)* gauge group that defines plane waves in the vacuum, the *O(2)* group being that of rotations in a plane, without reference to an orthogonal axis. In this conventional view, the physical fields are defined in the *O(2)* plane, and are transverse to the axis of propagation of the beam in the vacuum. Thus, for example, $B^{(1)} = B^{(2)*}$ is a plane wave of magnetic flux density that propagates in free space. By a careful examination of the conjugate product, $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, the conventional view just described has recently been changed fundamentally [5—12] because of the existence in the circular basis (1), (2), (3) of the Lie algebra:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*}, \quad \boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)} = iB^{(0)}\boldsymbol{B}^{(1)*}, \quad \boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)} = iB^{(0)}\boldsymbol{B}^{(2)*}, \tag{2.6.1}$$

where $\boldsymbol{B}^{(3)}$ is the spin field of vacuum electromagnetism, and $B^{(0)}$ is the scalar amplitude of the magnetic flux density of the beam. The conjugate product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is the basis of magneto-optical phenomena [7], of which there are several well known examples [13], and so $\boldsymbol{B}^{(3)}$ is an experimental observable. It magnetizes material matter, for example a plasma of electrons set up in helium by microwave pulses [14], and the magnetization, $\boldsymbol{M}^{(3)}$, set up by $\boldsymbol{B}^{(3)}$ is proportional to the *square root* of the beam power density (I_0), or intensity, in W m^{-2}. The required experimental conditions for the unequivocal isolation of the characteristic $I_0^{1/2}$ profile of $\boldsymbol{B}^{(3)}$ have been determined precisely [6] by solving the relativistic Hamilton-Jacobi equation of one electron (e) in the classical electromagnetic field, represented by the four-potential A_μ.

The various consequences of $\boldsymbol{B}^{(3)}$ have been worked into several branches of contemporary electromagnetic field theory [5—12], but in this Letter, its effect is explored on electroweak theory, which unifies electromagnetism with the weak field [15]. It is shown in Sec. 6.2 that the existence of the observable $\boldsymbol{B}^{(3)}$ in electromagnetism means that the gauge group symmetry must be enlarged to *O(3)*. In Sec. 6.3 it is shown that this means that the massive bosons of *GWS* acquire an additional physical dimension, the (3) dimension, and their concomitant fields are no longer purely transverse. In other words, the observable $\boldsymbol{B}^{(3)}$ of the electromagnetic sector is made up of conjugate products of intermediate vector boson field components. The latter are therefore also experimental observables. In Sec. 6.4, finally, it is shown that the observable $\boldsymbol{B}^{(3)}$ in electromagnetism

does not affect the ability of *GWS* to predict the correct intermediate vector boson masses.

6.2 The O(3) Gauge Group of Vacuum Electromagnetism

The defining Lie algebra (1) is that of the non-Abelian group of rotations, *O(3)*, in three dimensional space [6,15]. Since $\boldsymbol{B}^{(3)}$ is a physical observable, the gauge group of vacuum electromagnetism is also *O(3)*, and not the *O(2)* of the conventional view [15]. There is a physical magnetic flux density, $\boldsymbol{B}^{(3)}$, orthogonal to the plane of definition of the plane waves $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$. The photon, therefore, can no longer be regarded as a particle without mass, because special relativity [15] shows that such a particle can have only two (transverse) degrees of polarization. The Wigner little group [16] for the photon as particle also becomes *O(3)*, and not the obscure *E(2)*, the group of rotations and translations in a plane. The inference of photon mass leads in turn to the replacement [6] of the d'Alembert with the Proca equation, which leads to the replacement of $B^{(0)}$ in Eqs. (2.6.1) by the very slowly exponentially decaying $B^{(0)} \exp(-\xi Z)$ where ξ is the photon rest wavenumber [6] and Z is distance along the direction of propagation of radiation in vacuo. The range of electromagnetism is therefore not infinite, as discussed by Vigier [17].

The inference of an *O(3)* gauge group leads, furthermore, to a generalization of the vacuum Maxwell equations [6] to take into account the existence of a physical third axis (3) in free space. The usual plane wave relations are supplemented by an equation formally linking $\boldsymbol{B}^{(3)}$ and the imaginary and unphysical electric field strength $i\boldsymbol{E}^{(3)}$ in free space

$$\nabla \times \left(i\boldsymbol{E}^{(3)}\right) = -\frac{\partial \boldsymbol{B}^{(3)}}{\partial t} = \boldsymbol{0}\,. \tag{2.6.2}$$

The defining Lie algebra for $i\boldsymbol{E}^{(3)}$ links it to the ordinary plane wave $\boldsymbol{E}^{(1)} = \boldsymbol{E}^{(2)*}$ through the cyclically symmetric,

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = -E^{(0)}\left(i\boldsymbol{E}^{(3)}\right)^*, \qquad \boldsymbol{E}^{(2)} \times \left(i\boldsymbol{E}^{(3)}\right) = -E^{(0)}\boldsymbol{E}^{(1)*}, \qquad (2.6.3)$$
$$\left(i\boldsymbol{E}^{(3)}\right) \times \boldsymbol{E}^{(1)} = -E^{(0)}\boldsymbol{E}^{(2)*} .$$

In contrast to the Lie algebra of magnetic fields, Eqs. (2.6.1), the conjugate product of polar vectors $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$ cannot form a real polar vector, only the axial $\boldsymbol{B}^{(3)}$ [5],

$$\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)} = c^2\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = ic^2B^{(0)}\boldsymbol{B}^{(3)*}, \qquad (2.6.4)$$

so $i\boldsymbol{E}^{(3)}$ in Eqs. (2.6.4) is mathematically an axial vector. It is therefore unphysical because a physical electric field is a polar vector, and indeed there are no known effects of the putative physical $\boldsymbol{E}^{(3)}$. In contrast, $\boldsymbol{B}^{(3)}$ is a real, axial vector, i.e., has the necessary symmetry for a physical magnetic field. The latter is therefore an experimental observable, *the first classical vacuum field to be inferred since Maxwell.* It is the spin field (i.e., phase free magnetic field) fundamentally responsible for *all* magneto-optic phenomena. For example, the well known inverse Faraday effect [13,14] can be understood [18] at visible frequencies in terms of the conjugate product, which is now understood to be the product $iB^{(0)}\boldsymbol{B}^{(3)*}$, and this is now recognized to be the second order component. There is also a first order component of the inverse Faraday effect due to $\boldsymbol{B}^{(3)}$ itself [6]. This dominates at microwave frequencies with an $I_0^{1/2}$ dependence under well defined conditions [19].

Since unified field theory such as *GWS* is based conventionally on the assumption that the electromagnetic gauge group is *O(2) (= U(1))*, it has to be re-examined as follows in the light of $\boldsymbol{B}^{(3)}$.

6.3 The Effect of $\boldsymbol{B}^{(3)}$ on GWS

The enlargement of the *O(2)* sector of *GWS* to *O(3)* must occur in such a way that it maintains the ability of *GWS* to predict the correct intermediate boson masses of the well known CERN experiment [15]. Obviously, the observed masses cannot change with the belated realization that $\boldsymbol{B}^{(3)}$ exists in the vacuum, and so $\boldsymbol{B}^{(3)}$ cannot affect the boson masses. In *GWS*, the potential four-vector A_μ is expressed in terms of the massive bosons $W_{3\mu}$ and X_μ which are components of the electromagnetic field,

$$A_\mu = W_\mu^3 \sin\theta_W + X_\mu \cos\theta_w . \qquad (2.6.5)$$

Here θ_w is the Weinberg angle, which is fixed experimentally. So the extent to which W_μ^3 and X_μ can contribute to A_μ is also fixed experimentally. In an abstract isospin space [15], the physical part of W_μ^3 is the 3 component of this abstract space. X_μ on the other hand is an isospin scalar [15]. In the four dimensional space-time of special relativity, however, both W_μ^3 and X_μ are four-vectors, and can therefore be written in Minkowski notation as

$$W_\mu^3 := \left(\boldsymbol{W}^3, iW^{3(0)}\right), \qquad X_\mu := \left(\boldsymbol{X}, iX^{(0)}\right). \qquad (2.6.6)$$

The space parts can be expressed in the circular basis (1), (2), (3) giving

$$\boldsymbol{A}^{(1)} = \boldsymbol{W}^{3(1)} \sin\theta_w + \boldsymbol{X}^{(1)} \cos\theta_w , \qquad (2.6.7)$$
$$\boldsymbol{A}^{(2)} = \boldsymbol{W}^{3(2)} \sin\theta_w + \boldsymbol{X}^{(2)} \cos\theta_w ,$$

for $\boldsymbol{A}^{(1)}$ and its complex conjugate $\boldsymbol{A}^{(2)}$. Therefore $\boldsymbol{B}^{(3)}$ can be expressed in terms of transverse components of the massive bosons W_μ^3 and X_μ by

$$\boldsymbol{B}^{(3)} = -i\frac{\kappa^2}{B^{(0)}}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}, \qquad (2.6.8)$$

where κ is the wavenumber of the electromagnetic beam; from this we infer that the bosons W_μ^3 and X_μ themselves have longitudinal components which define Lie algebras akin to Eqs. (2.6.1) and (2.6.3). Since W_μ^3 and X_μ are parts of A_μ in *GWS*, they are plane waves, e.g.,

$$\boldsymbol{W}^{3(1)} = \frac{W^{3(0)}}{\sqrt{2}} (i\boldsymbol{i} + \boldsymbol{j}) e^{i\phi} , \qquad \boldsymbol{W}^{3(2)} = \frac{W^{3(0)}}{\sqrt{2}} (-i\boldsymbol{i} + \boldsymbol{j}) e^{-i\phi} , \tag{2.6.9}$$

and so

$$\boldsymbol{W}^{3(1)} \times \boldsymbol{W}^{3(2)} = iW^{3(0)}\boldsymbol{W}^{3(3)*} = -W_3^{(0)} \left(i\boldsymbol{W}^{3(0)} \right)^* , \quad \text{et. cyclicum} \tag{2.6.10}$$

is a Lie algebra akin to (3), $\boldsymbol{W}^{3(1)}$ and $\boldsymbol{W}^{3(2)}$ being polar vectors, parts of $\boldsymbol{A}^{(1)}$ and $\boldsymbol{A}^{(2)}$ respectively. Similarly,

$$\boldsymbol{X}^{(1)} \times \boldsymbol{X}^{(2)} = -X^{(0)} (i\boldsymbol{X}^{(3)})^* \tag{2.6.11}$$

is a Lie algebra. Thus, both W_μ^3 and X_μ are described by *O(3)* gauge geometry. We have therefore succeeded in enlarging the gauge geometry of GWS to include $\boldsymbol{B}^{(3)}$ self-consistently.

6.4 Boson Masses and $\boldsymbol{B}^{(3)}$

With the advent of $\boldsymbol{B}^{(3)}$ in the electromagnetic field, the W_μ^3 and X_μ bosons acquire three states of circular polarization, (1), (2) and (3). The extra state of polarization does not affect the mass of the boson. For example, we have

$$W_\mu^3 W_\mu^3 = \boldsymbol{W}^{3(1)} \cdot \boldsymbol{W}^{3(1)} + \boldsymbol{W}^{3(2)} \cdot \boldsymbol{W}^{3(2)} + \boldsymbol{W}^{3(3)} \cdot \boldsymbol{W}^{3(3)} - W^{3(0)2} , \tag{2.6.12}$$

which contains the additional term $\boldsymbol{W}^{3(3)} \cdot \boldsymbol{W}^{3(3)} - W^{3(0)2}$, a part of the additional term $\boldsymbol{A}^{3(3)} \cdot \boldsymbol{A}^{3(3)} - A^{3(0)2}$ in electromagnetism. However, this term vanishes because $|\boldsymbol{A}^{(3)}| = A^{(0)}$. The mass of W_μ^3 appears as a premultiplier of $W_\mu^3 W_\mu^3$ in the appropriate Lagrangian [15], and from this we infer that the extra (3) polarization makes no difference to the mass of the boson concomitant with W_μ^3. The only way in which the mass could be affected were if the premultiplier were for some reason different for transverse and longitudinal terms. This does not seem very likely because mass is a scalar Lorentz invariant. Four-vector products such as $W_\mu^3 W_\mu^3$ and $X_\mu X_\mu$ are also Lorentz invariants.

In conventional *GWS* [15], the photon mass is modeled to zero, but the concept of spontaneous symmetry breaking of the vacuum is used within non-Abelian, abstract isospin space [15], to provide the intermediate vector bosons with mass. The advent of $\boldsymbol{B}^{(3)}$, however, means that the photon must also be massive in *GWS*. This is a direct result of the experimental observable $\boldsymbol{B}^{(3)}$ which was related in Sec. 6.3 to the vector bosons. The latter acquire in turn the polarization (3), which cannot exist in a massless photon. This implies that *GWS* (and grand unified theory such as *SU(5))* must accommodate finite photon mass, for example as in the work of Huang

[1]. This illustrates how $\boldsymbol{B}^{(3)}$ has highly non-trivial repercussions throughout contemporary unified and grand unified field theory. In the electromagnetic sector, the Higgs mechanism is well known to be compatible with gauge invariance, and leads to finite photon mass through spontaneous symmetry breaking of the vacuum. The acquired photon mass is inevitably accompanied [15] by the acquisition of a third, physical polarization, manifest in $\boldsymbol{B}^{(3)}$. This type of result is, however, modeled out in *GWS* to force the result that photon mass is identically zero. With the advent of $\boldsymbol{B}^{(3)}$ such a procedure is invalidated and must be replaced by a mechanism which self-consistently accounts for photon mass.

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