Paper 7

The Physical Meaning of $\boldsymbol{B}^{(3)}$

The physical meaning is discussed, using simple concepts, of the novel longitudinal field $\boldsymbol{B}^{(3)}$ (the Evans-Vigier field) of vacuum electromagnetism. In words without equations, it is explained why the physical $\boldsymbol{B}^{(3)}$ is not accompanied by a physical electric field. The source of $\boldsymbol{B}^{(3)}$; its mode of propagation; and its symmetry and energy characteristics are explained in physical terms rather than mathematical.

Key words: $\boldsymbol{B}^{(3)}$ field, physical meaning of.

7.1 Introduction

In recent months [1—10] it has become clear that the conventional view of vacuum electromagnetism is incomplete, because there exists in the vacuum the Evans-Vigier field, $\boldsymbol{B}^{(3)}$. This is a novel, classical, magnetic field in free space, the first to be inferred since Maxwell. As such, it has its quantum mechanical counterpart, the photomagneton [2]. The purpose of this short note is to describe the physical meaning of $\boldsymbol{B}^{(3)}$ without abstract mathematics, because $\boldsymbol{B}^{(3)}$ is a remarkable development, having been hidden in Maxwell's equations for well over a hundred years. In Sec. 7.2, its origin is described in the plane waves of vacuum electromagnetism. In Sec. 7.3, it is explained in physical terms why $\boldsymbol{B}^{(3)}$ propagates through the vacuum, and finally, in Sec. 7.4, its symmetry and energy characteristics are explained in physical terms as simply as words allow. A short discussion section explains how $\boldsymbol{B}^{(3)}$ can be isolated from the ever-present plane waves which are its source in the vacuum, and thereby observed experimentally.

7.2 The Origin of the Evans-Vigier Field

The magnetic component of the plane wave, $\boldsymbol{B}^{(1)}$, from Maxwell's equations is a complex quantity in general and so has a complex conjugate, $\boldsymbol{B}^{(2)}$, which is also a solution of Maxwell's equations. This can be thought of in terms of a complex, circular, representation of three dimensional space, a representation which is entirely equivalent to the usual real Cartesian. This picture is analogous to, but not the same as, a complex spinor representation used routinely for fermions such as the electron. The vectors $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ are therefore components of the complete vector field in this circular representation [11]. They are, however, only two components out of a possible three, because we are dealing with three dimensional space. The third component is the Evans-Vigier field, denoted as $\boldsymbol{B}^{(3)}$ because (3) is the third axis associated with (1) and (2). The component $\boldsymbol{B}^{(3)}$ is generated by the vector product of $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ in analogy with $\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k}$, et cyclicum, of the Cartesian representation, where **i**, **j**, and **k** are the usual Cartesian unit vectors along *X*, *Y* and *Z* respectively.

Very simply, therefore, the physical source of $\boldsymbol{B}^{(3)}$ is the cross product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$. This inference is cyclically symmetric [6—10], the physical source of $\boldsymbol{B}^{(1)}$ is $\boldsymbol{B}^{(2)} \times \boldsymbol{B}^{(3)}$ and that of $\boldsymbol{B}^{(2)}$ is $\boldsymbol{B}^{(3)} \times \boldsymbol{B}^{(1)}$. The three components $\boldsymbol{B}^{(1)}$, $\boldsymbol{B}^{(2)}$ and $\boldsymbol{B}^{(3)}$ are physical magnetic fields and all three are axial vectors. The cross product of two axial vectors is another axial vector, so that this view is self-consistent. The source of the Evans-Vigier field is therefore the plane wave components $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$.

It can be shown [10] that $\boldsymbol{B}^{(3)}$ can be expressed consistently in a variety of ways. The source of a magnetic field in classical electrodynamics [12] is described by the Biot-Savart-Ampère (BSA) law, and, indeed, $\boldsymbol{B}^{(3)}$ can be written [10] in this form, i.e., as the vector cross product of a transverse momentum, $\boldsymbol{p}^{(1)}$, with an electric component, $\boldsymbol{E}^{(2)}$, of the plane wave. Another inference of classical electrodynamics is that a magnetic field is the curl of a vector potential, and, indeed, $\boldsymbol{B}^{(3)}$ can be written as $-\nabla^{(1)} \times \boldsymbol{A}^{(2)}$, where $\nabla^{(1)}$ is a well defined curl operator [10] and $\boldsymbol{A}^{(2)}$ a plane wave potential [10]. Furthermore, the *BSA* and *curl A* forms of $\boldsymbol{B}^{(3)}$ are equivalent to the various *double field* forms typified by $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$. This analysis, developed elsewhere [10] shows that in the classical sense, $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is indeed a source for a magnetic field. There are also various other forms of $\boldsymbol{B}^{(3)}$, tabulated in the literature [10], and its existence has been demonstrated from first principles using the Dirac equation [7] of relativistic quantum field theory, and the Hamilton-Jacobi equation [6] of relativistic classical field theory. Intuitively, the complicated language and mathematical analysis behind these demonstrations can be reduced to a consideration of the helical motion of the tip of a magnetic or electric field propagating in free space. The field is $\hat{C}$ negative, like charge, and so this motion is intuitively analogous to that of a current through a solenoid, producing a magnetic field. It is well known [13] in the classical theory of fields that a circularly polarized electromagnetic wave drives an electron in a circle, so the field itself is in a sense, charged, (i.e., $\hat{C}$ negative, where $\hat{C}$ is the charge conjugation operator) otherwise there would be no effect on the electron. The photon, on the other hand, is considered to be an uncharged quantum of energy. Therefore the photon and field must always be considered concomitantly.

7.3 Propagation Through the Vacuum of the Evans-Vigier Field

The cross product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ removes the electromagnetic phase, ϕ [1—10], so that $\boldsymbol{B}^{(3)}$ has been described as a static magnetic field. More accurately, it is a phase free magnetic field, and propagates through the vacuum with its source, the plane wave components $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$. Therefore, a pulse of light carries with it the source of a $\boldsymbol{B}^{(3)}$ field through the vacuum, and so does a continuous beam. The pulse is detectible experimentally because it has travelling intensity, or power density (W m^{-2}), and the cross product $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is physically interpretable as being proportional to the antisymmetric part of the light intensity tensor [3,14] itself. This is always non-zero, and so is $\boldsymbol{B}^{(3)}$. Therefore $\boldsymbol{B}^{(3)}$ propagates because the intensity of light propagates. The intensity of light, even in the vacuum, is a tensor [3,14,15], this being an early inference by Placzek in the theory of non-linear optics [14]. (Intensity is quadratic in the electric field of the plane wave.) If light intensity did not have an antisymmetric, mathematically imaginary, tensor component, there would be no inverse Faraday effect [16], in which light magnetizes material matter, a process which is free of the electromagnetic phase as first inferred by Pershan [16] using general arguments. The inverse Faraday effect has been observed experimentally in glasses and liquids [17] and in an electron plasma [18] using respectively visible and microwave radiation. Since $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ is the source of $\boldsymbol{B}^{(3)}$, the inverse Faraday effect is due to $\boldsymbol{B}^{(3)}$, and it is concluded that $\boldsymbol{B}^{(3)}$ is the fundamental field of magneto-optics [1—10]. Intuitively, it is expected that magnetization be due to a magnetic field, and magnetization by light is due to the magnetic field $\boldsymbol{B}^{(3)}$, a consistent physical result. We conclude that the antisymmetric part of light intensity manifests itself physically as a magnetic field, the Evans-Vigier field.

7.4 Considerations of Symmetry and Energy Conservation

The cross product of two axial vectors is another axial vector [19], and the cross product of two polar vectors is also an axial vector. Thus, both $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ and the cross product of electric plane wave components, $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$, are proportional to $\boldsymbol{B}^{(3)}$ [6]. It can also be shown that the cross product of vector potentials, $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$, is similarly proportional to $\boldsymbol{B}^{(3)}$, a result which leads to a self-consistent representation of $\boldsymbol{B}^{(3)}$ using *O(3)* gauge theory rather than the conventional *O(2)* = *U(1)* symmetry group [19] for the electromagnetic sector in contemporary field theory. The various technical ramifications of the *O(3)* gauge group are developed elsewhere [6]. These include the important inference that the photon as particle must have mass, because it is three dimensional in nature, not two, as in the conventional *O(2)* gauge group. Here we are concerned with a simpler inference of symmetry, that $\boldsymbol{B}^{(3)}$ cannot be accompanied by a real $\boldsymbol{E}^{(3)}$, an inference which follows from the fact that a real polar vector cannot be formed from the conjugate products $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$, $\boldsymbol{E}^{(1)} \times \boldsymbol{E}^{(2)}$, or $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$. It can be shown [6] that $\boldsymbol{B}^{(3)}$ is accompanied, formally, by a pure imaginary $-i\boldsymbol{E}^{(3)}$, which being imaginary and first order, is not a physical field. Consistently, no first order experimental effect of a putative $\boldsymbol{E}^{(3)}$ has been reported. This is again an intuitively comfortable result because we do not expect a solenoid to produce an electric field in its axis, only a magnetic field. In this intuitive view, light is, loosely speaking, an optical solenoid producing $\boldsymbol{B}^{(3)}$, an axial vector about its axis of propagation. We have therefore referred to $\boldsymbol{B}^{(3)}$ as the spin field [6—10] to distinguish it from the wave field components $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$.

When considering the effect of $\boldsymbol{B}^{(3)}$ on electromagnetic energy density, however, it is necessary to consider vector magnitudes. In the circular representation (1), (2), (3), this means that we must consider dot products $\boldsymbol{B}^{(i)} \cdot \boldsymbol{B}^{(i)*}$, where * denotes complex conjugate and where *i* runs from 1 to 3, not from 1 to 2 as in the conventional view [12]. (Recall that in the conventional view there are only wave components, $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$, and these exist in a flat, two dimensional, world.) The quantum of light energy, $\hbar\omega$, the dictionary photon [19] must then be expressed in terms of the sum $\boldsymbol{B}^{(1)} \cdot \boldsymbol{B}^{(1)*} + \boldsymbol{B}^{(2)} \cdot \boldsymbol{B}^{(2)*} + \boldsymbol{B}^{(3)} \cdot \boldsymbol{B}^{(3)*}$. This does not change the Planck constant, however, because the same quantum of energy, $\hbar\omega$, is merely redistributed among (or thought of in terms of) three vector components, the overall magnitude remaining the same [10]. (One photon remains one photon, and can be detected experimentally as such. We infer that it is concomitant with three field components rather than two as thought conventionally.) Similarly, the quantum of light energy can be thought of in terms of the sum of three electric field components $\boldsymbol{E}^{(1)} \cdot \boldsymbol{E}^{(1)*} + \boldsymbol{E}^{(2)} \cdot \boldsymbol{E}^{(2)*} + i\boldsymbol{E}^{(3)} \cdot \left(i\boldsymbol{E}^{(3)}\right)^*$, where, now, $-i\boldsymbol{E}^{(3)}$ is multiplied by its own conjugate and becomes real and therefore physical. In the vacuum, the sum of magnetic field components is proportional to the sum of electric field components, both being proportional to $\hbar\omega$. The quantum of energy is again redistributed among three concomitant field components rather than two, its magnitude, and that of Planck's constant h, being unchanged. This energy analysis and others like it [6] illustrates the need for $-i\boldsymbol{E}^{(3)}$ as well as $\boldsymbol{B}^{(3)}$. Since $\hbar\omega$ is unchanged it is concluded that $\boldsymbol{B}^{(3)}$ does not affect the fundamentals of the old quantum theory, e.g. the Einstein theory of absorption and spontaneous emission, and the light quantum hypothesis itself. Thus spectra remain frequency dependent and discrete, $\boldsymbol{B}^{(3)}$ is a magnetizing field that is phase free, and frequency independent. The theory of the inverse Faraday effect is the same in structure at microwave or visible frequencies far from optical resonance. At or near resonance, Woźniak *et al.* [20] have shown that useful additional frequency dependent features occur but the optical property fundamentally responsible for magnetization by light remains the conjugate product, which is now understood [1—10] to be $iB^{(0)}\boldsymbol{B}^{(3)*}$, where $B^{(0)}$ is the field amplitude. It should be clearly understood that although the conjugate product is imaginary, $\boldsymbol{B}^{(3)}$ itself is real, and physical. This is, at the root, a consequence of space geometry itself [6].

7.5 Discussion

It has just been inferred that $\boldsymbol{B}^{(3)}$ is real and physical, so must produce real and physical effects which are observable experimentally. Conversely, $-i\boldsymbol{E}^{(3)}$ is unphysical, and cannot produce observable effects at first order. In order to separate out, or isolate, $\boldsymbol{B}^{(3)}$ experimentally, and thus to prove its existence, it is necessary to demonstrate its characteristic square root power density ($I_0^{1/2}$) dependence [6]. The classical, but relativistic, theory of the orbital angular momentum of the electron in the electromagnetic field is adequate [6] to show that the $I_0^{1/2}$ dependence can be expected to dominate experimentally using microwave pulses of sufficient intensity, or power density. An increase in power density of about two orders of magnitude over that used by Deschamps *et al.* [18] should be sufficient. The $I_0^{1/2}$ dependence of the magnetization can be due only to the vacuum Evans-Vigier field, because first order magnetization effects due to the plane waves $\boldsymbol{B}^{(1)}$ and $\boldsymbol{B}^{(2)}$ disappear on average. In general, the interaction of $\boldsymbol{B}^{(3)}$ with one electron is relativistic in nature, at visible frequencies the magnetizing effect of light is dominated [6] by I_0 acting at first order, at microwave frequencies, with sufficient power density by $I_0^{1/2}$. This result explains why the measurements on the inverse Faraday effect to date [17,18] have shown an I_0 dependence of the magnetization. The $I_0^{1/2}$ dependence has also emerged from an interesting analysis by Chiang [21] of the influence of ion motion in the inverse Faraday effect. His figure one shows the expected linear dependence of magnetization on $\boldsymbol{B}^{(3)}$, but Chiang did not make the key inference that $\boldsymbol{B}^{(3)}$ is generated [6] from $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}/(iB^{(0)})$ in free space. Chiang's analysis however agrees qualitatively with that given in this discussion, as the power density increases, the quadratic dependence of plasma magnetization on I_0 *evolves into an* $I_0^{1/2}$ *dependence*, which eventually saturates, because the maximum orbital angular momentum that the photon can transfer to the electron in a perfectly elastic collision is ħ [10]. We have also reached this conclusion independently [10], and physically, it means that the angular momentum of the photon, ħ, has been completely transferred to the electron using enormous beam power densities. An experimental investigation of these effects is necessary, because data to date have been confined to the quadratic region, (I_0 dependence of the magnetization).

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