Paper 10

The Interaction of the Evans-Vigier Field with Atoms

The theoretical study is initiated of the interaction of the Evans-Vigier field, $\boldsymbol{B}^{(3)}$, with atomic matter, represented by atoms such as H in which there is net electronic spin angular momentum only, and by atoms in which there is both spin and orbital electronic angular momentum. In H it is inferred from the Dirac equation that the net spin of the ground state electron should interact directly with $\boldsymbol{B}^{(3)}$, so that the Zeeman splitting due to such an interaction should be proportional to the *square root* of the power density of the beam, whatever its frequency. In atoms or molecules in which there is also net orbital angular momentum, the effect of $\boldsymbol{B}^{(3)}$ at first and second order is treated approximately using the classical Hamilton-Jacobi equation.

Key words: Evans-Vigier field, atomic interaction with.

.0.1 Introduction

It has been shown recently [1,2] that the interaction of the classical :lectromagnetic field with one electron is governed entirely by the Evans-/igier field, $\boldsymbol{B}^{(3)}$. Thus far, this has been shown in two ways, using the :lassical but relativistic Hamilton-Jacobi equation for the orbital electronic ngular momentum, and the Dirac equation for the spin electronic angular nomentum, which has no classical meaning [3]. Other methods of lemonstration can be used, for example using the Dirac equation in its Iamilton-Jacobi form. If the newly inferred $\boldsymbol{B}^{(3)}$ were zero, there would be 10 observable interaction between the electron and the field, contrary to :xperience. It is well known experimentally [4] that intense microwave ›ulses magnetize an electron plasma, a process which the classical Iamilton-Jacobi equation ascribes *entirely* to $\boldsymbol{B}^{(3)}$, acting at first and second ›rders in $B^{(0)}$, the scalar magnitude of the magnetic flux density of the ircularly polarized microwave pulse. For sufficiently low frequencies and ntense pulses, the magnetization is dominant at first order in B^0, and is herefore proportional to $I_0^{1/2}$, where I_0 is the power density of the pulse in V m^{-2}.

If for simplicity we consider the interaction of one electron with the lectromagnetic field in relativistic quantum field theory [3], the Dirac quation shows [1] that the permanent magnetic dipole moment ($\boldsymbol{m}$) set up y the intrinsic electronic spin forms an interaction Hamiltonian $H_s = -\boldsymbol{m}\cdot\boldsymbol{B}^{(3)}$. The magnetization due to $\boldsymbol{m}$ is $N\boldsymbol{m}$ if there are N electrons n a plasma. It is a permanent magnetization whose average value is zero, 1ot one induced by the electromagnetic field. In atoms such as H however, his type of interaction Hamiltonian leads to an optical Zeeman effect for all eam frequencies, and Sec. 10.2 is an account of the fundamentals of this ffect, which is detectible with electron spin resonance [5]. Section 10.3 xtends the discussion to atoms (and molecules) with net orbital as well as pin electronic angular momentum. Finally, a discussion is given in the 3orn-Oppenheimer approximation of fundamental magnetic dipole :ansitions involving $\boldsymbol{B}^{(3)}$.

10.2 Optical Zeeman Effect in H Due to $\boldsymbol{B}^{(3)}$

In atomic H, the single electron is bound to the nucleus in an orbital, and its net orbital angular momentum is quenched to zero [6]. It therefore has only a net spin angular momentum ($\boldsymbol{S}$) from the Dirac equation. In a conventional uniform magnetic field, the Zeeman effect occurs due to the electronic spin angular momentum, and is essentially an observable splitting of the atomic absorption spectrum of H [7]. In a free electron, on the other hand, there are no atomic absorption lines, and the Zeeman effect in an electron plasma is not observable in this way. In order to understand the interaction of electromagnetic radiation with atomic or free electrons, the Dirac equation is necessary, because without it, there is no spin quantum number S. Therefore we begin our discussion with a summary of the Dirac equation for one electron (e) in the electromagnetic field represented by the four-potential A_μ, and explain the emergence [1] of the Evans-Vigier field from the first principles of relativistic quantum field theory. Thereafter the discussion is extended to the Dirac equation of the electron of the H atom in the electromagnetic field, with emphasis on $\boldsymbol{B}^{(3)}$. This should be regarded as only the first step towards the rigorous understanding of the interaction of $\boldsymbol{B}^{(3)}$ with atomic matter. A fuller and more detailed understanding will rely on numerical methods, because the Dirac equation becomes analytically intractable.

The Dirac equation for the interaction of the free electron with the electromagnetic field is [1]

$$\gamma_\mu \left(p_\mu + eA_\mu\right)\psi\left(\boldsymbol{p}\right) = -m_0 c\psi\left(\boldsymbol{p}\right), \tag{2.10.1}$$

where γ_μ is the Dirac matrix [3] and ψ the Dirac four-spinor. In Minkowski notation,

$$\gamma_\mu = \left(\gamma, i\gamma^{(0)}\right), \qquad A_\mu = \left(A, \frac{i\phi}{c}\right),$$

$$p_\mu = \left(p, \frac{iEn}{c}\right), \tag{2.10.2}$$

and m_0 is the electron's mass. Using standard methods of solution [1] Eq. (2.10.1) reduces to the energy eigenequation

$$\hat{W}u = Hu, \tag{2.10.3}$$

where u is a Dirac four-spinor in the standard representation [1—3], and where the Hamiltonian eigenvalue H is

$$H = \frac{1}{2m_0}(\sigma \cdot (\boldsymbol{p} + e\boldsymbol{A}))^2 - e\phi . \tag{2.10.4}$$

Here σ is a Pauli spinor [1—3]. It can be shown [1] that the part of H that describes the interaction of the electron's intrinsic spin with the electromagnetic field is

$$H_s = \frac{i\sigma}{2m_0} \cdot (\boldsymbol{p} + e\boldsymbol{A}) \times (\boldsymbol{p} + e\boldsymbol{A}) = \frac{e\hbar}{2m_0}\sigma \cdot \boldsymbol{B}^{(3)}$$
$$= -\boldsymbol{m} \cdot \boldsymbol{B}^{(3)} , \tag{2.10.5}$$

where $\boldsymbol{B}^{(3)}$ is the Evans-Vigier field [8—12] of vacuum electromagnetism. Here $\boldsymbol{S} = (\hbar\sigma)/2$ is the electronic spin angular momentum, and e/m_0 the charge to mass ratio of the electron. Therefore $\boldsymbol{B}^{(3)}$ is to the vacuum electromagnetic field as $\boldsymbol{S}$ is to the electron, an irremovable component. If $\boldsymbol{B}^{(3)}$ were zero, then $\boldsymbol{S}$ could not interact with the electromagnetic field, in contradiction with the structure of the Dirac equation itself.

To extend this analysis to the H atom requires an extra term V in the Hamiltonian describing the fact that the electron is bound in an orbital to the nucleus. In the Born-Oppenheimer approximation [13] the Hamiltonian is split into a part dealing with the isolated atom (i.e., the way in which the electron is bound to the nucleus) and an interaction term. The latter can be expressed in terms of atomic property tensors, such as the magnetic dipole moment of the one free electron,

$$\boldsymbol{m} = -\frac{e}{2m_0}\boldsymbol{S} . \tag{2.10.6}$$

In this approximation, the interaction of the electron in the H atom with the applied electromagnetic field is described in the same way as that for the free electron of Eq. (2.10.4), i.e., through the dot product of $\boldsymbol{m}$ and the $\boldsymbol{B}^{(3)}$ of circularly polarized electromagnetic radiation in vacuo. In the H atom, however, there are observable electronic spectra in the visible and ultra-violet regions of the electromagnetic range of frequencies [14], spectra which appear as discrete absorption or emission lines. These do not occur in the electron plasma, because they are due essentially to atomic structure [7]. These spectral features are now known with great precision, and can be used to measure the effect of the $\boldsymbol{B}^{(3)}$ field through its Zeeman effect [7].

The Zeeman effect of $\boldsymbol{B}^{(3)}$ in atomic H occurs because the Dirac equation is based on spinors, which imply that the eigenvalues of the H atom's electronic spin angular momentum are $\hbar/2$ and $-\hbar/2$. In $\boldsymbol{B}^{(3)}$ these are no longer degenerate, and transitions between them are possible. These were first measured in atomic H in an ordinary magnetic field by Beringer and Heald [15] about forty years ago, and should be measurable for $\boldsymbol{B}^{(3)}$ with contemporary technology. If so, the $\boldsymbol{B}^{(3)}$ Zeeman effect should depend on $I_0^{1/2}$ because $\boldsymbol{B}^{(3)}$ depends on $I_0^{1/2}$. In the same way as the ordinary Zeeman effect, due to an ordinary magnetic field, depends on its flux density, so does the Zeeman effect due to the Evans-Vigier field, which has all the known properties [1,2] of a magnetic field. We refer to this predicted phenomenon as the optical Zeeman effect. In atomic H it should be measurable with the use of pulses of high intensity circularly polarized pump radiation and contemporary synchronized detection. For example the ESR signal of H should be shifted by $\boldsymbol{B}^{(3)}$, and the shift should be proportional to $I_0^{1/2}$. Similar optical Zeeman effects should be observable in other atoms with net ground state electron spin, such as the alkali metal vapors. (The original Zeeman effect was observed in sodium vapor.) In order to observe them it is necessary to use circularly polarized pulses of very high intensity, because $\boldsymbol{B}^{(3)}$ is zero [1] in linearly or incoherently polarized radiation, and because

$$B^{(0)} = \left(\frac{I_0}{\epsilon_0 c^3} \right)^{\frac{1}{2}}, \tag{2.10.7}$$

where ϵ_0 is the vacuum permittivity and c the speed of light in vacuo. Therefore for a power density of 1.0 W cm^{-2}, (10^4 W m^{-2}), the magnitude of $B^{(0)}$ is only about 10^{-5} T, ten times smaller than the Earth's mean magnetic field, producing a very small Zeeman shift.

Since $\boldsymbol{B}^{(3)}$ emerges from the Dirac equation itself, it is non-zero, and is an observable. If definitive experimental evidence to the contrary is obtained, then the Dirac equation will have failed at a basic level.

10.3 Atoms and Molecules with Orbital Electronic Angular Momentum

The equilibrium of one electron in the electromagnetic field can be considered classically with the relativistic Hamilton-Jacobi equation [1,2]. Consider the condition,

$$\omega = \frac{e}{m_0} B^{(0)}, \tag{2.10.8}$$

where e/m_0 is the charge to mass ratio of the electron and where $B^{(0)}$ is the scalar magnitude of the magnetic flux density of the beam. Under condition (2.10.8), ω is both the angular frequency of the beam and the orbital angular

frequency of the electron in equilibrium with the beam. For the electromagnetic beam, ω is κc where κ is the magnitude of the wave-vector, and where c is the speed of light. For the electron, the de Broglie matter-wave equation [1] gives

$$\omega = \frac{1}{\hbar} \left(m_0^2 c^4 + \hbar^2 \kappa^2 c^2 \right)^{\frac{1}{2}}. \tag{2.10.9}$$

Under condition (2.10.8), using Eq. (2.10.9), we obtain

$$m_0^2 c^4 + \hbar^2 \kappa^2 c^2 = \hbar^2 \frac{e^2}{m_0^2} B^{(0)2}, \tag{2.10.10}$$

which is a *cyclotron condition* for equilibrium of the electron in the field. In the limit,

$$\kappa \gg \frac{m_0 c}{\hbar}, \tag{2.10.11}$$

(where κ now refers to the matter wave of the electron) we obtain the result,

$$\omega \sim \kappa c \sim \frac{e B^{(0)}}{m_0}, \tag{2.10.12}$$

which, using the relation between $A^{(0)}$ and $B^{(0)}$ of the wave,

$$A^{(0)} = \frac{B^{(0)}}{\kappa}, \tag{2.10.13}$$

is the charge quantization condition [1]. The electron and photon become indistinguishable in the limit (2.10.11) in which the cyclotron frequency of the electron is the angular frequency of the beam.

This simple illustration shows that the beam can be thought of as driving the electron in an orbit, which can be described in terms of classical, relativistic mechanics. This is quite different from the interaction process of the intrinsic spin angular momentum with the field, a process which has no classical interpretation. In an atom in which the electron has both orbital and spin angular momentum, the Zeeman splitting pattern due to $\boldsymbol{B}^{(3)}$ is therefore affected by both types of angular momentum appearing in the appropriate Hamiltonian operator of the Dirac equation. We expect phenomena to order $I_0^{1/2}$ and I_0 in the optical Zeeman spectrum in such atoms, and in general, these can be understood only by solving the Dirac equation numerically.

A qualitative understanding of the problem can be attained, however, by considering an atom with one electron, an electron which has orbital as well as spin angular momentum, and by splitting off the interaction Hamiltonian of the electron with the field from the Hamiltonian describing the way in which the electron is bound to the nucleus. This type of approximation is the basis of semi-classical radiation theory [13], in which atomic property tensors are treated quantum mechanically, and the field classically. In the non-relativistic approximation several predicted phenomena of the Evans-Vigier field have been described [8—12]. In the fully relativistic treatment, however, major new features emerge — at microwave frequencies the interaction Hamiltonian (and therefore the $\boldsymbol{B}^{(3)}$ induced Zeeman splitting) becomes proportional to $B^{(0)}$, and therefore to I_0. At visible frequencies the process is dominated by the term in I_0, because the classical Hamilton-Jacobi equation of the free electron in the field shows these properties. This result explains why shifts caused by visible lasers of atomic frequencies appear experimentally to be dominated [16] by an I_0 dependence. It is clear that the new theoretical understanding provided by $\boldsymbol{B}^{(3)}$ implies the need for a fundamental re-appraisal of the interpretation of spectra such as these.

10.4 Discussion

The existence of $\boldsymbol{B}^{(3)}$ in vacuo means that there must be a novel magnetic dipole interaction, which is not considered in the usual Born-Oppenheimer approximation. In the usual semi-classical approach to radiation theory [13], the latter leads to interaction Hamiltonians which include magnetic dipole terms such as

$$H_{int} = -\boldsymbol{m}\cdot\boldsymbol{B}(t) . \tag{2.10.14}$$

In this expression, however, $\boldsymbol{B}(t)$ is time dependent and originates in the plane wave $\boldsymbol{B}^{(1)} = \boldsymbol{B}^{(2)*}$, not in the $\boldsymbol{B}^{(3)}$ field itself. The novel interaction term $H_s = -\boldsymbol{m}\cdot\boldsymbol{B}^{(3)}$,, however, is *fundamental* in atomic and molecular spectroscopy whenever a circularly polarized field is used, i.e., whenever the Evans-Vigier field is non-zero in vacuo. Some effects of H_s have been discussed in Secs. 10.2 and 10.3, and should be observable with contemporary technology. Magnetization by circularly polarized electromagnetic radiation [1] also depends on an interaction term of this type, and this can be understood classically for the free electron interacting with the field through the relativistic Hamilton-Jacobi equation. The Dirac equation in its Hamilton-Jacobi form [17], or an equivalent quantum equation, must be used to extend this understanding to atoms, so that a fully consistent theory emerges in relativistic quantum mechanics. It is already clear, however, that if $\boldsymbol{B}^{(3)}$ were zero, the Hamilton-Jacobi equation itself would give a meaningless conclusion.

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Paper 11

The Derivation of the Majorana Form of Maxwell's Equations from the B Cyclic Theorem

It is demonstrated that the *B* Cyclic theorem (equivalence principle) of the new electrodynamics gives Majorana's form of Maxwell's equations in the vacuum. This demonstration provides a link between the new and received views of vacuum electrodynamics, showing that the equations of motion can be derived from the equivalence principle, assuming only the correspondence principle of quantum mechanics. Therefore the *B* Cyclic theorem is quantized to give the Maxwell equations in Majorana's form.

11.1 Introduction

In the past few years it has become clear that a major advance in electrodynamics has occurred. The electromagnetic field is now thought to have longitudinal components in the vacuum [1—7], one of which, conveniently referred to as the $\boldsymbol{B}^{(3)}$ field, being phase free and observable empirically, for example in magneto-optics. This longitudinally directed