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Paper 19

Electromagnetism in Curved Space-time

A suggestion is developed for a theory of electromagnetism in curved space-time, a theory based on a novel *anti*symmetric Ricci tensor which is postulated to be directly proportional to the $G_{\mu\nu}$ tensor of Evans and Vigier, and which therefore deals self-consistently with the experimentally observable $\boldsymbol{B}^{(3)}$ field of magneto-optics.

Key words: Electromagnetism; general relativity; $\boldsymbol{B}^{(3)}$ field.

In this note, a brief summary is given of the essentials of a novel theory [1—3] of electromagnetism, a theory necessitated by the tiny experimental magneto-optic effects [4—8] which need for their self-consistent explanation the $\boldsymbol{B}^{(3)}$ field [9—15] in *curved* space-time. The essence of our argument here is that $\boldsymbol{B}^{(3)}$ can be obtained straightforwardly from the Riemann tensor [6—18] by using the contraction indicated by

$$R^{(A)}_{\mu\nu} := R^{\lambda}_{\lambda\mu\nu} . \tag{2.19.1}$$

This produces an *antisymmetric* Ricci tensor $R^{(A)}_{\mu\nu}$ to which the electromagnetic field tensor $G_{\mu\nu}$ introduced by Evans and Vigier [9,10] is directly proportional,

$$R^{(A)}_{\mu\nu} = \frac{e}{\hbar} G_{\mu\nu} . \tag{2.19.2}$$

Here $e/\hbar$ is a universal constant, the ratio of the quanta of charge and action or angular momentum). The Ricci tensor $R^{(A)}_{\mu\nu}$ is defined through affine parameters (Christoffel symbols) in the usual way [16—18],

$$R^{(A)}_{\mu\nu} = \partial_\mu \Gamma^\lambda_{\lambda\nu} - \partial_\nu \Gamma^\lambda_{\lambda\mu} + \Gamma^\rho_{\lambda\mu}\Gamma^\lambda_{\rho\nu} - \Gamma^\rho_{\lambda\nu}\Gamma^\lambda_{\rho\mu} , \tag{2.19.3}$$

and the $\boldsymbol{B}^{(3)}$ field [9—15] is proportional directly to the part of the Ricci ensor quadratic in the affine parameter. The latter is used to define the vector potential in curved space-time,

$$\Gamma^\kappa_{\lambda\mu} = \frac{e}{\hbar} M_\lambda A^\kappa_\mu , \tag{2.19.4}$$

where M_μ is a rotation generator [9,10] and where $eA^\kappa_\lambda/\hbar$ is an energy-momentum tensor of electromagnetism. If $\lambda = \kappa$ we obtain,

$$A_\mu := M_\lambda A^\lambda_\mu , \tag{2.19.5}$$

and the $G_{\mu\nu}$ tensor becomes,

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu + \frac{e}{\hbar} A^2 (M_\mu M_\nu - M_\nu M_\mu) . \tag{2.19.6}$$

Restricting attention to $\mu = 1$, $\nu = 2$, the $\boldsymbol{B}^{(3)}$ field in curved space-time s obtained as,

$$B_3 := \frac{e}{\hbar} A^2 (M_1 M_2 - M_2 M_1) , \tag{2.19.7}$$

or in the complex basis ((1), (2), (3)) [9—15],

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar} \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} . \tag{2.19.8}$$

Maxwellian electrodynamics is recovered if and only if the rotation generators commute, i.e., if the quadratic term is arbitrarily abandoned, so that

$$G_{\mu\nu} \rightarrow F_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{2.19.9}$$

This means that Maxwellian electrodynamics is a linear approximation in which terms quadratic in A are missing. In the basis ((1), (2), (3)) the Maxwellian approximation means that the right hand side in the equation,

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)} \boldsymbol{B}^{(3)*} , \tag{2.19.10}$$

is zero. This is geometrically incorrect, and contradicts the experimental observation of $\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)}$ in magneto-optics [4—8]. Maxwellian electrodynamics is adequate therefore for low intensity light, but becomes internally inconsistent when magneto-optics is understood in terms of a self-consistent theory in curved space-time.

The $\boldsymbol{B}^{(3)}$ allows an understanding of electrodynamics and gravitation as being proportional to respectively the antisymmetric and symmetric components of the same Riemann tensor, i.e., in terms of curvilinear space-time geometry. If account is taken of $\boldsymbol{B}^{(3)}$ one can no longer logically adhere to a flat space-time for electromagnetism and a curved space-time for gravitation, and in magneto-optics, one is actually observing the spinning of space-time itself. Thus, as pointed out by Roy [11], $\boldsymbol{B}^{(3)}$ becomes the relict magnetic field in cosmology, responsible for circular polarization in the 2.7 K background radiation [19]. These are the essentials of electromagnetism in curved space-time rather than in flat space-time, and these ideas are being developed in detail elsewhere [20—22].

Acknowledgments

York University, Toronto, and the Indian Statistical Institute are thanked for visiting professorships, and the participants of the first Vigier Symposium are thanked for many interesting discussions.

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