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# INCONSISTENCIES OF THE U(1) THEORY OF ELECTRODYNAMICS: STRESS ENERGY MOMENTUM TENSOR

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The internal gauge space of electrodynamics considered as a U(1) gauge field theory is a scalar. This leads to the result that in free space, and for plane waves, the Poynting vector and energy vanish. This result is consistent with the fact that U(1) gauge field theory results in a null third Stokes parameter, meaning again that the field energy vanishes in free space. A self consistent definition of the stress energy momentum tensor is obtained with a Yang Mills theory applied with an O(3) symmetry internal gauge space. This theory produces the third Stokes parameter self consistently in terms of the self-dual Evans-Vigier fields **B**(3).

Key words: inconsistencies of U(1) gauge field theory, stress energy-momentum tensor.

## 1. INTRODUCTION

In Heaviside Maxwell electrodynamics the field energy, Poynting vector and Maxwell stress tensor are incorporated in one tensor, the stress energy momentum tensor [1]. In order to obtain a non-null energy and field momentum (Poynting vector), the method of averaging [2,3] is used. The Poynting vector, for example, becomes proportional to $\mathbf{E} \times \mathbf{B}^*$, where $\mathbf{E}$ is the electric field strength of the field and $\mathbf{B}$ its magnetic flux density. In this note it is shown that this method is inconsistent with electrodynamics considered as a U(1) gauge field theory, but consistent with electrodynamics considered as a Yang Mills theory with an O(3) internal gauge symmetry with a complex internal gauge space ((1),(2),(3)) based on the existence of circular polarization in radiation at a fundamental (one photon) level. The U(1) gauge field theory of electrodynamics produces a null third Stokes parameter, which again produces a null field energy in free space. The O(3) theory of electrodynamics produces a non-null third Stokes parameter in terms of the self-dual Evans-Vigier $\mathbf{B}^{(3)}$ [4–10], and so produces self-consistently a non-null field energy in free space.

## 2. THE U(1) COVARIANT DERIVATIVE AND NULL FREE SPACE POYNTING VECTOR AND FIELD ENERGY

In general gauge field theory for any gauge group the first tensor is defined through the commutator of covariant derivatives, giving the result [11]:

$$G_{\mu\nu} := \partial_\mu A_\nu - \partial_\nu A_\nu - ig[A_\mu, A_\nu], \tag{1}$$

where the commutator is non-zero in general. Here $A_\mu$ is the connection, or potential, and is defined in general through the gauge group symmetry. The field tensor $G_{\mu\nu}$ is covariant for all gauge groups, and the general gauge field theory is compatible with special relativity for all gauge group symmetries. In the general theory, the homogeneous and inhomogeneous Maxwell equations in vacuo are generalized respectively to

$$D^\nu \widetilde{G}_{\mu\nu} := 0, \quad D^\nu G_{\mu\nu} = 0, \tag{2}$$

where $D^\nu$ denotes the covariant derivative pertinent to the gauge group symmetry being used and $\widetilde{G}^{\mu\nu}$ is the dual of $G_{\mu\nu}$. In the U(1) gauge theory the commutator in Eq. (1) vanished because the U(1) group only has one structure constant and the internal symmetry of the gauge theory is a scalar symmetry. The covariant derivative in U(1) is

$$D^\nu = \partial^\nu + igA^\nu, \tag{3}$$

where $g$ is a proportionality constant. Therefore Eqs. (2) reduce to:

$$(\partial^\nu + igA^\nu)\widetilde{F}_{\mu\nu} = 0, \tag{4}$$

$$(\partial^\nu + igA^\nu)F_{\mu\nu} = 0, \tag{5}$$

which become the free-space homogeneous and inhomogeneous Maxwell-Heaviside equations if and only if:

$$A^\nu \widetilde{F}_{\mu\nu} = 0, \tag{6}$$

$$A^\nu F_{\mu\nu} = 0, \tag{7}$$

or, in vector notation,

$$\begin{aligned} \mathbf{A} \cdot \mathbf{B} = 0, \quad \mathbf{A} \times \mathbf{E} = \mathbf{0}, \\ \mathbf{A} \cdot \mathbf{E} = 0, \quad \mathbf{A} \times \mathbf{B} = \mathbf{0}. \end{aligned} \tag{8}$$

For plane waves, and using the usual U(1) relation

$$\mathbf{B} = \boldsymbol{\nabla} \times \mathbf{A}, \tag{9}$$

the vector potential $\mathbf{A}$ is proportional to $\mathbf{B}$, and so

$$\mathbf{B} \times \mathbf{E} = \mathbf{0}. \tag{10}$$

If we attempt to define the free-space field energy and momentum in terms of the products $\mathbf{B} \cdot \mathbf{B}$ and $\mathbf{B} \times \mathbf{E}$ the results are zero in U(1) gauge field theory. In order to obtain the well known field energy and Poynting vector (12) of the free electromagnetic field, products such as $\mathbf{B} \cdot \mathbf{B}^*$ and $\mathbf{B} \times \mathbf{E}^*$ have to be used. This procedure, although commonplace, and referred to in the textbooks as time averaging [12], introduces phenomenology extraneous to U(1), because it introduces the complex internal gauge space ((1),(2),(3)).

The fundamental inconsistencies of electrodynamics regarded as a U(1) gauge field theory are summarized therefore as follows: (1) If the U(1) covariant derivative is used, the field energy, momentum, and third Stokes parameter vanish. (2) If the phenomenological "times averaging" procedure is implemented, the resultant Poynting vector is proportional to $\mathbf{E} \times \mathbf{B}^*$ and perpendicular to the plane of definition of U(1) (the group homomorphic with O(2), the group of rotations in a plane). This result is internally inconsistent because if O(2) defines the plane, there can be no physical quantity in free space perpendicular to that plane.

## 3. STRESS ENERGY-MOMENTUM TENSOR IN YANG MILLS THEORY WITH O(3) INTERNAL GAUGE SYMMETRY

Yang Mills theory [13] was originally intended to generalize electrodynamics and in this theory the potentials and field tensor are written in an internal gauge space. If this space is of O(3) symmetry and is defined with the complex basis ((1),(2),(3)), with unit vectors $\mathbf{e}^{(1)}, \mathbf{e}^{(2)}, \mathbf{e}^{(3)}$ the potentials and field strength tensor becomes vectors in this internal space, so there are three components of the potential and of the field strength tensor [14]. The stress energy-momentum tensor in the O(3) symmetry theory is therefore

$$T_\mu^\nu = \epsilon_0 \left( \mathbf{G}^{\nu\rho} \cdot \mathbf{G}_{\rho\mu} \; - \frac{1}{4} \mathbf{G}^{\rho\sigma} \cdot \mathbf{G}_{\rho\sigma} \right), \tag{8}$$

and self-consistently defines, for example, the energy of the field as

$$U = \epsilon \cdot \left( E^{1(1)} E_1^{(2)} + E^{2(1)} E_2^{(2)} + E^{1(2)} E_1^{(1)} E^{2(2)} E_2^{(1)} + E^{3(2)} E_3^{(2)*} \right). \tag{9}$$

The energy is finite and made up of complex conjugate products. The Poynting vector is similarly defined as:

$$T_1^0 = \epsilon_0 (\mathbf{G}^{02} \cdot \mathbf{G}_{21} + \mathbf{G}^{02} \cdot \mathbf{G}_{31}), \tag{10a}$$

$$T_2^0 = \epsilon_0 (\mathbf{G}^{01} \cdot \mathbf{G}_{12} + \mathbf{G}^{02} \cdot \mathbf{G}_{32}), \tag{10b}$$

$$T_3^0 = \epsilon_0 (\mathbf{G}^{01} \cdot \mathbf{G}_{13} + \mathbf{G}^{02} \cdot \mathbf{G}_{23}), \tag{10c}$$

and is finite. There also emerges the self-duality condition for longitudinal components:

$$E^{2(2)}E_3^{(3)*} = c^2 B^{3(3)}B_3^{(2)*}. \tag{11}$$

The $\mathbf{B}(3)$ component of this theory is defined [4-10] in terms of a finite third Stokes parameter:

$$\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)} \tag{12}$$

giving, self-consistently, a finite field energy through the well-known relation

$$S_0 = \pm S_3 \tag{13}$$

between the zero-order ($S_0$) and third-order ($S_3$) Stokes parameters for circular polarization in free space.

## 4. SELF-DUALITY

The longitudinal magnetic component of this Yang Mills theory, sometimes referred to [4-10] as the Evans-Vigier field, has the property of being self dual. This can be expressed through the fact that $\mathbf{B}^{(3)}$ is the dual to an imaginary $i\mathbf{E}^{(3)}c$. Empirical data from the third Stokes parameter and magneto-optics show that $\mathbf{B}^{(3)}$ is nonzero and physical. At second order, however, the duality of $\mathbf{B}^{(3)}$ to $i\mathbf{E}^{(3)}/c$ can be expressed through Eq. (11), because the product of $i\mathbf{E}^{(3)}$ with its complex conjugate is real-valued and finite.

The self duality of $\mathbf{B}^{(3)}$ is unique to O(3) gauge theory. Such a concept does not occur in U(1) gauge theory.

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14. E.g., as defined in Ref. 11, Chap. 10.