# ON WHITTAKER'S ANALYSIS OF THE ELECTROMAGNETIC ENTITY, PART IV: LONGITUDINAL MAGNETIC FLUX AND TIME-LIKE POTENTIAL WITHOUT VECTOR POTENTIAL AND WITHOUT ELECTRIC AND MAGNETIC FIELDS

## ABSTRACT

The condition is defined under which there exists in vacuo Whittaker's longitudinal magnetic flux and a scalar time-like potential, but no vector potential and no electric and magnetic fields. This fundamentally changes the theory of the electromagnetic entity, for example photons emerge straightforwardly from canonical quantization of the scalar time-like potential when no vector potential and no fields are present.

## INTRODUCTION

In the fourth paper of this series, {1-3} it is demonstrated that the electromagnetic entity is fundamentally due to a scalar longitudinal time-like potential which co-exists in vacuo under well defined conditions with the magnetic fluxes ***f*** and ***g*** of Whittaker {4, 5}, which are longitudinally directed in vacuo. The condition is defined under which these quantities are non-zero, but where the vector potential and all electric and magnetic fields are zero. The electromagnetic entity under this condition therefore exists as a stream of photons, each carrying a magnetic flux $\hbar/e$. The magnetic flux density corresponding to this magnetic flux is the Evans-Vigier field, $\boldsymbol{B}^{(3)}$ {6, 10} on the O(3) level of gauge theory applied to electrodynamics. In the U(1) theory, there are no electric and magnetic fields at all, yet there is a stream of photons carrying magnetic flux. Each photon carries a flux $\hbar/e$. The vector quantities ***f*** and ***g*** defined by Whittaker are longitudinal magnetic fluxes. On the U(1) level, there is no longitudinal magnetic flux density corresponding to these fluxes. This is possible only in a mathematical sense, when the lateral extent of the plane wave is indeterminate. For every physical situation, there exists the $\boldsymbol{B}^{(3)}$ field, and this means that an O(3) gauge theory or similar must be used to describe the electromagnetic entity in vacuo and in field matter interaction.

## DEFINITION OF THE CONDITION.

It has been shown {1-3} that the transverse part of the vector potential $A$ is physical and gauge invariant, following Whittaker's logic {4, 5}, and if it is assumed for the sake of argument that it is a plane wave, the **complete** vector potential is defined in terms of the Whittaker flux ***g*** by:

$$\boldsymbol{A} = -\nabla \times \boldsymbol{g} + \frac{i}{c}\dot{\boldsymbol{g}} \tag{1}$$

which can be written for convenience as:

$$\boldsymbol{A} = A_X \boldsymbol{i} + A_Y \boldsymbol{j} + \frac{i}{c}\dot{G}\boldsymbol{k}. \tag{2}$$

The square of the complete vector is:

$$\boldsymbol{A}^2 = A_X^2 + A_Y^2 - \frac{\dot{G}^2}{c^2} \tag{3}$$

$$A^2 = (\nabla \times \boldsymbol{g})\cdot(\nabla \times \boldsymbol{g}) - \frac{1}{c^2}\dot{\boldsymbol{g}}\cdot\dot{\boldsymbol{g}} \tag{4}$$

and vanishes under the condition:

$$(\nabla \times \boldsymbol{g})\boldsymbol{\cdot}(\nabla \times \boldsymbol{g}) = \frac{1}{c^2}\dot{\boldsymbol{g}}\boldsymbol{\cdot}\dot{\boldsymbol{g}} \tag{5}$$

Under this condition therefore:

$$|\boldsymbol{A}| = 0 \tag{6}$$

but Whittaker's magnetic flux $\boldsymbol{g}$ is non–zero. It has been shown that the transverse part of the vector potential:

$$\boldsymbol{A}_T \equiv -(\nabla \times \boldsymbol{g}) \tag{7}$$

is gauge invariant and physical {1, 3}, and so under condition (5), $\boldsymbol{g}$ itself is gauge invariant and physical. If the **complete** vector potential $\boldsymbol{A}$ is used, all magnetic fields on the U(1) level vanish:

$$|\boldsymbol{B}| = |\nabla \times \boldsymbol{A}| = 0 \tag{8}$$

and because of the relation between Stratton's potential $\boldsymbol{S}$ and $\boldsymbol{A}$:

$$|\boldsymbol{S}| = ic|\boldsymbol{A}| \tag{9}$$

all electric fields, defined by

$$|\boldsymbol{E}| \equiv -|\nabla \times \boldsymbol{S}| \tag{10}$$

vanish in the U(1) level.

Condition (4) can be defined as:

$$\boldsymbol{A} = \frac{A^{(0)}}{\sqrt{2}}\left(\boldsymbol{j}\cos\phi - \boldsymbol{i}\sin\phi\right) + \kappa\left(X\cos\phi - Y\sin\phi\right)\boldsymbol{k} \tag{11}$$

$$\phi \equiv \omega t - \kappa Z \tag{11a}$$

and it is easily checked that

$$\boldsymbol{B} = \nabla \times \mathrm{A} = 0; \qquad \mathrm{E} = 0. \tag{12}$$

We have

$$|A| = \frac{A^{(0)}}{\sqrt{2}}\left[1 + \kappa^2 X^2\left(1 - \sin 2\phi\right)\right]^{\frac{1}{2}} \tag{13}$$

so $|A| \rightarrow 0$ as $\kappa - \infty$ for finite $X$, because:

$$\sin 2\phi = 1 + \frac{1}{\kappa^2 X^2}. \tag{14}$$

The maximum value of the sine function is 1.0, so to approach this condition as closely as possible, we maximize $\kappa$. In the ultraviolet region, $\kappa^2$ is about $10^{20}$ cm$^{-2}$.

Under this condition, the vacuum beam consists entirely of the time-like photon $\phi_L$. Upon interaction with matter, the d'Alembert equation:

$$\Box A = -\mu_0 j \tag{15}$$

is obtained and $A$, and $E$ and $B$ reappear, because the solutions to the d'Alembert equation is the well known Liènard-Wiechert potential:

$$A(x_0,t) = \frac{\mu_0}{4\pi}\int\frac{\left[j(x'_a)\right]dV'}{r(x_a,x'_a)} \tag{16}$$

where $j(x'_a)$ is a distribution at the point $x'_a$ and where the rectangular bracket symbol means that the variables are taken at a retarded time $t'$. The time

$$t'(x_a,x'_a) = t \pm \frac{r}{c} = t \pm \frac{\kappa}{\omega}r \tag{17}$$

corresponds to a shifting of the origin by an amount equal to the time it takes a light signal to propagate from point $x'_a$ to point $x_a$. In the presence of a non-zero $\left[j(x'_a)\right]$, $A$ is never zero. The balance condition (4) however is one in which $A$ approaches zero and in which there are no field $E$ and $B$ for circularly polarized beams. In the presence of charge and current density, the balance condition no longer holds and the solution for $G$ is no longer:

$$G = \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{(\omega t - \kappa Z)} \tag{18}$$

Also, in field-matter interaction, $f$ is no longer equal to $ig$.

It has been shown in the third paper of this series, that canonical quantization to photons can be carried out straightforwardly from the scalar time-like potential defined by {1-3}

$$\phi_L \equiv -\omega\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i\phi} \tag{19}$$

and which obeys the massless Klein-Gordon equation regarded as an equation for a real classical field. Canonical quantization results in an ensemble of massless photons obeying Bose-Einstein statistics {1-3}. This procedure remains valid even when the complete $A$ vanishes, and with it all U(1) field components.

When this stream of photons forms the time-like potential and interacts with matter, condition (4) is no longer necessarily true and fields may appear again. This seems to be what happens in the photo-electric effect, where in the quantum theory photons create a voltage or e.m.f. upon interaction with matter. This is the basis of the photoelectric industry and of great importance for energy acquisition.

## DISCUSSION

Using the logic developed by Whittaker, it has been shown straightforwardly that there can exist a physical time-like photon, the photon obtained by canonical quantization from the longitudinal potential (19). This photon exists when the complete vector potential's magnitude is zero, and when there are no U(1) electric and magnetic fields present at all. The only other entity present is Whittaker's magnetic flux $g$, which is directed longitudinally in vacuo, along the axis of propagation. These results are obtained from the Maxwell-Heaviside equations using the Lorenz condition, i.e. from the d'Alembert and Laplace equations used by Whittaker. They show several major flaws in the contemporary perception of the electromagnetic entity in vacuo and field matter interaction. Namely:

1) The potentials $\boldsymbol{A}$ and $\boldsymbol{S}$ are physical and gauge invariant, there is no gauge freedom as thought at present.

2) The photon is obtained from a physical time-like and structured scalar potential, $\phi_L$, concomitant with a physical longitudinal vector potential.

3) The problems of canonical quantization in the Coulomb and Lorenz gauges are caused by ignoring the existence of the physical time-like and longitudinal potentials.

4) In order to obtain a self-consistent magnetic flux density, $\boldsymbol{B}^{(3)}$, from Whittaker's $\boldsymbol{g}$, we must upgrade electrodynamics to an O(3) gauge theory, in order to define self-consistently the magnetic flux density $\boldsymbol{B}^{(3)}$ associated with the magnetic flux $g$. Condition (14) removes the indeterminacy in the lateral extent of a plane wave, and so makes physical sense out of this mathematical abstraction. Every beam of light, after all, has a finite radius in the laboratory.

## ACKNOWLEDGMENTS

Many colleagues world wide are acknowledged for internet discussions over the past few years, and individual institutes of AIAS (Alpha Foundation's Institute for Advanced Study) for funding for individual fellows and emeriti.

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