ELECTRODYNAMICS

ELECTRODYNAMICS AS A NON-ABELIAN GAUGE FIELD THEORY

ABSTRACT

The classical theory of electrodynamics is developed as a non-Abelian gauge field theory, offering a route to field unification through the emergence of the fundamental magnetizing field, $B^{(3)}$ observable in magneto-optics. The new theory argues that the potentials are physical constructs and not a mathematical convenience as proposed by Heaviside. The structure of the theory represents a return to the original concepts of Faraday and Maxwell, while retaining the Heaviside equations in their original form.

INTRODUCTION

Electrodynamics is generally thought to be the most perfect theory in physics and, in its quantized form, (quantum electrodynamics) describes data such as the Lamb shift with great accuracy. The theory, which has its origins in the nineteenth century, is therefore a profound achievement of communal thought. Towards the end of the twentieth century, however, it is beginning to be realized that there are several flaws inherent in its linear structure. The Maxwell equations, in the received view, are four linear differential equations in components of the electromagnetic field. The latter is thought to transmit energy/momentum from one oscillating charge to another through the vacuum. The other three foundational fields of nature – the gravitational, weak and strong – are non-linear.

In order to construct a unified field theory, it is beginning to be realized that the electromagnetic field sector must be reconstructed carefully as non-linear in its components, and updated to absorb ideas about the structured vacuum provided by the contemporary theory of the other three foundational sectors mentioned already. Several of these ideas stem from the phenomenon of Aharonov and Bohm.

Maxwell's linear field equations emerge from contemporary gauge field theory if the electromagnetic sector is given a particular linear symmetry labeled in group theory by U(1) or O(2). In the intervening vacuum between one moving charge and another, everything takes place in the plane perpendicular to the direction in which the field is traveling. Electromagnetic radiation, in the received view, is thought to consist of plane waves. The O(2) group describes rotation in a plane. Rotation in three dimensional space, however, is described by a different group, O(3), whose "twin group", or isomorphic group, is labeled SU(2), not U(1). Contemporary gauge field theory is based on group theory and produces the received Maxwell equations if and only if the description of electrodynamics is governed by the two dimensional U(1) three dimensional O(2), even in the three dimensional world of everyday experience.

The great Maxwellian theory therefore runs into a profound paradox, which is revealed through non-linear radiative phenomena such as those of magneto optics – circularly polarized radiation can magnetize matter, at its simplest one electron. These and other phenomena defy description with a field theory whose electromagnetic sector has planar and linear symmetry, so the solution of this paradox requires the equivalent of the Maxwell field equations in non-linear O(3) or SU(2) symmetry. These are given in the concluding section of this paper, and open the way to a unified field theory based on a structured vacuum as inferred from general relativity.

The natural philosophy of the classical electromagnetic field, as developed by Heaviside and the Maxwellians of the late nineteenth century, are crystalized into four differential equations which are known as the vector Maxwell equations[1]. These are, in S.I. units,

$$\nabla \times \boldsymbol{B} = 0, \tag{1}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} = 0, \tag{2}$$

$$\nabla \times \boldsymbol{D} = \rho, \tag{3}$$

$$\nabla \cdot \boldsymbol{H} = \boldsymbol{J} + \frac{\partial \boldsymbol{D}}{\partial t}, \tag{4}$$

and are respectively the Gauss; Faraday; Coulomb and Ampère-Maxwell laws. In this system:

$$\boldsymbol{D} = \varepsilon_0 \boldsymbol{E} + \boldsymbol{P}, \quad \boldsymbol{B} = \mu_0 \left(\boldsymbol{H} + \boldsymbol{M} \right).$$

Here, $\boldsymbol{E}$ is the electric field strength (V m^{-1}); $\boldsymbol{D}$ is the displacement (C m^{-2}); ρ is the charge density (C m^{-3}); $\boldsymbol{H}$ is the magnetic field strength (A^{-1}); $\boldsymbol{B}$ is the magnetic flux density (T or Wb m^{-2}); $\boldsymbol{J}$ is the current density (A m^{-2}); $\boldsymbol{P}$ is the polarization and $\boldsymbol{M}$ is the magnetization. The permittivity and permeability in vacuo in S.I. units are:

$$\varepsilon_0 = 8.854188 \times 10^{-12} \quad \text{J}^{-1}\text{C}^{-2}\text{m}^{-1}$$

and

$$\mu_0 = 4\pi \times 10^{-7} \quad \text{Js}^2\text{C}^{-2}\text{m}^{-1}.$$

Equations (1) and (2) can be combined into the homogeneous tensor equations, and eqns. (2) and (3) into the inhomogeneous tensor equations. The former is an identity in the fundamental fields $\boldsymbol{E}$ and $\boldsymbol{B}$ which can be expressed in terms of the scalar and magnetic vector potentials, ϕ and A,

$$\boldsymbol{B} = \nabla \times \text{A} \tag{5a}$$

$$\boldsymbol{E} = -\frac{\partial \boldsymbol{A}}{\partial t} - \nabla \phi. \tag{5b}$$

The quantities that appear in the inhomogeneous Maxwell equations are $\boldsymbol{D}$ and $\boldsymbol{H}$, and these are equations rather than identities of the classical theory, which allows no magnetic monopoles.

In contravariant covariant tensor notation[2], eqns. (1) to (4) become

$$\partial_\mu \tilde{\boldsymbol{F}}^{\mu\nu} \equiv 0 \tag{6a}$$

$$\partial_\mu \boldsymbol{H}^{\mu\nu} \equiv \boldsymbol{J}^\nu \tag{6b}$$

$$\tilde{F}^{\mu\nu} \equiv \begin{bmatrix} 0 & -cB^{(1)} & -cB^{(2)} & -cB^{(3)} \\ cB^{(1)} & 0 & E^{(3)} & -E^{(2)} \\ cB^{(2)} & -E^{(3)} & 0 & E^{(1)} \\ cB^{(3)} & E^{(2)} & -E^{(1)} & 0 \end{bmatrix} \tag{6c}$$

where $F^{\mu\nu}$ is the dual of the anti-symmetric field tensor F,

$$\tilde{F}^{\mu\nu} \equiv \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}. \tag{7}$$

Equation (6a) is a Jacobi identity,

$$\partial^\lambda F^{\mu\nu} + \partial^\mu F^{\mu\lambda} + \partial^\nu F^{\lambda\mu} \equiv 0. \tag{8}$$

In this notation, the basic four vectors are:

$$x^\mu = (x^0, x^{1,}x^2, x^3) = (ct, X, Y, Z)$$
$$x_\mu = (x_0, x_1, x_2, x_3) = (ct, -X, -Y, -Z) \tag{9}$$

and the basic derivatives are

$$\frac{\partial}{\partial x^\mu} \equiv \partial_\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, \frac{\partial}{\partial X}, \frac{\partial}{\partial Y}, \frac{\partial}{\partial Z} \right) = (\partial_0, \partial_1, \partial_2, \partial_3)$$
$$\frac{\partial}{\partial x_\mu} \equiv \partial^\mu = \left(\frac{1}{c}\frac{\partial}{\partial t}, -\frac{\partial}{\partial X}, -\frac{\partial}{\partial Y}, -\frac{\partial}{\partial Z} \right) = (\partial^0, \partial^1, \partial^2, \partial^3). \tag{10}$$

It follows that

$$E_1 = -E_X, \qquad E_2 = -E_Y, \qquad E_3 = -E_Z,$$
$$E^1 = E^X, \qquad E^2 = E^Y, \qquad E^3 = E^Z. \tag{11}$$

and so forth, and that:

$$F^{\mu\nu} = \begin{bmatrix} 0 & -E^{(1)} & -E^{(2)} & -E^{(3)} \\ E^{(1)} & 0 & -cB^{(3)} & cB^{(2)} \\ E^{(2)} & cB^{(3)} & 0 & -cB^{(1)} \\ E^{(3)} & -cB^{(2)} & cB^{(1)} & 0 \end{bmatrix} \tag{12a}$$

$$F_{\mu\nu} = \begin{bmatrix} 0 & -E_{(1)} & -E_{(2)} & -E_{(3)} \\ E_{(1)} & 0 & cB_{(3)} & -cB_{(2)} \\ E_{(2)} & -cB_{(3)} & 0 & cB_{(1)} \\ E_{(3)} & cB_{(2)} & -cB_{(1)} & 0 \end{bmatrix} \tag{12b}$$

$$\tilde{F}^{\mu\nu} = \begin{bmatrix} 0 & -cB^{(1)} & -cB^{(2)} & -cB^{(3)} \\ cB^{(1)} & 0 & E^{(3)} & -E^{(2)} \\ cB^{(2)} & -E^{(3)} & 0 & E^{(1)} \\ cB^{(3)} & E^{(2)} & -E^{(1)} & 0 \end{bmatrix} \tag{12c}$$

$$\tilde{F}_{\mu\nu} = \begin{bmatrix} 0 & -cB_{(1)} & -cB_{(2)} & -cB_{(3)} \\ cB_{(1)} & 0 & -E_{(3)} & E_{(2)} \\ cB_{(2)} & E_{(3)} & 0 & -E_{(1)} \\ cB_{(3)} & -E_{(2)} & E_{(1)} & 0 \end{bmatrix} . \tag{12d}$$

The symbol $\varepsilon^{\mu\nu\rho\sigma}$ is the fully anti-symmetric tensor in four dimensions with $\epsilon^{0123} = 1$. Using the rules of tensor algebra with summation of repeated indices, the reader should check that eqn. (6a) gives eqns. (1) and (2) and that eqn. (6b) gives eqns. (3) and (4) with the definitions

$$H^{\mu\nu} \equiv \begin{bmatrix} 0 & -D^{(1)} & -D^{(2)} & -D^{(3)} \\ D^{(1)} & 0 & -H^{(3)}/c & H^{(2)}/c \\ D^{(2)} & H^{(3)}/c & 0 & -H^{(1)}/c \\ D^{(3)} & -H^{(2)}/c & H^{(1)}/c & 0 \end{bmatrix} . \tag{13}$$

The scalar and vector potentials can be combined into a potential four-vector such that

$$F^{\mu\nu} \equiv \partial^\mu A^\nu - \partial^\nu A^\mu \tag{14a}$$

where

$$A^\mu \equiv (\phi, cA). \tag{14b}$$

The reader should check that eqns. (14) give eqns. (5). This is a concise summary of the great Maxwellian theory of classical electrodynamics, one of the most elegant and profound in all natural philosophy.

CLASSICAL LIMITATIONS OF THE MAXWELLIAN THEORY

A vast amount of empirical data has been described with precision by eqns. (6). The classical limitations of the theory are however well known. Ritz[3], for example, argued precisely and elegantly against it on the grounds that Maxwell's ether was mechanically unsound, and that the third law of Newton is not complied with. This is the well known radiation reaction problem treated at length in a classic mid-twentieth century text such as that by Jackson[4].

Recently, it has been argued by Barrett[5] and Bearden[7] that there are intrinsic assumptions which make the theory far more restrictive than hitherto realized, even within its own classical framework, i.e., without quantization being considered at all. Both have argued against it on the grounds that A^μ is not gauge invariant in the Maxwell theory as developed by Heaviside and is therefore unphysical and unobservable. They therefore criticize what has been regarded as the basis of the classical theory – its gauge freedom. In the standard vector notation, this problem can be isolated quite easily as follows. It is possible in the rules of vector algebra to add to A and to ϕ an **arbitrary** variable ψ such that[4]

$$A \to A - \nabla\psi, \qquad\qquad \phi \to \phi + \frac{\partial\psi}{\partial t}, \qquad\qquad (15)$$

without affecting E and B in Eq. (5). In tensor notation this **gauge transformation** can be expressed as

$$A_\mu \to A_\mu + \partial_\mu(c\psi), \qquad\qquad (16)$$

which is conveniently written as

$$A_\mu \to A_\mu + \partial_\mu\phi. \qquad\qquad (17)$$

In the standard development, the gauge freedom allowed by Eq. (17) is used to assert the Lorenz gauge condition. In vector algebra,

$$\nabla \cdot A + \frac{1}{c^2}\frac{\partial\phi}{\partial t} = 0, \qquad\qquad (18)$$

and in tensor algebra,

$$\partial_\mu A^\mu = 0. \qquad\qquad (19)$$

Using eqns. (10) and (14b), it is seen that eqns. (18) and (19) are the same in S.I. units. The importance of the **arbitrary** eqn. (19) is that it allows the derivation from eqn. (14a) of the d'Alembert wave equation which can be written symmetrically in ϕ and A. Upon quantization this leads to the idea of a massless particle, the photon, and in classical electrodynamics to the idea of advanced and retarded solutions.

Criticisms of these well known ideas have been based recently[5-8] on the fact that eqns. (1) and (4) are vector equations in fields with no boundary conditions stated. They were derived not by Maxwell but by Heaviside and the Maxwellians of the late nineteenth century by "murdering the potentials", i.e., by relegating A^μ to a subsidiary mathematical role. Evidently, the arbitrary nature of ϕ in the gauge transform, Eq. (17), means that A^μ **cannot be physical** in Heaviside's interpretation of Maxwell. The latter used quaternionic (SU(2)) algebra in twenty equations in which the potentials were physical. Faraday argued[5-7] for a prototypically physical potential which he named the "electrotonic state". Bearden[7] has correctly argued that the Lorenz condition is equally arbitrary in the Heaviside formulation because the Lorenz condition is one on ϕ and A, which are both arbitrary quantities in this view. A combination of such quantities, howsoever

apparently useful, is arbitrary and unphysical. Bearden refers to the Lorenz condition as symmetrical regauging[7], a process which loses information about the vacuum flux, from which energy can be obtained. The transverse gauge, in which ϕ is set to zero, shows that the quantity ϕ can become independent of A without any change in the transverse field components, for example vacuum plane waves. If one takes Heaviside's viewpoint, this is explained on the grounds that only fields are physical. However, Schwarzschild[8] has shown that eqns. (1) to (4) can be replaced **completely** by delayed action at a distance equations involving only **classical** potentials, which are therefore just as physical as fields. Whittaker[9-11] has shown that the scalar potential is structured and the terminology "scalar potential" is therefore contradictory.

Barrett[5,6] has demonstrated that the interpretation of several physical effects relies on a physical and classical A^{μ}, in Heaviside's view a contradiction in terms. These phenomena include the Sagnac; Aharonov-Bohm; topological phase; de Haas van Alphen; quantum Hall and Ehrenberg-Siday effects. These important criticisms lead to the necessity of a major philosophical revision of classical electrodynamics because the Heaviside vector equations (1) to (4) on which they are based are interpreted using a non-physical A^{μ}. This contradicts experimental evidence[5,6] and another explanation must be found for the fact that eqns. (1) to (4) have appeared to be so successful. Such an advance is also needed because, in quantum mechanics, the vector potential is primary and all. It should also be so in classical mechanics if we are to achieve any kind of intellectual consistency.

ABELIAN AND NON-ABELIAN CLASSICAL GAUGE FIELD THEORY

The four equations (1) to (4) are linear in the fields and differential operators and can be derived from an Abelian-Lagrangian through the Euler-Lagrange equation of motion.[2] The Abelian rotation group O(2) is the group of rotations in two dimensions[2], and is homomorphic with the group of complex numbers of the form $e^{i\alpha}$, the group U(1). Similarly O(3) is holomorphic with SU(2). In contemporary gauge field theory, the electromagnetic field is the gauge field that guarantees invariance under local U(1) transformations. The gauge transform becomes synonymous with the U(1) covariant derivative,[2]

$$D_{\mu} \equiv \partial_{\mu} + i\frac{e}{\hbar}A_{\mu} \equiv \partial_{\mu} - igA_{\mu}, \tag{20}$$

which is the minimal prescription expressed in coordinate space rather than momentum space, where it is

$$p_{\mu} \rightarrow p_{\mu} - eA_{\mu}, \tag{21}$$

Therefore the electromagnetic field adds eA_{μ} to the four-momentum p_{μ}, a process which occurs in a scalar inner space. Comparing (20) and (21) gives the well known ansatz on the four-momentum in quantum mechanics,

$$p_{\mu} \rightarrow i\hbar\partial_{\mu}, \qquad En \rightarrow i\hbar\frac{\partial}{\partial t}p \rightarrow \hbar\nabla \tag{22}$$

so this introduces a physical A_{μ}, because the ansatz is one of quantum mechanics in which A_{μ} is **physical**. This is self-contradictory in U(1) electrodynamics because, in this theory, A_{μ} is unphysical classically and we cannot apply an ansatz on an unphysical quantity (the classical A_{μ}) to obtain a physical quantity (the "quantized" A_{μ}). Yet this is the long accepted wisdom because it is a useful calculating device.

To attempt to remove this trouble, a more general form of gauge theory can be introduced into classical electrodynamics by borrowing ideas from high energy physics, where the field can be any field, or unified field, and in which derivatives f_{μ} are replaced by covariant derivatives, D_{μ}, defined for any group. In this general gauge theory, the potentials are affine connections and are physical, universal influences.[2, 12-15]

The gauge transform is a physical coordinate transformation; the field tensor $G_{\mu\nu}$ is a commutator of

covariant derivatives for any field and group, and is rigorously covariant under a gauge transform. The field tensor and vector potentials are written in an internal space superposed on space-time, and in general, the theory leads to a non-Abelian Lagrangian and non-Abelian relations between physical fields and physical potentials. A gauge transform of the potential in any group symmetry is a coordinate transform, a physical, covariant process. These results are obtained elegantly from a round trip with covariant derivatives. The same process leads to a generally valid Jacobi identity first derived by Feynman[2]:

$$D^v \tilde{G}_{\mu v} \equiv 0, \tag{23}$$

and we can show[12] that this equation leads back to the Gauss and Faraday Laws (eqns. (1) and (2)) with new information, specifically the appearance under all conditions of a fundamental, real, phaseless, irrotational, gradientless and time-independent magnetic field[12-15],

$$B^{(3)} \equiv -ig A^{(1)} \times A^{(2)}, \tag{24}$$

where g is a constant and where $A^{(1)} \times A^{(2)} = A \times A^*$, and A^* is the complex conjugate of A. The magnetic field $B^{(3)}$ is observed as the physical quantity $A \times A^*$ in magneto-optical experiments. This demonstrates the superiority of the new theory (an O(3) symmetry theory) over the older U(1) symmetry theory because, in the latter, A and A^* are random, and $A \times A^*$ is unphysical. In fact, in the rigorous field theory that leads to eqn. (24), $A \times A^*$ is identically **zero** in U(1) **linear, Abelian**, classical electrodynamics[2].

The meaning of "gauge transformation" is changed fundamentally during the course of this generalization. In the older view, it introduces a serious internal inconsistency because the gauge transform produces in the minimal prescription[2],

$$p_\mu \to p_\mu - e A_\mu + e \partial_\mu \Phi, \tag{25}$$

and since ϕ is **undefined**, then p_μ **becomes random or arbitrary** after U(1) gauge transformation. This is a catastrophe for the classical, linear theory because eqn. (25) is a violent contradiction of observation. The same gauge transformation in quantum mechanics can be written as

$$\left(\partial_\mu + i \frac{e}{\hbar} A_\mu \right) \psi \to \partial_\mu (\psi + \Phi) + i \frac{e}{\hbar} A_\mu \psi, \tag{26}$$

and is normally interpreted as the addition of an arbitrary phase factor into the wave function. Thus, for example, the Dirac equation is said to be gauge invariant in the presence of an electromagnetic field. However, this long held interpretation of gauge transformation does not change the fact that the numerical value of the still classical A_μ in, for example, the Dirac equation, is changed **arbitrarily and classically** from A_μ to $A_\mu + \partial_\mu \Phi$. Additionally, canonical quantization in the Lorenz gauge is beset with difficulties as is well known, and leads to a dubious "elimination" of longitudinal photons with the Gupta-Bleuler condition[2],

$$\langle \psi | \partial_\mu A^\mu | \psi \rangle = 0. \tag{27}$$

This has to be used because

$$\partial_\mu A^\mu | \psi \rangle \neq 0, \tag{28}$$

even in the vacuum. Lastly, the Lorenz condition itself violates the fundamental covariant commutation ansatz of the canonical quantization process[2]. The validity of the whole quantization procedure depends on the famous gauge fixing term $-\left(\lambda/2 \right)\left(f_\mu A^\mu \right)^2$, which is arbitrary and unphysical in the same classical

theory that is being subjected to attempted quantization. These well known difficulties of canonical quantization of U(1) electrodynamics arise from attempts to make the assumed particulate photon concomitant with the transverse plane waves of the received view while keeping A_μ manifestly covariant. In fact, this is a hopelessly confused theory in an area central to modern physics.

Electromagnetism in the linear, classical U(1) theory is simply an augmentation of the space-time translation generator $P_\mu \equiv p_\mu / \hbar = i\partial_\mu$ of the Poincaré group[2,15],

$$P_\mu \to P_\mu + gA_\mu.$$ (29)

In this view, the electromagnetic field produces **only a translation** in Minkowski, or flat, space-time, in which

$$\left[P_\mu, P_\nu \right] = 0, \qquad \left[A_\mu, A_\nu \right] = 0$$ (30)

and if gauge theory is looked upon as a round trip with covariant derivatives, the above corresponds to a U(1), Abelian group theory, with fields always linearly related to potentials. Therefore the linear, Maxwellian electrodynamics are built on the assumption that the space-time translation generators gA_μ commute.

In the O(3) gauge theory on the other hand (Yang-Mills theory[2,6] applied to electromagnetism[5,6,12-15]),

$$\left[A_\nu^{(1)}, A_\mu^{(2)} \right] = \frac{i}{g} B_{\nu\mu}^{(3)*},$$ (31)

and the minimal prescription is extended to give eqn. (31) together with

$$\left[A_\nu^{(1)}, A_\mu^{(1)} \right] = \left[A_\nu^{(2)}, A_\mu^{(2)} \right] = 0.$$ (32)

From the structure of Eq. (31) the complex potentials can take on the role of space-time rotation generators[2,13-15] as well as **space-time translation generators** (eqns. (32)). The fundamental reason for the properties (31) to (33) is that the potentials $A_\nu^{(1)}$ and $A_\mu^{(2)}$ are complex with two states of polarization: the electromagnetic field is circularly polarized in general.

In this O(3) gauge theory, the covariant derivative and minimal prescription are structured in such a way that Eq. (23) becomes

$$\partial^\nu \tilde{G}_{\mu\nu}^{(i)} = 0, \qquad i = 1, 2, 3,$$ (33a)

$$B^{(1)} \times B^{(2)} = iB^{(0)} B^{(3)*},$$ (33b)

and it can be shown that eqn. (33b) is identical with eqn. (31). Equation (33a) with $i = 3$ is consistent with the fact that $B^{(3)}$ is time independent and divergentless,

$$\nabla \cdot B^{(3)} = 0, \qquad \frac{\partial B^{(3)}}{\partial t} = 0.$$ (33c)

Equation (33a) with $i = 1$ or 2 gives eqns. (1) and (2).

Finally, there is a **fundamental** philosophical difference between U(1) and O(3) electrodynamics which can be found in the interpretation of "gauge transformation". In the non-Abelian gauge field theory, gauge transformation is a coordinate transformation[2] as in general relativity, and therefore a classical and physical process. For example, if (3) is aligned with Z, rotation around Z leaves $B^{(3)}$ unchanged, while $A^{(1)} = A^{(2)*}$ are rotated by a rotation matrix or generator. This is frame rotation, **an O(3) gauge transformation** under which $B^{(3)}$ is invariant. More generally, the classical $B^{(3)}$ is covariant, but always physical, under an O(3) gauge transform, or coordinate transform. A Lorentz boost in Z leaves $B^{(3)}$ invariant if (3) is aligned with Z, and so on. This concept of a physical gauge transform leads to a straightforward quantization method[13-15], based on angular momentum. For example, eqn. (33b) is a classical angular momentum relation, a relation between rotation generators of O(3). As shown by Atkins[1], this type of relation produces virtually everything in quantum mechanics. **The difficulties with U(1) canonical field quantization disappear**.

CONCLUSION: THE O(3) FIELD EQUATIONS

The four Maxwell equations in O(30 symmetry replace eqns. (1) to (4) in the required non-linear and non-Abelian theory of electrodynamics. They are as follows:

The Homogeneous Field Equations

$$D_\mu \tilde{G}^{\mu\nu} = 0, \tag{34}$$

The Inhomogeneous Field Equations

$$D_\mu G^{\mu\nu} = \frac{J^\nu}{\varepsilon_0}, \tag{35}$$

with,

$$\tilde{G}^{\mu\nu} = \frac{1}{2}\varepsilon^{\mu\nu\rho\sigma} \tilde{G}_{\rho\sigma}. \tag{36}$$

In the presence of material magnetization and polarization,

$$D_\mu H^{\mu\nu} = J^\nu. \tag{37}$$

These equations can be expressed in any convenient basis, such as the O(3) basis labeled ((1), (2), (3)), based on the fact that radiation is circularly polarized, and they replace the U(1) Maxwell equations (1) to (4). In order to progress to a unified, non-linear field theory, these equations must be the starting point for the description of any electromagnetic phenomenon at the classical level.

ACKNOWLEDGEMENTS

REFERENCES

1.   Atkins, P.W. 1983. *Molecular Quantum Mechanics*. Oxford: Oxford University Press, 2nd edn.
2.   Ryder, L.H. 1997. *Quantum Field Theory*. Cambridge: Cambridge University Press, 2nd Edn.

3.   Ritz, W. 1908. Recherches critiques sur l'électrodynamique générale. *Annales de Chimie Physique* 13(8), 145-209; Fritzius, R.S. 1998. Translation: Critical researches in general electrodynamics. *Apeiron*; Evans, M.W. 1998. Commentary on Ritz's work as translated by Fritzius. *Apeiron*.

4.   Jackson, J.D. 1962. *Classical Electrodynamics*. New York: Wiley.

5.   Barrett, T.W. 1993. Electromagnetic Phenomena Not Explained by Maxwell's Equations. In Lakhtakia, A., ed., Essays on the Formal Aspects of EM Theory. Singapore: World Scientific pp. 6 ff.

6.   Barrett, T.W. 1995. Sagnac Effect: A consequence of conservation of action due to gauge field conformal invariance in a multiply-joined topology of coherent fields. In T.W. Barrett and D. M. Grimes (eds.), *Advanced Electrodynamics, Foundations, Theory and Applications*. Singapore: World Scientific pp. 278 ff.

7.   Bearden, T.E. 1997.

8.   Schwarzschild, K. 1903. Gottinger Nachr., Math.-Phys. Klasse, p. 126; discussed in ref. (3), pp. 156 ff.

9.   Whittaker, E.T. 1989. *A History of the Theories of Aether and Electricity*. New York: Dover

10. Whittaker, E.T. 1903. On the partial differential equations of mathematical physics. *Mathematical Annals* 57, 333-355..

11. Whittaker, E.T. 1904. On an expression of the electromagnetic field due to electrons by means of two scalar potential functions. *Proceedings, London Mathematical Society* 1(2), 367-372.

12. Evans, M.W. 1998. Classical Electrodynamics as a Non-Abelian Gauge Field Theory, *Foundations of Physics*, in press.

13. Evans, M.W. and Kielich, S., eds. 1992, 1993, 1997. *Modern Nonlinear Optics*. New York: Wiley.

14. Evans, M.W. and Hasanein, A.A. 1994. *The Photomagneton in Quantum Field Theory*. Singapore: World Scientific.

15. Evans, M.W., Vigier, J.P. Et al. 1994-1998. *The Enigmatic Photon, Vol. 1-5*. Dordrecht: Kluwer Academic Publishers.