ON THE INTERPRETATION OF THE CHARGE *e* IN NON-ABELIAN ELECTRODYNAMICS: CONSERVATION OF CAUSALITY

ABSTRACT

In non-Abelian theories of electrodynamics, the elementary charge e appears in the basic definition of the field tensor, pre-multiplying a non-zero commutator of potentials. In this paper, the meaning of the universal constant e is interpreted as a coupling constant between source and field, so causality is conserved, the field is always generated by a source, an electron, and the "signature" of this source, e, is always present in the field tensor and concomitant non-Abelian field equations. There can be no source-free regions, as in Abelian (or Maxwell-Heaviside) electrodynamics. The idea of a source-free region means that there is an effect (the field) without a cause (the source), and this is in conflict with causality, a basic self-inconsistency of Abelian electrodynamics.

INTRODUCTION

Contemporary gauge theory derived from an attempt by Yang and Mills in 1955 {1} to generalize electrodynamics to a non-Abelian gauge theory in which there is an internal gauge symmetry which is not U(1). Recently, ideas about non-Abelian electrodynamics have been developed extensively {2-12} and found to be more self-consistent and far richer in structure and concept than the received Abelian theory whose sector symmetry is U(1), and which is represented by the famous Maxwell-Heaviside equations. It has been shown {2-12} that electrodynamics possesses non-Abelian internal gauge indices, labeled (1), (2) and (3), which derive from circular polarization. Therefore electrodynamics can be represented both classically and quantum mechanically by a non-Abelian gauge theory which possesses several advantages over the received Maxwell-Heaviside equations. The latter usually appear within the standard model as a U(1) sector, a gauge theory where the cross product $\boldsymbol{A} \times \boldsymbol{A}^* = \boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ is identically zero by definition. This is self-inconsistent because this conjugate product is a well known observable in the third Stokes parameter and in magneto-optics, typified {2-12} by the inverse Faraday effect and radiatively induced fermion resonance.

In this paper, the non-Abelian structure of electrodynamics is shown to possess a critical advantage over its received Abelian counterpart in that it conserves causality under all conditions. In Maxwell-Heaviside electrodynamics, causality is lost whenever it is asserted that there may exist in nature a source-free region in which the electromagnetic field propagates in the vacuum without a source. There is therefore an effect, but no source, or cause of the effect. In non-Abelian electrodynamics, the universal constant e, which measures the coupling between field and source, or effect and cause, is always present in the field tensor and is the "signature" of the fact that the field emanates originally from a source (an electron) even though this may be distant to quasi-infinity. This result is a clear indication of the advantage of non-Abelian ideas about electrodynamics over Abelian ideas of all kinds where there can be source-free fields in which e is absent. In Section 2, some ideas from general gauge field theory are used to show the presence of e in the non-Abelian field tensor, one component of which is the so-called Evans-Vigier field $\boldsymbol{B}^{(3)}$, which is longitudinally directed in vacuo and which is the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ multiplied by the universal constant $-ie/\hbar$. In section 3, the field equations are discussed and shown also to conserve causality under all conditions.

NON-ABELIAN ELECTRODYNAMICS

In contemporary gauge theory, the field tensor is derived from a closed loop in Minkowski space-time using covariant derivatives {13}, or parallel transport. A general n-dimensional field ψ is transported around the closed loop and there is interaction between ψ and the covariant derivative which contains Feynman's "universal influence", the potential A_μ, which in general has non-Abelian internal indices {2-13}. The round trip journey shows that the commutator of covariant derivatives is non-zero, and given by:

$$\left[D_\mu, D_\nu\right] = -ig\left(\partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right]\right). \tag{1}$$

The factor g is a coupling constant indicating interaction between fields in this process, which is mathematically quite general. If the coupling constant g were zero, the commutator would vanish, a self-contradiction. The general field tensor is given by {13}:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - ig\left[A_\mu, A_\nu\right] \tag{2}$$

where the commutator of potentials is in general non-zero, and may contain non Abelian internal indices {2-13}. The reduction to the received view of electrodynamics is made by asserting erroneously that the commutator $[A_\mu, A_\nu]$ is zero for electrodynamics, leaving the familiar Abelian field tensor:

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu \tag{3}$$

where the potentials have no internal indices. This is the familiar four-curl of the Maxwell-Heaviside electrodynamics. This is an erroneous procedure however because A_μ in electrodynamics is in general complex and the commutator $[A_\mu, A_\nu]$ is non-zero - it is a physical observable in the third Stokes parameter and in magneto-optics {2-12}. In non-Abelian electrodynamics, the quantity $A^{(1)} \times A^{(2)}$ is self-consistently non-zero {2-12} and describes the physical observable. The complete electrodynamical field tensor is therefore:

$$G^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu + g\varepsilon_{abc} A^b_\mu A^c_\nu \quad \text{et cyclicum} \tag{4}$$

in the complex basis ((1), (2), (3)) {2-12}. One of the components of this tensor is the longitudinally directed magnetic field {14}:

$$\boldsymbol{B}^{(3)} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)} \tag{5}$$

which contains the universal constant $g = e/\hbar$ as a premultiplier. The interpretation of the universal constant e is that it is the coupling factor between the source of the field (an electron) and the field. It does not mean that mass is present in the field, and does not mean that the field acts as its own source. Causality is always conserved because the coupling constant $e/\hbar$ between the field and the source is always present in non-Abelian electrodynamics. In the Abelian case, which is obtained by **incorrectly** asserting that the commutator $[A_\mu, A_\nu]$ is zero, the coupling constant e is not present, indicating that the Abelian electromagnetic field is free of its source. This is cause without effect and a violation of causality because an electromagnetic field must derive always from a source.

It is well known from special relativity {15} that two charges e co-moving at c neither attract nor repel, so two classical non-Abelian electromagnetic beams do not interact. This conclusion is predicated by the assumption that the mass of the photon is zero.

THE FIELD EQUATIONS

When classical electrodynamics are written in non-Abelian form with an internal O(3) symmetry gauge space ((1), (2), (3)), the field equations become:

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{6}$$

$$D_\mu \boldsymbol{H}^{\mu\nu} = \boldsymbol{J}^\nu \tag{7}$$

respectively the homogeneous and inhomogeneous field equations. The symbol D_μ denotes the O(3) symmetry covariant derivative {2-12} and the field tensors $\tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0}$ and $\boldsymbol{H}^{\mu\nu}$ are vector quantities in the internal gauge space. The four-current $\boldsymbol{J}^\nu$ is similarly a vector in the internal gauge space. The consequences of these equations have been developed extensively {2-12}, and no conflict occurs with experimental data. Indeed, the equations are much more self-consistent than their Abelian counterparts - the received Maxwell-Heaviside equations {16}.

It is important to note that the homogeneous field equation (6) - a Feynman Jacobi identity {13} - contains the universal constant $e/\hbar$, because it can be written as:

$$\left(\partial_\mu + \frac{e}{\hbar}\boldsymbol{A}_\mu \times\right)\tilde{\boldsymbol{G}}^{\mu\nu} \equiv \boldsymbol{0} \tag{8}$$

The presence of e in this equation is the "signature" of the fact that the field $\tilde{\boldsymbol{G}}^{\mu\nu}$ originates in a source, an electron, which may be situated quasi-infinitely away from the field. In the Abelian counterpart of eqn. (6), the familiar equation:

$$\partial_\mu \tilde{F}^{\mu\nu} \equiv 0 \tag{9}$$

the coupling constant $e/\hbar$ does not appear, and the field is interpreted as free of its source. This violates causality as discussed already. A field must always have a source with which it interacts through the universal constant $e/\hbar$ which does not depend on time or distance.

The inhomogeneous equation (7) is interpreted as generating a field in the vicinity of a four current $\boldsymbol{J}^\nu$. The universal constant $e/\hbar$ is interpreted in the same way, as coupling the source and field (Dirac field and electromagnetic field in quantized form).

DISCUSSION

When it comes to extending the classical theory of non-Abelian electrodynamics to its counterpart in quantum electrodynamics, {12} the presence of the source (or electron) becomes critically important to prevent the occurrence of charged photons. It is possible to develop a self-consistent and powerful non-Abelian theory of quantum electrodynamics if the source to field coupling is present. This coupling is represented by the universal constant $e/\hbar$. A large number of new insights emerge from such a non-Abelian theory of quantum electrodynamics, which is again based on the fact that the commutator $[A_\mu, A_\nu]$ does not vanish in electrodynamics.

In conclusion, the presence of e in the field tensor and homogeneous field equation of classical non-Abelian electrodynamics means that causality is always conserved, there is always a source to the field. This is a major conceptual advance over the concept of source free field in Abelian electrodynamics.

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