TOPOLOGICAL EXPLANATION OF THE SAGNAC AND MICHELSON EFFECTS

ABSTRACT

The Sagnac and Michelson effects are explained using the topological magnetic field $B^{(3)}$ from the non-Abelian Stokes Theorem applied to classical electrodynamics. Conventional Maxwell-Heaviside theory can explain neither effect. The Sagnac formula with platform in motion is obtained as a gauge transformation.

INTRODUCTION

A phase difference is seen in the Sagnac interferometer {1-5} with platform at rest, and this shifts according as to whether the platform is rotated clockwise (C) or anticlockwise (A). In the Michelson interferometer, a phase shift is observed by the recombination of two beams at a beam splitter after reflection from two orthogonal mirrors {6-8}. In this paper, both effects are explained using a topological magnetic field in classical electrodynamics through the use of a non-Abelian Stokes Theorem {9,10}. In Section 2, it is shown that the conventional Maxwell-Heaviside theory does not explain the observed Sagnac or Michelson phase shifts and in Section 3, a topological explanation is given for both effects. The Sagnac formula is derived straightforwardly as a non-Abelian gauge transformation.

FAILURE OF THE MAXWELL-HEAVISIDE THEORY

The vacuum d'Alembert equation is invariant under motion reversal symmetry (T); parity inversion symmetry (P); and gauge transformation. It is also metric independent {9, 10}. The Maxwell-Heaviside equations have the same properties in the vacuum. These equations are invariant under a U(1) gauge transformation in general gauge field theory {11}, and such a procedure adds a random phase to the electromagnetic phase. The C loop is generated from the A loop in the Sagnac effect by the T operator. In the Michelson interferometer, the beam traveling from mirror to beam splitter is generated from the beam traveling from beam splitter to mirror by the P operation, which for normal incidence and for a perfectly reflecting mirror, is equivalent precisely to reflection.

These invariance properties of the d'Alembert and Maxwell-Heaviside equations in vacuo mean that exactly the same equations describe the A and C loops in the Sagnac effect, with exactly the same solutions, including phase. There can be no phase difference with platform at rest, contrary to observation {1-5} and the Maxwell-Heaviside equations fail to describe the Sagnac effect with platform at rest. The equations are invariant to rotation {9, 10} and also fail to describe the Sagnac effect with platform in motion.

The equations are similarly invariant under P, and for perfect normal incidence and perfect reflection, there is no phase shift in the Michelson interferometer, contrary to observation {6-8}. The phase arriving back at the beam splitter from either arm is exactly the same as the original phase, and this result is independent of the length of either arm of the Michelson interferometer. Moving one mirror with respect to the other has no effect in the Maxwell-Heaviside theory, obviously contrary to observation {6-8}.

These are two major failures of the Maxwell-Heaviside theory, invariant under U(1) gauge transformation {11}.

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The results described in section 2 are due to a major failure in the underlying U(1) gauge symmetry {11} of conventional Maxwell-Heaviside electrodynamics. If we adopt an O(3) gauge symmetry, {12} the Sagnac effect becomes a round trip in Minkowski space-time described by a non-Abelian Stokes Theorem. It is convenient to adopt the notation given by Broda {13}:

$$P \exp\left(i \oint_{\partial S=C} A_i(x)dx^i \right) = P' \frac{i}{2}\left(\int_S F_{ij}(x)dx^i \wedge dx^j \right) \tag{1}$$

where P denotes path ordering, and P' surface ordering. The field tensor is described by:

$$F_{ij} = \partial_i A_j - \partial_j A_i - ig\left[A_i, A_j \right] \tag{2}$$

and the gauge potential is a connection of the form:

$$A_i = A_i^a T^a \tag{3}$$

with O(3) commutation relations:

$$\left[T^a, T^b \right] = i\varepsilon^{abc} T^c \tag{4}$$

Eqn. (1) can be reduced straightforwardly to the form:

$$\oint_1 \kappa_z dZ - \oint_2 \kappa_z dZ = g \oint_S B^{(3)} \cdot dS \tag{5}$$

which is P and T negative. The integrals on the left hand side are path ordered, and on the right hand side appears a surface ordered area integral over the magnetic field $B^{(3)}${12, 14-23}, which is topological in origin. The coefficient g is a coupling constant between source and field, and, in this classical theory, has the units of $\kappa/A^{(0)}$ where κ is the wavevector, and $A^{(0)}$ the magnitude of the vector potential related to the magnitude of $B^{(3)} = B^{(0)}k$ by $B^{(0)} = \kappa A^{(0)}$.

Eqn. (5) gives the phase seen experimentally in the Sagnac effect. From the definition of $B^{(3)}$ in the underlying O(3) gauge theory, we obtain:

$$\oint_C \kappa_z dZ - \oint_A \kappa_z dZ = g \oint_{\cdot S} B^{(3)} \cdot dAr \tag{6}$$
$$= \kappa^2 Ar$$

where Ar is the area of the Sagnac loop. This result is independent of the shape of the loop and is a number, independent of whether the observer is on the platform, as in the Michelson-Gale experiment {24}, or off it.

The underlying O(3) gauge theory is gauge covariant, and an observable phase shift is generated by reorientation in the internal gauge space, which in this case is the physical space of three dimensions {11 - 23}. As a result, a rotation of the platform is a gauge transformation, which results straightforwardly in {23}:

$$\omega \rightarrow \omega \pm \Omega \tag{7}$$

using the rules of O(3) gauge transformation. Here Ω is the angular frequency of rotation of the platform. Using eqn. (7) in eqn. (6) gives the extra phase shift $(A - C)$ seen in the Sagnac effect by rotating the platform:

$$\Delta\phi_{A-C} = \frac{4\omega\Omega A r}{c^2} \tag{8}$$

Using $\Delta\phi = \omega\Delta t$, we obtain the well known Sagnac formula {1-5}:

$$\Delta t_{A-C} = \frac{4\Omega A r}{c^2} \tag{9}$$

which has been verified experimentally to very high precision {24}. The O(3) theory provides a precise explanation, the U(1) theory fails completely.

The Michelson effect is similarly given by eqn. (5), where the path ordered integrals on the left hand side are P negative, equal and opposite, and generate the observed phase shift. Moving one mirror with respect to the other increases Z in one arm, generating the well known Michelson interferogram, the basis for the contemporary Fourier transform infrared spectrometer.

DISCUSSION

In the underlying O(3) gauge field theory, the topological magnetic field $B^{(3)}$ is defined in the complex basis ((1), (2), (3)) {12} as:

$$B^{(3)*} \equiv -igA^{(1)} \times A^{(2)} \tag{10}$$

and is directed longitudinally in the axis of propagation of the light. It is the fundamental spin {14-22} of the electromagnetic field and its theory has been well developed {12-22} recently. In the Sagnac effect, the ordered surface integral is P and T negative, and the surface is the area of the Sagnac loop, as observed {1-5}. In the Michelson effect, the area integral is P and T negative and defined by the phase difference on the left hand side.

It is concluded that an O(3) gauge theory applied to classical electrodynamics gives a precise explanation of the Sagnac and Michelson effects, while a U(1) gauge theory fails completely in both cases. This suggests strongly that classical electrodynamics is in general a non-Abelian gauge field theory {12-22}.

ACKNOWLEDGMENTS

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