CALCULATION OF THE SAGNAC EFFECT IN MATTER WAVES USING AN O(3) VACUUM TOPOLOGY

ABSTRACT

The Sagnac effect is given precisely by a simple application of gauge theory assuming an O(3) vacuum topology. The result unifies the kinematic and electrodynamic treatments of the Sagnac effect and is self-checked by a simple calculation using special relativity. The result is that the Sagnac effect exists for all matter waves and is the same for all matter waves to high precision. This is strong empirical evidence for the fact that electrodynamics and dynamics can be constructed on an O(3) vacuum topology.

INTRODUCTION

It has been demonstrated recently by Hasselbach et al. {1} that the Sagnac effect exists in matter waves, using electrons. In this paper, it is shown that it should exist in all matter waves, and should be the same for all matter waves to high precision. The calculation is predicated upon the assumption of an O(3) vacuum topology. A gauge transform in this topology produces the Sagnac effect straightforwardly to a precision of one part in 10^{23} {2}, and shows that the effect is topological in origin, and independent of the type of matter wave being used. The topological explanation holds for the photon with and without mass, the electron, neutron, atom, molecule and all matter. It is self-checked for massive particles by a simple calculation in standard special relativity, which gives the same result. However, standard special relativity in the Einstein vacuum runs into difficulties {3} when dealing with the Sagnac effect in the massless photon, whereas our novel topological explanation in the O(3) vacuum explains it straightforwardly with a holonomy based on O(3) covariant derivatives in gauge theory {4-6}. The same result precisely is obtained in O(3) electrodynamics, suggesting that the vacuum structure is governed by the O(3) rotation group for dynamics and electrodynamics. The vacuum is probably more accurately described by the Poincaré group, whose little group is the O(3) group for a particle with mass. These are new concepts in dynamics and electrodynamics.

TOPOLOGICAL EXPLANATION

We start with a structured vacuum of O(3) symmetry in the complex basis ((1), (2), (3)). This is neither Newton's nor Einstein's vacuum, but is suggested by recent gauge theoretical developments in electrodynamics {4-12}. The same structured vacuum applies to both electrodynamics and to dynamics. In this vacuum, the energy momentum tensor is also a vector in the internal gauge space {12} ((1), (2), (3)):

$$\begin{aligned} p^{\mu} &= p^{\mu(1)}e^{(1)} + p^{\mu(2)}e^{(2)} + p^{\mu(3)}e^{(3)} \\ &= \hbar\left(\kappa^{\mu(1)}e^{(1)} + \kappa^{\mu(2)}e^{(2)} + \kappa^{\mu(3)}e^{(3)}\right) \end{aligned} \tag{1}$$

where

$$\omega^2 = c^2\kappa^2 + \frac{m_0^2 c^4}{\hbar^2}. \tag{2}$$

Here, ω is the angular frequency of a matter wave, κ is its wavenumber magnitude, c is a universal constant, which for a massless photon is the speed of light, m_0 is the rest mass of the particle corresponding to the matter wave, and $\hbar$ is the Dirac constant. The rest mass m_0 can be the photon rest mass, which is estimated {13} to be less than 10^{-68} kg.

In condensed notation, both p_μ and κ_μ are governed by a gauge transformation {12}:

$$p_\mu \rightarrow S p_\mu S^{-1} - i\left(\partial_\mu S\right) S^{-1} \tag{3}$$

and similarly for κ_μ. For a Sagnac platform spinning about the orthogonal Z axis, the rotation generator S is {4-12}:

$$S = \exp\left(i J_Z \left(\alpha\left(x^\mu\right)\right)\right) \tag{4}$$

where α is an angle in the plane of the Sagnac platform. By special relativity, it is a function of the Minkowski coordinates x^μ. Here, J_Z is the Z rotation generator of the O(3) group which is the group symmetry of the internal gauge space, in this case, the structured vacuum. From eqn. (3), we obtain the following result {4-12}:

$$\kappa^{O(3)} \rightarrow \kappa^{O(3)} \pm \partial^0 \alpha \tag{5}$$

which is the same as

$$\omega \rightarrow \omega \pm \Omega \tag{6}$$

This is a topological result given by the structure of the vacuum, and it is true for all matter waves, including the wave associated with the massless photon, the electromagnetic wave {14}. It is also true for the photon with rest mass, m_0, as recently pointed out by Vigier {13}. There is no reason to suppose that a particle is massless, as first indicated by de Broglie.

The holonomy difference with platform at rest for anticlockwise (A) and clockwise (C) loops (round trips in Minkowski spacetime with O(3) covariant derivatives) in the Sagnac effect in the O(3) vacuum is {4-12}:

$$\Delta\gamma = \exp\left(i 2 \kappa^2 A r\right) \tag{7}$$

where, from eqn. (2):

$$\kappa^2 = \frac{\omega^2}{c^2} - \frac{m_0^2 c^4}{\hbar^2}. \tag{8}$$

The extra holonomy difference due to the rotating platform is from eqn. (6):

$$\Delta\Delta\gamma = \exp\left(i \frac{4\omega\Omega A r}{c^2}\right) \tag{9}$$

giving the observable phase difference:

$$\Delta\Delta\phi = \cos\left(\frac{4\omega\Omega A r}{c^2}\right) \tag{10}$$

for all matter waves. This result has been tested in a Michelson-Gale experiment to a precision of one part in 10^{23} {2}. It does not depend on the rest mass of the particle, and so should be the same for all matter waves. This prediction has recently been verified experimentally by Hasselbach et al. {1} for electrons, and in a calculation by Vigier {13} for photons with mass. It allows for the fact that a photon may have mass. The same result as eqn. (10) has also been obtained recently using electrodynamics {4-12}, so the topological description unifies the dynamic and electrodynamic descriptions of the Sagnac effect for the first time in eighty five years.

EXPLANATION IN STANDARD SPECIAL RELATIVITY

Let the tangential velocity of the disc be v_1 and the velocity of the particle be v_2 in the laboratory frame {5}. When the particle and disc are moving in the same direction, the velocity of the particle is $v_2 - v_1 = v_3$ relative to an observer on the periphery of the disc. Vice versa, the relative velocity is $v_2 + v_1 = v_4$. The special theory of relativity states that time for the two particles will be dilated to different extents, so the time dilation difference relative to the observer on the periphery of the disc is:

$$\Delta\mathfrak{I} = \left(1 - \frac{v_3^2}{c^2}\right)^{-1/2} - \left(1 - \frac{v_4^2}{c^2}\right)^{-1/2} = 2\frac{v_2 v_1}{c^2} + \ldots \tag{11}$$

using the binomial theorem. When the disc is stationary {16}:

$$t = \frac{2\pi r}{v_2} \tag{12}$$

where r is the radius of the disc. So the observable time difference of the Sagnac effect is:

$$\Delta\Delta t = t\Delta\mathfrak{I} = \frac{4\pi r v_1}{c^2} = \frac{4\Omega A r}{c^2} \tag{13}$$

where Ω is the angular frequency of the disc and Ar is the area of the Sagnac platform. It is well known {4-12} that this is a frame invariant result, the same to an observer on and off the disc. The observable phase change is therefore:

$$\Delta\Delta\phi = \cos\left(\frac{4\omega\Omega A r}{c^2}\right) \tag{14}$$

which is the same as eqn. (10). This result is true for any particle velocity v_2. However, this method cannot be applied to a photon without mass, because c is a universal constant and it is not possible to add or subtract a velocity to c. The topological explanation in section 2 is therefore more generally applicable and is a powerful result of gauge theory in the O(3) vacuum rather than the Einstein vacuum.

CONCLUSION

The dynamical and electrodynamical explanations of the Sagnac effect have been unified by assuming an O(3) symmetry for the structured vacuum, the symmetry of the little group of the Poincaré group for a particle with mass, including the photon with mass. The result is the same for all matter waves, and agrees with the same calculation in special relativity provided that the rest mass m_0 is not zero in the latter calculation. When the rest mass is zero, the latter method is not applicable, but the topological result always holds to extremely high precision. It is possible that the vacuum structure is that of the Poincaré group, whose little group for finite m_0 is O(3).

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