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OBSERVATION OF THE ONE PHOTON EVANS-VIGIER FIELD IN THE TOPOLOGICAL PHASE

ABSTRACT

The dynamical and topological phases are inter-related by the theorem:

$$\phi = \exp(i\kappa \cdot \boldsymbol{r}) = \exp\left(i\frac{e}{\hbar}\Phi\right)$$

where Φ is the magnetic flux generated in free space by the Evans-Vigier field, $\boldsymbol{B}^{(3)}$. For one photon, $\Phi = \hbar/e$, the ratio of the Planck constant to the elementary charge, and the topological phase becomes the photon helicity, ± 1. More generally, $\Phi = B^{(3)}Ar$, where Ar is the beam cross-section. Therefore the well observed topological phase is conclusive empirical evidence for the $\boldsymbol{B}^{(3)}$ field in free space for one photon, and for a higher symmetry non-Abelian structure for electrodynamics on the classical and quantum levels. The non-Abelian symmetry of electrodynamics is gauge covariant and denies gauge invariance, allowing for the existence of photon mass and a preferred frame of reference in free space.

INTRODUCTION

The topological phase {1-5} denies gauge invariance for the free electromagnetic field, and is a non-Abelian, gauge covariant, phenomenon of nature. It is observed empirically for one photon {6,7}. It originates in the topology of connections on a fiber bundle: a non-Abelian Stokes theorem provides the net comparative phase change in the internal direction of a photon or particle traversing a closed path {4}. The Maxwell-Heaviside theory of electrodynamics cannot fully determine the phase of polarization in momentum space, and is therefore incomplete. The topological phase, discovered originally by Pancharatnam {8}, is one sign of this incompleteness. Experiments {9} show that interferometry in general is a manifestation of the Pancharatnam (P) phase, which has the same effect and origin as the dynamical phase {9} in non-Abelian electrodynamics, such as O(3) electrodynamics {10-12}. There are several important observable differences between the P phase and the dynamical phase {9}.

In this paper, it is shown that the topological phase originates for one photon in the Evans-Vigier field {10-12}, the archetypical spin field component of O(3) electrodynamics. This type of electrodynamics is specifically non-Abelian at the classical and quantum levels, and so is able to account for the observed one photon P phase from the first principles of gauge field theory {13, 14}. It is therefore able to describe the photon's Berry phase {3}, which is related to its P phase, as originating in the $\boldsymbol{B}^{(3)}$ field. The higher symmetry of O(3) electrodynamics has several advantages over Maxwell-Heaviside electrodynamics, features which have been developed extensively in the literature {15-20}.

In Section 2, the non-Abelian Stokes Theorem is developed in terms of O(3) covariant derivatives. Section 3 reduces the general theorem for one photon to the simple result:

$$h \equiv \kappa \cdot \boldsymbol{r} = \frac{e}{\hbar}\Phi \tag{1}$$

$$\Phi = B^{(3)}Ar \tag{2}$$

where κ is the wave-vector at the path coordinate r, and where Φ is the magnetic flux carried by the photon through the vacuum, the ratio of the Dirac constant to the elementary charge e. In the free electromagnetic field:

$$\Phi = B^{(3)} Ar \tag{2}$$

where Ar is the beam cross-section in square meters and $B^{(3)}$ the scalar magnitude of the Evans-Vigier field. For one photon:

$$h = \pm 1, \tag{3}$$

the photon's helicity.

THE ELECTROMAGNETIC PHASE IN TERMS OF A NON-ABELIAN STOKES THEOREM.

The electromagnetic phase in O(3) electrodynamics is given by:

$$\phi = \exp\left(\oint D_\mu dx^\mu\right) = \exp\left(\iint \left[D_\mu, D_\nu\right] d\sigma^{\mu\nu}\right) \tag{4}$$

a closed loop with covariant derivatives {13} in the Minkowski space-time of special relativity. The line and surface integrals involve O(3) covariant derivatives {10-20}:

$$D_\mu = \partial_\mu - i\frac{e}{\hbar}A_\mu \tag{5}$$

and the vector potential is defined in terms of O(3) rotation generators J^a:

$$A_\mu \equiv J^a A_\mu^a \tag{6}$$

where

$$\left[J_1, J_2\right] = iJ_3. \tag{7}$$

In eqn. (4) therefore, dx^μ is a line element in Minkowski space-time, and $d\sigma^{\mu\nu}$ an element of hypersurface on the Poincaré sphere. The closed loop (or round trip {10-20}) in space-time generates the free electromagnetic field:

$$G_{\mu\nu} = i\frac{\hbar}{e}\left[D_\mu, D_\nu\right]. \tag{8}$$

Theorem (4) incorporates in one novel equation the Maxwell-Heaviside and Wu-Yang phases {21}. The former appears from the fundamental de Broglie (wave particle) dualism of quantum mechanics:

$$\begin{aligned} \partial_\mu &= -i\kappa_\mu \\ D_\mu &= -i\left(\kappa_\mu + \frac{e}{\hbar}A_\mu\right) \end{aligned} \tag{9}$$

and the Wu-Yang phase is part of the covariant derivative. The complete O(3) electromagnetic phase is therefore summarized by the integral of the electromagnetic field tensor over the hyper-surface on the Poincaré sphere:

$$\phi = \exp\left(-i\frac{e}{\hbar}\iint G_{\mu\nu} d\sigma^{\mu\nu}\right) \tag{10}$$

where the field tensor contains a non-zero commutator of potentials:

$$G_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu - i\frac{e}{\hbar}\left[A_\mu, A_\nu\right]. \tag{11}$$

The only non-vanishing (non-oscillatory) part of this integral **is the topological phase**:

$$\phi = \exp\left(-i\frac{e}{\hbar}\iint -i\frac{e}{\hbar}\left[A_\mu, A_\nu\right] d\sigma^{\mu\nu}\right). \tag{12}$$

This does not exist in the Maxwell-Heaviside theory because the latter is an Abelian gauge theory of U(1) symmetry, unable therefore, to describe the empirically observable topological phase. These concepts are well established in non Abelian gauge field theory in general, {10-20} but do not occur in the Maxwell-Heaviside theory, whose linear Abelian nature prohibits the description of the non-linear, non-Abelian, topological phase.

TOPOLOGICAL PHASE AND $B^{(3)}$ FIELD FOR ONE PHOTON.

The general theorem (4) introduced for the first time in this paper is reduced in this section using the wave particle momentum dualism {15-20}:

$$\hbar\kappa = eA^{(0)} \tag{13}$$

where $A^{(0)}$ is the scalar magnitude of a longitudinal, phase free, vector potential peculiar to O(3) electrodynamics. It is again undefined in the Maxwell-Heaviside theory and in consequence, this theory violates Newton's Third Law in radiation/matter interaction such as the Compton effect. In O(3) electrodynamics, the Compton effect is understood straightforwardly {15-20} from the correspondence of the quantized momentum for one photon, $\hbar\kappa$, and its classical equivalent $eA^{(0)}$. The latter does not exist in the Maxwell-Heaviside theory.

If we restrict consideration to a plane wave propagating in the $Z = 3$ axis, with a beam cross-section defined in the X - Y (1-2) plane, we obtain from eqn. (13):

$$\oint A_3 dx^3 = -i\iint \frac{\kappa}{A^{(0)}}\left[A_1, A_2\right] d\sigma^{12} \tag{14}$$

Using eqns. (6) and (7):

$$\oint dx^3 \equiv \oint dr = \kappa\iint d\sigma^{12} = \kappa\iint dAr \tag{15}$$

and so

$$\boldsymbol{\kappa}\boldsymbol{\cdot}\boldsymbol{r} = \kappa\oint dr = -i\frac{e}{\hbar}\iint \frac{\kappa}{A^{(0)}}\left[A_1, A_2\right] dAr = \frac{e}{\hbar}B^{(3)}Ar, \tag{16}$$

where for a plane wave in the complex basis ((1), (2), (3)) defined by circular polarization {15-20}:

$$\boldsymbol{B}^{(3)*} = -i\frac{e}{\hbar}\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}. \tag{17}$$

Eqn. (17) is self-consistent with the B Cyclic theorem of O(3) electrodynamics {10-12}:

$$\boldsymbol{B}^{(1)} \times \boldsymbol{B}^{(2)} = iB^{(0)}\boldsymbol{B}^{(3)*} \tag{18}$$

through the duality relation (13). Therefore the magnetic flux in vacuo is, from eqn. (16):

$$\Phi = B^{(2)}Ar = -i\iint \frac{\kappa}{A^{(0)}}\left[A_1, A_2\right] d\sigma^{12} \tag{19}$$

and originates in the σ^{12} component of the hyper-surface on the Poincaré sphere. The physical origin of Φ is the interference between the (1) and (2) (complex conjugate) components of the circularly polarized beam. These exist on the single photon level, and so the topological phase for one photon originates in the latter's $\boldsymbol{B}^{(3)}$ field. We have therefore demonstrated the physical origin of the topological phase.

The magnetic flux carried by one photon is:

$$\Phi = \frac{\hbar}{e} \tag{20}$$

and so h becomes the helicity {1-5, 10-20}:

$$h = \pm 1 \tag{21}$$

for a plane wave in circular polarization propagating in Z. This result is self-consistently regained from eqns. (13) and (15) as follows:

$$\begin{aligned} r &= \kappa\, Ar = \frac{1}{\kappa}; \\ \kappa \bullet \boldsymbol{r} &= \pm 1. \end{aligned} \tag{22}$$

DISCUSSION

The $\boldsymbol{B}^{(3)}$ field has been observed as the P phase in various forms of interferometry {9}: the P phase is distinguished experimentally by the fact that it is achromatic and depends only on polarization; is non-additive (has no $\boldsymbol{r}$ dependence); is unbounded (depends on cyclic and therefore periodic changes in the state of polarization); and can be observed in unseparated beams. The dynamical phase is chromatic, additive, bounded and needs separate beams in interferometry in order to become effective. The P phase and concomitant $\boldsymbol{B}^{(3)}$ field arise from cycling in the polarization state of light while keeping the direction of the beam fixed, and are equivalent to a gauge potential in the parameter or momentum space. They are due to parallel transport in the presence of a gauge field {4, 5, 21} of O(3) internal symmetry and are equivalent to an optical Aharonov-Bohm effect {1-5}. The Berry phase is related to the P phase {4, 5} and can be observed in a Faraday effect due to $\boldsymbol{B}^{(3)}$ - the rotation of a plane of polarization of light transmitted through a fiber wound helically on a cylinder {22}. This phenomenon is due to cycling in the direction of a beam of light so that the tip of the spin vector of a photon in this beam traces out a closed curve on the sphere of spin directions {4, 5}.

The fundamentally non-Abelian nature of these phenomena means that the Maxwell-Heaviside equations become:

$$\begin{aligned} D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} &\equiv \boldsymbol{0} \\ D_\mu \boldsymbol{H}^{\mu\nu} &= \boldsymbol{J}^{\nu} \end{aligned} \tag{23}$$

where the field tensors $\tilde{\boldsymbol{G}}^{\mu\nu}$ and $\boldsymbol{H}^{\mu\nu}$, and four current $\boldsymbol{J}^{\nu}$ are vectors in the O(3) symmetry internal gauge space. A systematic development of these equations of O(3) electrodynamics is given elsewhere {15-20}. The key importance of the topological phase is that it indicates conclusively the empirical existence of the Evans-Vigier field of the photon. The latter does not exist in the Maxwell-Heaviside theory, whose field equations are written in a scalar (U(1)) internal gauge space. In this space, the commutator defining the P phase in eqn. (12) is zero by definition. The O(3) gauge group is preferred to the SU(2) gauge group because the latter is simply connected and does not support an Aharonov-Bohm effect or topological phase {4, 5, 13, 14} because the mappings are deformable to a constant map. The O(3) group is doubly connected and supports the Aharonov-Bohm {13, 14} and topological phase effects. The O(3) group is homomorphic with SU(2) but is its covering group.

The adoption of the O(3) gauge symmetry for the whole of electrodynamics is therefore indicated by the empirical existence of $\boldsymbol{B}^{(3)}$ in the topological phase on the one photon level. This course of action brings with it several fundamental conceptual advances in electrodynamics and unified field theory {15-20}. The most important of these is that electrodynamics is a non-Abelian gauge field theory, and this is indicated fundamentally {4, 5} by the fact that a classical polarized wave is always constituted of two vectorial components, for example (1) and (2) in circular polarization, which give rise to $\boldsymbol{B}^{(3)}$ and the topological phase through a non-zero commutator of potentials. Without this commutator, there is no topological phase. As soon as a non-Abelian structure is adopted for electrodynamics, the theory loses gauge invariance {4, 5}. The field tensor is gauge covariant in the vacuum and a gauge transformation becomes a physical rotation. In the Maxwell-Heaviside theory, the potentials are not gauge invariant on the classical level and are not regarded as physical for this reason {4, 5}. In the O(3) electrodynamics, the gauge transformation of the potentials is a well defined physical process giving rise to a characteristic inhomogeneous term responsible for physical effects on the classical level {4, 5, 13, 14}. An example is the Sagnac effect {23}.

Another major conceptual advance is the realization in O(3) electrodynamics that the constant e, like $\hbar$, is a fundamental constant of physics that exists under all conditions, in the field as well as on the electron. This realization straightforwardly unifies the Maxwellian concept of material charge being a result of the field {24}, and the Lorentzian concept of the field being the result of charged matter. The fundamental de Broglie duality (13) follows from O(3) electrodynamics and allows a simple classical explanation of the Compton effect, as well as of the topological phase in experiments such as the Sagnac effect {23} and interferometry in general.

The general adoption of O(3) electrodynamics would have far reaching consequences throughout physics and cosmology. For example, the loss of gauge invariance means that the adoption of a preferred frame becomes possible, bringing with it a theory of electrodynamics in which the mass of the particulate photon is non-zero, as in the Einstein/de Broglie/Vigier theory of light {15-20}. On the cosmological scale, the photon mass has several observable consequences {15-20}, and may be a fundamental cause for the abandonment of the Big Bang Theory. Finite photon mass also means the ultimate adoption of a new form of special relativity, as argued recently by Vigier {25} and Selleri et al. {26}. Finally, in this paper, it is argued that such a consequence means that there is a need for a reassessment of the Michelson-Morley experiment, whose results are non-null as also argued by Vigier {27}.

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