LORENTZ INVARIANCE

LORENTZ TRANSFORM OF INHOMOGENEOUS O(3) EQUATION

Inhomogeneous Field Equation

$$D_\mu \boldsymbol{H}^{\mu\nu} = \boldsymbol{J}^\nu$$
$$\partial_\mu \boldsymbol{H}^{\mu\nu(3)*} = \boldsymbol{J}^{\nu(3)*} + igA_\mu^{(1)} \times \boldsymbol{H}^{\mu\nu(2)}$$

Use the constitutive relation:

$$\boldsymbol{H}^{\mu\nu(3)*} = \varepsilon \boldsymbol{G}^{\mu\nu(3)*}$$

$$\Rightarrow \partial_\mu \boldsymbol{H}^{\mu\nu(3)*} = 0 = \partial_\mu{}' \boldsymbol{H}^{\mu\nu(3)*\prime}$$

This is Lorentz invariant ($\nu = 0$).

$$\boldsymbol{J}^{\nu(3)*} = -igA_\mu^{(1)} \times \boldsymbol{H}^{\mu\nu(2)} = J^{\nu(3)} e^{(3)*}$$

The four vector $J^{\nu(3)}$ transforms properly under a Lorentz transformation.

***Z* Boost (Jackson's notation)**

$$J_\mu{}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & \gamma & i\gamma\beta \\ 0 & 0 & -i\gamma\beta & \gamma \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ J_Z \\ iJ_0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \gamma J_Z - \gamma\beta J_0 \\ i(\gamma J_0 - \gamma\beta J_Z) \end{bmatrix}$$

***Y* Boost (Jackson's notation)**

$$J_\mu{}' = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \gamma & 0 & i\gamma\beta \\ 0 & 0 & 1 & 0 \\ 0 & -i\gamma\beta & 0 & \gamma \end{bmatrix} \begin{bmatrix} 0 \\ 0 \\ J_Z \\ iJ_0 \end{bmatrix} = \begin{bmatrix} 0 \\ -\gamma\beta J_0 \\ J_Z \\ i\gamma J_0 \end{bmatrix}$$