INCONSISTENCIES OF THE U(1) THEORY OF ELECTRODYNAMICS: THE COMPTON EFFECT

ABSTRACT

The fundamental gauge field theory of O(3) electrodynamics leads to the longitudinally directed vector potential $A^{(3)}$, which appears in the definition of the vacuum field tensor. This leads to a straightforward classical quantum equivalence for the Compton effect:

$$e\left|A^{(3)}\right| = \hbar\kappa .$$

In U(1) electrodynamics, there is no $A^{(3)}$ and so there is no classical Compton effect. The non-Abelian or O(3) theory of electrodynamics is therefore self-consistent, while the U(1) theory is self-inconsistent. Some consequences are discussed.

INTRODUCTION

The fundamental gauge theory of O(3) electrodynamics {1-5} leads to the longitudinally directed vector potential $A^{(3)}$, which appears in the vacuum field tensor definition. The component $A^{(3)}$ does not appear in U(1) electrodynamics but has substantial advantages in field/particle theory as developed in this paper.

In U(1) electrodynamics the linear momentum of radiation is defined through the Poynting vector:

$$\langle p\rangle = \varepsilon_0 c \int \boldsymbol{E} \times \boldsymbol{B}\, dV \tag{1}$$

Paradoxically, this quantity is zero, because the cross product $\boldsymbol{E} \times \boldsymbol{B}$ is zero for plane waves {6}. The only way to obtain a non-zero Poynting vector is to introduce the conjugate product $\boldsymbol{E} \times \boldsymbol{B}^*$. This is equivalent in the complex basis ((1), (2), (3)) {1-5} to the conjugate product $\boldsymbol{E}^{(1)} \times \boldsymbol{B}^{(2)}$. The basis ((1), (2), (3)), however, is that for an internal gauge space of an O(3) symmetry electrodynamics {7} in which $A^{(3)}$ is not zero. This is a Yang-Mills theory applied to electrodynamics as intended originally {8}. Section 2 details some problems with the conventional definition of the Poynting vector while Section 3 suggests a novel classical/quantum momentum equivalence based on O(3) electrodynamics and the existence in free space of the longitudinal vector potential $A^{(3)}$. Finally, a short discussion section analyses these results in terms of some interesting phase shift effects observed recently by Anastasovski et alia {9}.

INCONSISTENCIES OF THE POYNTING VECTOR IN U(1) ELECTRODYNAMICS

Even if the Poynting vector is modified to the usual textbook definition for average beam momentum:

$$\langle p\rangle = \varepsilon_0 c \int \boldsymbol{E} \times \boldsymbol{B}^*\, dV \tag{2}$$

the average momentum becomes proportional to beam intensity and the conjugate product $\boldsymbol{E} \times \boldsymbol{B}^*$ removes the frequency, and so $<p>$ is not linearly proportional to frequency as in the Planck-Einstein quantization of radiation into photons with energy and linear and angular momentum. The well known photoelectric and Compton effects for example lead to the quantum theory, but are paradoxical in classical U(1) electrodynamics as just discussed {6}. The problem of radiation reaction {10, 11} is paradoxical on the classical level itself, because it leads to runaway, non-Newtonian, solutions in the Abraham-Lorentz equation {10, 11}. It is suggested that O(3) electrodynamics removes the paradoxes from field theory as follows.

THE CONCEPT OF LONGITUDINAL FREE SPACE POTENTIAL

In O(3) electrodynamics, {1-5} there exists a free space (i.e. vacuum) linear momentum:

$$p^{(3)} = eA^{(3)} \tag{3}$$

which is longitudinally directed as required, i.e. is directed in the propagation axis of the radiation beam. The momentum ($p^{(3)}$) is defined through the minimal prescription applied to the beam in the vacuum, and the elementary charge e is regarded {1-5} as a universal constant. The quantum classical equivalence then shows that the classical momentum becomes:

$$p^{(3)} = eA^{(3)} = \hbar\kappa \tag{4}$$

through Planck-Einstein quantization. It becomes the momentum of the photon, proportional to frequency as observed in the photoelectric and Compton effects. It can be shown {1-5} that eqn. (4) is also the self-consistent result of O(3) symmetry gauge field theory. The result in eqn. (4) has a manifestly covariant form:

$$\begin{aligned} p^{\mu(3)} &= eA^{\mu(3)} = \hbar\kappa^{\mu} \\ A^{\mu(3)} &= \frac{1}{c}\left[A^{(0)}, cA^{(3)}\right]. \end{aligned} \tag{5}$$

It is well known that the energy-momentum of radiation in U(1) electrodynamics is defined through an integral over the tensor $T^{\mu\nu}$ and for this reason is not generally covariant. To make it so requires the use of special hypersurfaces as attempted for example by Fermi and Rohrlich {12}. The root cause of this problem is Poynting's Theorem {12}. The O(3) energy momentum (5) in contrast, is generally covariant in O(3) electrodynamics.

The problem of radiation reaction {10, 11} leads in U(1) electrodynamics to the Abraham-Lorentz equation in which the radiation reaction force is not Newtonian. It has unphysical runaway solutions {10, 11} because the force is proportional to the time derivative of acceleration, a problem addressed by Abraham, Lorentz and Poincaré among others throughout the twentieth century. In O(3) electrodynamics, the linear momentum $eA^{(3)}$ is classical and constant (conservation of momentum) until the radiation interacts with an electron as in the Compton effect {13}. Upon interaction, the frequency of the scattered radiation is found to be different from that of the incoming radiation. In O(3) electrodynamics, this is explained by the fact that action and reaction are equal and opposite and that force is the the of change of linear momentum. In this case:

$$p^{\mu(3)} = e\frac{\partial A^{\mu(3)}}{\partial\tau} \tag{6}$$

and so a change of linear momentum means a change of frequency through Planck-Einstein quantization. There is precise correspondence between linear classical momentum and quantized linear momentum as in eqn. (4). Note that both classical and quantized linear momentum are directed longitudinally as observed in the photoelectric and Compton effects and in the Lebedev effect {14}. There is no such correspondence in U(1) electrodynamics because the average momentum $<p>$ from Poynting's theorem vanishes if the product $\boldsymbol{E} \times \boldsymbol{B}$ is used, and can never be proportional to frequency on the classical level, the reason being that the conjugate product $\boldsymbol{E} \times \boldsymbol{B}^*$ removes the frequency. For this reason, there is no classical explanation {15} of the Compton and photoelectric effects in classical U(1) electrodynamics.

The photoelectric effect is the emission of electrons from metals upon ultra-violet irradiation {15}. Above a threshold frequency, the emission is instantaneous and independent of intensity. Below this threshold frequency, there is no emission however intense the radiation. This cannot be explained in classical U(1) electrodynamics, because beam energy is proportional to intensity in the Poynting Theorem, and not to frequency as observed. In classical O(3) electrodynamics, the phenomenon is explained straightforwardly by the following data, i.e.:

$$En = ecA^{(3)} = \text{constant time frequency} \tag{7}$$

and in Planck-Einstein quantization, the constant of proportionality is $\hbar$, which turns out to be the most fundamental universal constant in physics. The concomitant momentum relation, eqn. (4), is shown empirically by the Compton effect as argued already. Eqn. (4) means that above a threshold frequency, there is enough energy in the photon to cause electron emission in the photoelectric effect as observed and discussed for example by Atkins {15}. All of the energy and momentum of the photon are transferred to the electron in a collision above a certain threshold frequency because, at this point, the potential energy responsible for keeping the electron in place is exceeded. Atkins explains the phenomenon simply and decisively in terms of simple Newtonian logic {15}. If we attempt to apply this logic to $\langle p \rangle$ in eqn.(2), the Poynting momentum of classical U(1) electrodynamics, there is no threshold frequency possible on the classical level because $\langle p \rangle$ cannot be proportional to frequency, and we lose correspondence with the quantum theory, in which the momentum of the photon is proportional to the frequency as required empirically in the photoelectric effect. The mean Poynting momentum $\langle p \rangle$ and beam energy are both proportional to beam intensity, but the data in the photoelectric effect shows that they are independent of beam intensity. The momentum $p^{(3)} = eA^{(3)}$ of classical O(3) electrodynamics is not proportional to intensity, it is proportional to frequency through the **gauge** equation (4), which also leads to the B Cyclic theorem {1-5}, the fundamental angular momentum relation of O(3) electrodynamics. In the latter, Planck-Einstein quantization is straightforward.

In the Compton effect in O(3) electrodynamics, the observable change of wavelength {15} is:

$$\Delta\lambda = 2\left(\frac{eA^{(3)}}{mc}\right)\lambda_0 \sin^2\frac{\theta}{2} \tag{8}$$

where λ_0 is the wavelength of the incident beam, m is the electron mass, and θ is a scattering angle. If eqn. (4) is applied to this result, we recover the usual quantum description of the Compton effect.

The concept of $\boldsymbol{A}^{(3)}$ can also be used to suggest a way out of the Dirac paradox, {16} in which Dirac maintains that as long as we are dealing with transverse waves, we cannot bring in the Coulomb interaction between particles. Dirac suggested the use of longitudinal waves, but in O(3) electrodynamics, there is a force given by eqn. (6) whenever the beam interacts with an electron. This interaction results in a longitudinal force with a change of wavelength as just described in the Compton effect. This is not a Coulomb force however since $\boldsymbol{E}^{(3)}$ is zero in vacuo (there is no electric equivalent of the inverse Faraday effect).

Similarly, $A^{(3)}$ can be used to suggest a way out of the de Broglie paradox, {17}which points out that momentum and energy transform differently under Lorentz transformation from frequency, despite Planck-Einstein quantization. This paradox led de Broglie to postulate the existence of empty waves, which, however, have never been found in nature. It is therefore suggested that the Lorentz frequency transform always be applied to $eA^{(3)} = \hbar\frac{\omega}{c}e^{(3)}$ because this momentum is proportional to frequency empirically. If this momentum is interpreted as that of a particle traveling at the speed of light, the momentum becomes indeterminate (massless particle) or infinite (massive particle) unless it is **always** interpreted as being a constant ($\hbar$) multiplied by ω/c, which always exists empirically at the speed of light. The energy must evidently be interpreted in the same way, i.e. as a constant multiplied by frequency. The Lorentz transform applied to frequency produces the aberration of light as usual {18} in special relativity. In this interpretation, there is no de Broglie paradox and no need to postulate the existence of empty waves {17}.

DISCUSSION

Classical O(3) electrodynamics produces explanations from first gauge field theoretical principles, and without paradox, of the following:

1. third Stokes parameter in free space, through the modulus of $B^{(3)}$;
2. the inverse Faraday effect through $B^{(3)}$; the photoelectric effect, through $A^{(3)}$;
3. the Compton effect through $A^{(3)}$;
4. the radiation reaction problem, through $dA^{(3)}/dt$;
5. that the energy momentum flux vector $eA^{(3)}$ of radiation is generally covariant;
6. the Dirac paradox;
7. the de Broglie paradox;
8. the vacuum $B^{(3)}$; and
9. the vacuum $A^{(3)}$.

In the U(1) version of classical electrodynamics, we have the following inconsistencies:

1. the third Stokes parameter contradicts the gauge field principle being used ($A^{(1)} \times A^{(2)} = \mathbf{0}$) and is phenomenological, dating back to 1852 and thus predating the Maxwell-Heaviside equations;
2. the inverse Faraday effect is phenomenological and contradicts the gauge field principle $A^{(1)} \times A^{(2)} = \mathbf{0}$ in U(1);
3. there is no classical photoelectric effect in U(1);
4. there is no classical Compton effect in U(1);
5. radiation reaction in U(1) is non-Newtonian and unphysical;
6. the energy momentum vector of radiation is not generally covariant in U(1);
7. the Dirac paradox exists in U(1);
8. the de Broglie paradox exists in U(1);
9. there is no fundamental spin angular momentum, proportional to the $B^{(3)}$ field; and
10. there is no fundamental linear momentum $eA^{(3)}$ in U(1).

In respect of photon mass (m_0), one would expect an additional contribution to the rest energy and using the de Broglie Theorem {17}, there would be a threshold wavelength at which

$$\hbar\omega = m_0 c^2 \tag{9}$$

therefore at this wavelength, one would expect a Compton shift due to photon mass:

$$\Delta\lambda = 2\left(\frac{m_0}{m}\right)\lambda_0 \sin^2\frac{\theta}{2} \tag{10}$$

In principle, this agrees with the results of Anastasovski et al. {9} who also predict a phase shift due to photon mass, but of a different type than that given here.

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