INCONSISTENCIES OF THE
MAXWELL-HEAVISIDE THEORY OF ELECTRODYNAMICS:
THE AHARONOV-BOHM EFFECT

ABSTRACT

The Aharonov-Bohm (AB) effect is developed as a change of phase due to a change of orientation of an internal gauge space of O(3) symmetry. The AB effect is therefore interpreted as a gauge transformation of the pure vacuum arising from this change of orientation, and is therefore due to a physical inhomogeneous term arising from this gauge transform. Maxwell-Heaviside theory inconsistently treats the AB effect because, as argued by Barrett {1}, the theory has no convincing argument around the fact that the AB effect depends on a vector potential which is non-zero, but at the point of interaction of electromagnetic wave and electron, the magnetic field defined by $\nabla \times A$ is zero, and therefore so is the phase shift defined by the magnetic flux due to this field. The theory proposed here gets around this difficulty using non-Abelian electrodynamics.

INTRODUCTION

Concepts introduced and developed by Barrett {1-3} and others {4-14} have recently suggested extensions to the Maxwell-Heaviside theory of classical electrodynamics. These concepts are well summarized by Barrett{1-3} and are abstracted from his work as an introduction to this paper, in which the Aharonov-Bohm (AB) effect is found to be inconsistent with U(1) Yang-Mills gauge theory (Maxwell-Heaviside classical electrodynamics). The inconsistency in the received view of the effect {1} is that although the AB effect depends on the interaction of a vector potential A with an electron, the magnetic field $B = \nabla \times A$ is zero at the point of interaction, and the theoretical phase shift, defined by the integral over this magnetic field, is zero. This argument can always be used to counter the received view that the potential A is physical in U(1) theory, an argument which contradicts the fact that A in U(1) theory is defined only up to a gauge transformation and can be many valued for a single valued B.

In this paper, it is shown that if the gauge symmetry of the Yang-Mills theory is changed from U(1) to O(3), this problem is solved. The AB effect becomes one dependent on a gauge transform whose inhomogeneous term is physical {1-14}. The measured change in phase in the Aharonov-Bohm effect therefore becomes a change in orientation of the internal O(3) symmetry gauge space under a rotation. In other words, the effect is due to a gauge transformed vector potential in non-Abelian electrodynamics {1-14}, an explanation similar to the one used recently {15} to explain the Sagnac effect with platform in motion.

Barrett {1} has provided a summary of the fundamental reasoning behind such a description, and has similarly described the Aharonov-Bohm effect {1} in terms of an SU(2) gauge field theory using magnetic fields. The reasoning is that the most fundamental physical description is based on topology. Spacetime is endowed with a topological structure related to its physics, and for a known topology, the gauge theory description {1-14} is justified, giving the field equations as differential equations. Spacetime topology defines electromagnetic field equations, fields of force or of phase {1-3}. A set of field equations is valid only within a well defined topological description of the physical situation. In other work {4-8}, this philosophy has been extended slightly so that the physical situation includes the vacuum itself, resulting in the B Cyclic theorem and the O(3) electrodynamics {4-8}. In the O(3) description, which we apply in this paper, the potential differences are always physical, and can be detected for example in interferometry {8, 15-17} of all kinds. As pointed out by Barrett {1}, the Aharonov-Bohm effect is an interferometric effect.

There is a fundamental topological flaw in Maxwell-Heaviside theory (hereinafter referred to as "U(1)") because {2} the potential, as interpreted by Heaviside, has no physical meaning or effect classically. However, this can be true only in simply connected spaces, whereas the U(1) group space is multiply connected {18}. For a multiply connected manifold, the potential can have a physically discernible effect because the gauge factors can be different for different homotopy classes {2}. This paradox leaves only one avenue of progress open: the topology of spacetime means that we must use the correct multiply connected group to define the equations of motion of classical electrodynamics through gauge theory. Evidently, this group cannot be U(1) in any situation because U(1) is multiply connected, meaning that the potentials in U(1) may be physical. This conclusion contradicts the basic gauge invariance ansatz of U(1) theory {1-14}. This group may be SO(3) = SU(2)/Z2. We habitually refer to it as "O(3)".

As argued by Barrett, {2} any polarized classical wave (for example a circularly polarized electromagnetic plane wave) is made up of two vectorial components and this classical electromagnetic wave is a multiply connected field in O(3) form, homomorphic with SU(2) {18}. This topological argument alone is enough to justify the B cyclic theorem and the extension of classical electrodynamics {4-8} using Yang-Mills theory with an internal O(3) gauge group. In this view, the vacuum classical electromagnetic field and field/matter interaction are described by an O(3) Yang-Mills theory {4-8}, hereinafter referred to as "O(3)". The group space of O(3) (the sphere) is multiply connected, as required, and it is an essential topological requirement for the Aharonov-Bohm effect to exist {18}. To assert, as in the received view, that the vector potential is physical in U(1) is therefore self-contradictory. It can be physical only in O(3), even in the vacuum. In other words, the configuration space of the vacuum must be multiply connected {4-8}. The Aharonov-Bohm effect in O(3) is therefore due to a gauge transform in O(3), giving a physical inhomogeneous term. In U(1), such a gauge transform produces a random inhomogeneous term which can be any valued and which is therefore unphysical. The Aharonov-Bohm effect cannot therefore be defined by an integral over such a random function, as suggested by Ryder {18} using U(1) theory. This result is connected with the fact that the electromagnetic phase is arbitrary in U(1) as the result of gauge invariance. On the other hand, the phase factor {1-8} of O(3) is gauge invariant and physical, and a gauge transform in O(3) leads to a physical inhomogeneous term which, as argued in this paper, is responsible for the Aharonov-Bohm effect. This explanation follows the one first given by Barrett {1} in terms of fields rather than gauge transforms. The Aharonov-Bohm effect is therefore definitive evidence for O(3) electrodynamics, more properly SO(3) = SU(2)/Z2.

GAUGE TRANSFORMATION IN O(3)

The potential in O(3) electrodynamics is in general a twelve vector defined by {4-8}:

$$A_\mu \equiv A_\mu^X \boldsymbol{i} + A_\mu^Y \boldsymbol{j} + A_\mu^Z \boldsymbol{k} \tag{1}$$

where $\boldsymbol{i}, \boldsymbol{j}$ and $\boldsymbol{k}$ are Cartesian unit vectors such that:

$$\boldsymbol{i} \times \boldsymbol{j} = \boldsymbol{k} \tag{2}$$

in the internal gauge space. We define the Aharonov-Bohm effect in O(3) electrodynamics as a phase shift due to a rotation in the internal gauge space:

$$\psi' = \exp\left(iJ_a \Lambda_a\right)\psi \equiv S\psi \tag{3}$$

where ψ is a wave-function and where S is defined in a Cartesian basis as:

$$S_X = \exp\left(iJ_X \Lambda(X)\right) \tag{4}$$

$$S_Y = \exp\left(iJ_Y \Lambda(Y)\right) \tag{5}$$

$$S_Z = \exp\left(iJ_Z \Lambda(Z)\right) \tag{6}$$

where J_X, J_Y, J_Z are rotation generators {18} of the O(3) group, and where $\Lambda(X, Y, Z)$ is an angle, dependent on X, Y, and Z through special relativity. In shorthand notation, {18} special relativity dictates that the rotation (18) has the following effect on the potential, which is in general a tensor:

$$A'_\mu = SA_\mu S^{-1} - \frac{i}{g}\left(\partial_\mu S\right)S^{-1} \tag{7}$$

Focusing on the space components of eqn. (1), we obtain:

$$A = A_X i + A_Y j + A_Z k \tag{8}$$

where

$$A'_X \equiv -\frac{i}{g}\left(\partial_X S_X\right)S_X^{-1} = \frac{1}{g}\partial_X \Lambda(X)$$

$$A'_Y \equiv -\frac{i}{g}\left(\partial_Y S_Y\right)S_Y^{-1} = \frac{1}{g}\partial_Y \Lambda(Y) \tag{9}$$

$$A'_Z \equiv -\frac{i}{g}\left(\partial_Z S_Z\right)S_Z^{-1} = \frac{1}{g}\partial_Z \Lambda(Z)$$

are physical quantities, producing measurable effects, in this case the Aharonov-Bohm effect. The rotation generators and angles appearing in eqn. (9) are operators and functions in the group space (a sphere) of the gauge group O(3), and the Aharonov-Bohm effect in this view is due to the gauge transform of the true vacuum, $A_\mu = 0$, and therefore to the terms (9). The phase change of the electron wave-function is therefore given by the holonomy of the connection:

$$\Phi = \exp\left(ig\iint A'_\mu dx^\mu\right) \tag{10}$$

and therefore by:

$$\Phi = \exp\left(i\left(\int \partial_X \Lambda(X)dX + \int \partial_Y \Lambda(Y)dY + \int \partial_Z \Lambda(Z)dZ\right)\right). \tag{11}$$

The observable change in phase is therefore:

$$\Delta\delta = \cos\Phi = \frac{e}{\hbar}\int B\cdot dS \tag{12}$$

as observed experimentally {18}.

DISCUSSION

This is an internally consistent explanation of the Aharonov-Bohm effect because at the point of contact, the phase shift is produced by eqn. (11), i.e. by a gauge transform of the true vacuum that produces

a physical function which produces an observable effect. The received explanation {18} in U(1) is that the observed phase shift is given by:

$$\Delta\delta\left(U(1)\right) = \frac{e}{\hbar}\int B\cdot dS = \frac{e}{\hbar}\int \nabla\times A\cdot dS$$

$$= \frac{e}{\hbar}\oint_{2-1} A\cdot dr = \frac{e}{\hbar}\oint_{2-1} \nabla\chi\cdot dr \tag{13}$$

where

$$A \to A + \nabla\chi. \tag{14}$$

which is inconsistent for the reason already quoted in the introduction from Barrett, {1} and also because $\nabla\chi$ in U(1) is any valued, i.e. random and unphysical. In O(3), on the other hand, the Aharonov-Bohm effect is due to the physical inhomogeneous term produced by a gauge transformation as in section 2. This inhomogeneous term exists in regions of the vacuum where B is not present and where A_μ can be zero and is the true origin of the AB effect. The same inhomogeneous term is responsible for the Sagnac effect {15} and it is no coincidence that there is a link {2,3} between the two effects. This explanation follows the one first given {1} by Barrett using fields rather than potentials. It is concluded that the AB effect is due to vacuum topology and the Sagnac effect with platform in motion is an optical AB effect {4-8, 15}.

ACKNOWLEDGMENTS

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