SU(2) × SU(2) ELECTROWEAK THEORY II: CHIRAL AND VECTOR FIELDS ON THE PHYSICAL VACUUM

ABSTRACT

In this letter, a discussion is offered on how symmetry breaking of a theory with twisted bundle of two chiral SU(2) bundles leads to a set of gauge potentials from each group on the physical vacuum that are vector and chiral. The result is that symmetry breaking of this theory leads to massive $A^{(3)}$ transverse modes of the 3-photon along with electromagnetic photons plus the massive neutral and charged weakly interacting bosons. The electromagnetic sector is demonstrated to be a massless vector field and the remainder is a broken chiral field theory.

INTRODUCTION

The cyclic theory of electromagnetism has been demonstrated to be consistent with a SU(2) × SU(2) theory of electroweak unification {1}. It has been demonstrated that if we set $A^{(3)} = 0$ on the physical vacuum that a cyclic theory of electromagnetism is arrived at. This theory contains longitudinal $E^{(3)}$ and $B^{(3)}$ fields that are dual $E^{(3)} = B^{(3)}$, but where this duality is broken by current interactions. By setting $A^{(3)} = 0$, the transverse 3-modes of the theory have been completely eliminated by this arbitrary restriction of this gauge freedom. The elimination of these transverse 3-modes guarantees that photons are entirely defined by the $\sigma^{1,2}$ generators of the SU(2) theory of electrodynamics. Since the field defined by the σ^3 generators are longitudinal, this means they are irrotational $\nabla\bullet E(3) = \nabla\bullet B^{(3)} = 0$ and thus time independent. By Maxwell's equations, this means that there are no electromagnetic waves or photons associated with this field.

AXIAL-VECTOR SU(2) × SU(2) FIELDS: A FIRST LOOK

To start, we examine a putative model of a chiral-vector model at low energies to determine what sorts of processes may be involved with the broken symmetry of such a model. We start by naively considering a chiral-vector to see what sorts of structure may emerge at low energy without explicit consideration of the Higgs mechanism. The field theory starts out as a twisted bundle of two chiral groups SU(2) × SU(2) and emerges as a theory that is an axial-vector theory at low energy. We consider initially the situation where the theory is an axial-vector theory at low energy. We then consider the situation where there is a breakdown of chiral symmetry. This is then used to set up the more complete situation that involves the breakdown of the chiral theory at high energy into an axial-vector theory at low energy

In this paper, we relax the condition that $A^{(3)} = 0$. This statement would physically mean that the current for this gauge boson is highly non-conserved with a very large mass so that the interaction scale is far smaller than the scale for the cyclic electromagnetic field. In relaxing this condition, we will find that we still have a violation of current conservation.

With $A^{(3)} \neq 0$, we have the fields {1}

$$A'^{(1)}_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}\left(gA^{(3)}_{\mu} + g'b^{(3)}_{\mu} - gA^{(1)}_{\mu}\right)$$

$$Z^{0}_{\mu} = \frac{1}{\sqrt{g^2 + g'^2}}\left(gb^{(3)}_{\mu} + g'A^{(1)}_{\mu}\right) \tag{1}$$

$$\omega^{(3)}_{\mu} = \frac{g'}{\sqrt{g^2 + g'^2}}A^{(3)}_{\mu}.$$

One purpose here is to examine the $\omega^{(3)}_{\mu}$ connection: which will have a chiral component. This at first implies that the $B^{(3)}$ field is partly chiral, or that it is mixed with the chiral component of the other SU(2) chiral field in some manner to remove its chirality.

The theory of SU(2) electromagnetism, at high energy, is very similar to the theory of weak interactions in its formal structure. Further, it has implications for the theory of leptons. The electromagnetic interaction acts upon a doublet, where this doublet is most often treated as an element of a Fermi doublet of charged leptons and their neutrinos in the SU(2) theory of weak interactions.

Following in analogy with the theory of weak interactions, we let ψ be a doublet that describes an electron according to the 1-field and the 3-field. We start with the free particle Dirac Lagrangian and let the differential become gauge covariant.

$$\begin{aligned}\mathcal{L} &= \bar{\psi}\left(i\gamma^{\mu}D_{\mu} - m\right)\psi \\ &= \bar{\psi}\left(i\gamma^{\mu}\partial_{\mu} - m\right)\psi - gA^{b}_{\mu}\bar{\psi}\gamma^{\mu}\sigma_{b}\psi \\ &= \mathcal{L}_{free} + A^{b}_{\mu}J^{\mu}_{b}.\end{aligned} \tag{2}$$

where $\bar{\psi} = \psi^{\dagger}\gamma_{4}$. From here, we decompose the current J^{b}_{μ} into vector and chiral components,

$$J^{b}_{\mu} = \psi^{\dagger}\gamma_{4}\gamma_{\mu}(1+\gamma_{5})\sigma^{3}\psi = V^{b}_{\mu} + \chi^{b}_{\mu}. \tag{3}$$

This is analogous to the current algebra for the weak and electromagnetic interactions between fermions. We have the two vector current operators {2}

$$V^{a}_{\mu} = \frac{i}{2}\bar{\psi}\gamma_{\mu}\sigma^{a}\psi \tag{4}$$

and the two axial-vector current operators

$$\chi^{b}_{\mu} = \frac{i}{2}\bar{\psi}\gamma_{\mu}\gamma_{5}\tau^{b}\psi. \tag{5}$$

Here $\gamma_5 = i\gamma_1\gamma_2\gamma_3\gamma_4$, and τ^{b} are Pauli matrices. These define an algebra of equal time commutators:

$$\begin{aligned}\left[V^{a}_{4}, V^{b}_{\mu}\right] &= it^{abc}V^{c}_{\mu}, \\ \left[V^{a}_{4}, \chi^{b}_{\mu}\right] &= -it^{abc}\chi^{b}_{\mu}.\end{aligned} \tag{6}$$

If we set $\mu = 4$, we then have the algebra

$$\left[V_4^a, V_4^b\right] = i t^{abc} V_4^c, \qquad (7)$$
$$\left[V_4^a, \chi_4^b\right] = -i t^{abc} \chi_4^b.$$

If we set

$$Q_\pm^a = \frac{1}{2}\left(V_4^a \pm \chi_4^a\right) \qquad (8)$$

we then have the algebra

$$\left[Q_+^a, Q_+^b\right] = i t^{abc} Q_+^c$$
$$\left[Q_-^a, Q_-^b\right] = i t^{abc} Q_-^c \qquad (9)$$
$$\left[Q_+^a, Q_-^b\right] = 0.$$

This can be seen to define the SU(2) × SU(2) algebra.

The action of the parity operator on V_4^b and χ_4^b due to the presence of γ_5 in the axial-vector current

$$P V_4^b P^\dagger = V_4^b \qquad (10)$$
$$P \chi_4^b P^\dagger = -\chi_4^b.$$

As such, one SU(2) differs from the other by the action of the parity operator and the total group is the chiral group SU(2) × SU(2)$_p$.

We have at low energy half vector and half chiral vector theory SU(2) × SU(2)$_p$. Then, on the physical vacuum, we have the vector gauge theory described by $A^{(1)} = A^{(2)*}$ and $B^{(3)} = \nabla \times A^{(3)} + \left(i\frac{e}{\hbar}\right) \mathrm{A}^{(1)} \times \mathrm{A}^{(2)}$ and the theory of weak interactions with matrix elements of the form $\bar{\nu}\gamma_\mu\left(1-\gamma_5\right)e$ and are thus half vector and chiral on the level of elements of the left and right handed components of doublets. We then demand that on the physical vacuum that we have a mixture of vector and chiral gauge connections within both the electromagnetic and weak interactions due to the breakdown of symmetry. This will mean that the gauge potential $A^{(3)}$ will be massive and short ranged.

One occurrence is a violation of the conservation of the axial-vector current. We have that the 1 and 2-currents are conserved and invariant. On the high energy vacuum, we expect that the currents should obey

$$\partial^\mu J_\mu^b = 0, \qquad (11)$$

where $b \in \{1, 2\}$, which are absolutely conserved currents. However, for the $A_\mu^{(3)}$ fields, we have the non-conserved current equation {3}

$$\partial^\mu J_\mu^3 = i m_\psi \psi^\dagger \gamma_4 \gamma_5 \sigma^3 \psi, \qquad (12)$$

where inhomogeneous terms correspond to the quark-antiquark and lepton-antilepton pairs that are formed from the decay of these particles. This breaks the chiral symmetry of the theory. Then this current's action on the physical vacuum is such that when projected on a massive eigenstates for the 3-photon with transverse modes

$$\left\langle 0 \left| \partial^{\mu} J^{3}_{\mu} \right| X_{k} \right\rangle = \left(\frac{m^{2}}{\sqrt{\omega(k)\omega(k')}} \right) \left\langle X_{k'} \mid X_{k} \right\rangle e^{ikx}. \tag{13}$$

The mass of the chiral {1, 2}-bosons will then vanish, while the mass of the chiral 3-boson will be m. So rather than strictly setting $A^{(3)} = 0$, it is a separate chiral gauge field that obeys axial-vector non-conservation and only occurs at short ranges.

So now that we have an idea of what nature may look like on the physical vacuum, we need to examine how it is that we can have symmetry breaking and an SU(2) × SU(2)$_p$ gauge theory that gives rise to some of the above requirements of $B^{(3)}$ electromagnetism. A mixing of the two chiral SU(2) bundles at low energy is what will produce vector gauge bosons for the electromagnetic interaction. It is apparent that we need to invoke the mixing of two chiral gauge bosons in such a manner as to produce a vector theory of electromagnetism at low energy with a broken chiral theory of weak interactions.

CHIRAL AND VECTOR GAUGE THEORIES FROM VECTOR AND CHIRAL GAUGE THEORIES ON THE PHYSICAL VACUUM

The SU(2) × SU(2) theory should mimic the standard model with the addition of the $B^{(3)} = \left(\frac{e}{\hbar}\right) A^{(1)} \times A^{(2)}$ field at low energies. This means that we demand that a field theory that is completely chiral at high energy becomes a field theory that is vector and chiral in separate sectors on the physical vacuum of low energies. This means that a field theory that is chiral at high energy will combine with the other chiral field in the twisted bundle to produce a vector field plus a broken chiral field at low energy. Generally, this means that a field theory that has two chiral bundles at high energies can become vector and chiral within various independent fields that are decoupled on physical vacuum at low energies.

We consider a toy model where there are two fermions fields ψ and χ, where each of these fields consists of the two components right and left handed fields R_{ψ}, L_{ψ} and R_{χ}, L_{χ}. These fermi doublets have the masses m_1 and m_2. We then have the two gauge potentials A_{μ} and B_{μ} that interact respectively with the ψ and χ fields. In general with more Fermi fields this situation becomes more complex, where these two Fermi fields are degeneracies that spit into the multiplet of fermions known. In this situation there are four possible masses for these fields on the physical vacuum. These masses occur from Yukawa couplings with the Higgs field on the physical vacuum. These will give Lagrangians terms of the form $Y_{\phi} R^{\dagger}_{\psi} \phi L_{\chi} + H.C.$ and $Y_{\eta} L^{\dagger}_{\psi} \eta R_{\chi} + H.C.$, where now we have a two components $\phi^{(4)}$ field for the Higgs mechanism. These two components assume the minimal expectation values $\langle \phi_0 \rangle$ and $\langle \eta_0 \rangle$ on the physical vacuum. We then have the Lagrangian {4}

$$\begin{aligned} \mathcal{L} = & \bar{\psi}\left(i\gamma^{\mu}\left(\partial_{\mu} + igA_{\mu}\right) - m_1\right)\psi + \bar{\chi}\left(i\gamma^{\mu}\left(\partial_{\mu} + igB_{\mu}\right) - m_2\right)\chi \\ & - Y_{\phi} R^{\dagger}_{\psi} \phi L_{\chi} + H.C. - Y_{\eta} L^{\dagger}_{\psi} \eta R_{\chi} + H.C., \end{aligned} \tag{14}$$

that can be further broken into the left and right two component spinors

$$\begin{aligned}\mathcal{L} = & R_\psi^\dagger i\sigma^\mu\left(\partial_\mu + igA_\mu\right)R_\psi + L_\psi^\dagger i\sigma^\mu\left(\partial_\mu + igA_\mu\right)L_\psi \\ & + R_\chi^\dagger i\sigma^\mu\left(\partial_\mu + igB_\mu\right)R_\chi + L_\chi^\dagger i\sigma^\mu\left(\partial_\mu + igB_\mu\right)L_\chi \\ & - m_1 R_\psi^\dagger L_\psi - m_1 L_\psi^\dagger R_\psi - m_2 R_\chi^\dagger L_\chi - m_2 L_\chi^\dagger R_\chi \\ & - Y_\phi R_\psi^\dagger \phi L_\chi + Y_\chi^\dagger \phi^* R_\psi - Y_\eta L_\psi^\dagger \eta R_\chi + Y_\eta^* R_\chi^\dagger \eta^* L_\psi .\end{aligned} \tag{15}$$

The gauge potentials A_μ and B_μ are 2 × 2 Hermitian traceless matrices and the Higgs fields ϕ and χ are also 2 × 2 matrices. These expectations are real valued, and so we then expect that the non-zero contributions of the Higgs field on the physical vacuum are given by the diagonal matrix entries {4}

$$\langle\phi\rangle = \begin{bmatrix}\langle\phi^{(1)}\rangle & 0 \\ 0 & \langle\phi^{(2)}\rangle\end{bmatrix} \qquad \langle\chi\rangle = \begin{bmatrix}\langle\chi^{(1)}\rangle & 0 \\ 0 & \langle\chi^{(2)}\rangle\end{bmatrix}. \tag{16}$$

In a recently submitted paper these issues were not discussed {1}. There this matrix is proportional to the identity matrix and the matrix nature of the Higgs field was conveniently ignored. This means that the SU(2) × SU(2) electroweak theory shares certain generic features with the SU(2) × U(1) theory. The values of the vacuum expectations are such that at high energy the left handed fields R_χ and the right handed doublet field L_ψ couple to the SU(2) vector boson field B_μ, while at low energy the theory is one with a left handed SU(2) doublet R_ψ that interacts with the right handed doublet L_χ through the massive gauge fields A_μ. Then the mass terms from the Yukawa coupling Lagrangians will then give

$$m' = Y_\eta\left\langle\chi^1\right\rangle \gg m'' = Y_\eta\left\langle\chi^2\right\rangle \gg m''' = Y_\phi\left\langle\phi^1\right\rangle \gg m'''' = Y_\phi\left\langle\phi^2\right\rangle. \tag{17}$$

Further, if the SU(2) theory for B_μ potentials are right handed chiral and the SU(2) theory for A_μ potentials are left handed chiral then we see that a chiral theory at high energies can become a vector theory at low energies. The converse may also be true in another model.

In the switch between chirality and vectoriality at different energies there is an element of broken gauge symmetry. So far, we would have a theory of a broken gauge theory at low energy. However, there is a way to express this idea so that at low energy, we have a gauge theory accompanied by a broken gauge symmetry. To illustrate this let us assume we have a simple Lagrangian that couples the left handed fields ψ_l to the right handed boson A_μ and the right handed fields ψ_r to the left handed boson B_μ

$$\begin{aligned}\mathcal{L} = & \bar\psi_l\left(i\gamma^\mu\left(\partial_\mu + igA_\mu\right) - m_1\right)\psi_l + \bar\psi_r\left(i\gamma^\mu\left(\partial_\mu + igB_\mu\right) - m_2\right)\psi_r \\ & - Y_\phi \psi_l^\dagger \phi \psi_r - Y_\phi^* \psi_r^\dagger \eta \psi_l .\end{aligned} \tag{18}$$

If the coupling constant Y_ϕ is comparable to the coupling constant g, then Fermi expectation energies of the fermions occur at the mean value for the Higgs field $\langle\phi_0\rangle$. In this case the vacuum expectation of the vacuum is proportional to the identity matrix. This means that the masses acquired by the right chiral plus left chiral gauge bosons $A_\mu + B_\mu$ are zero, while the right chiral minus left chiral gauge bosons $A_\mu - B_\mu$ acquires masses approximately $Y_\phi\langle\phi_0\rangle$. The theory at low energies is a theory with an unbroken vector gauge theory plus a broken chiral gauge theory {4}. It is also the case that we demand that the charges of the two chiral fields $A^{1,2}$ and $B^{1,2}$ that add are opposite so that the resulting vector gauge bosons are chargeless.

Just as we have gauge theories that can change their vector and chiral character so also do the doublets of the theory. In so doing, this will give rise to the doublets of leptons and quarks plus doublets of very massive fermions. These massive fermions should be observable in the multi-TeV range of energy.

THE OCCURRENCE OF THE O(3) ELECTRODYNAMICS ON THE PHYSICAL VACUUM

The two parts of the twisted bundle are copies of SU(2) with a doublet fermion structures. However, one of the fermions has the extremely large mass $m' = Y_\eta \langle \chi^1 \rangle$ that is presumed to be unstable and not observed at low energies. So one sector of the twisted bundle is left with the same Abelian structure, but with a singlet fermion. This means that the SU(2) gauge theory becomes defined by the algebra over the basis elements $\hat{e}_i,\ i \in \{1,2,3\}$,

$$\left[\hat{e}_i, \hat{e}_j\right] = i\varepsilon_{ijk}\hat{e}_k. \tag{19}$$

We further need to examine the photon masses. We define the Higgs field by a small expansion around the vacuum expectations $\eta^1 = \xi^1 + \langle \eta_0^1 \rangle$ and $\eta^2 = \xi^2 + \langle \eta_0^2 \rangle$. The contraction of the generators σ^1 and σ^2 with the Higgs field matrix and right and left fields gives

$$\sigma^1 \cdot \eta R + \sigma^2 \cdot \eta L = 0, \tag{20}$$

which gives that the charges of the $A^{(1)}$ and $A^{(2)}$ fields are zero. These fields on the low energy vacuum can be thought of as massless fields composed of two gauge bosons, with masses $\sqrt{m' + m''} \gg M_Z$ and with opposite charges. These electrically charged fields can be thoughts of as $A^{\pm} = A^{(1)} \pm A^{(2)}$. These particles cancel each other and give rise to massless vector photon gauges fields. The field $A^{(3)}$ also has this mass. This massive field is also unstable and decays into particle pairs.

With the action of the more massive Higgs field, we are left with the gauge theory SU(2) × O(3), where the first gauge group acts on doublets and the last gauge group acts on singlets. Further on a lower energy scale, or equivalently long enough time scales, the field $A^{(3)}$ has decayed and vanished. At this scale the second gauge group is then represented by $O(3)_P$ meaning a partial group. This group describes Maxwell's equations along with the definition of the field $A^{(1)} \times A^{(2)}$.

From this point, we can then treat the action of the second Higgs field on this group in a manner described in {1}. If we set the second Higgs field to have zero vacuum expectation $\langle \phi^2 \rangle = 0$ then the symmetry breaking mechanism effectively collapses to this formalism which is similar to the standard SU(2) × U(1) model Higgs mechanism. We can then arrive at a vector electromagnetic gauge theory $O(3)_p$, p stands for partial, and a broken chiral SU(2) weak interaction theory. The mass of the vector boson sector is in the $A^{(3)}$ boson plus the $W^{\pm}$ and Z^0 particles.

THE SU(4) MODEL

It is possible to consider the two SU(2) group theories as being represented as the block diagonals of the larger SU(4) gauge theory. The Lagrangian density for the system is then

$$\mathcal{L} = \overline{\psi}\left(i\gamma^{\mu}\left(\partial_{\mu} + igA_{\mu}\right) - m_1\right)\psi - Y\overline{\psi}\phi\psi. \tag{21}$$

The gauge potentials A_μ now have 4 × 4 traceless representations. The scalar field theory that describes the vacuum will now satisfy field equations that involve all 16 components of the gauge potential. By selectively coupling these fields to the fermions it should be possible to formulate a theory that recovers a low energy theory that is the standard model with the $O(3)_p$ gauge theory of electromagnetism.

DISCUSSION

What has been presented is an outline of an SU(2) × SU(2) electroweak theory that can give rise to the non-Abelian O(3) theory of quantum electrodynamics on the physical vacuum. The details of the fermions and their masses has yet to be worked through, as well as the mass of the $A^{(3)}$ boson. This vector boson as well as the additional fermions should be observable within the 10 TeV range of energy. This may be accessible by the CERN Large Hadron Collider within the next decade.

The principal purpose here has been to demonstrate what sort of electroweak interaction physics may be required for the existence of an O(3) theory of quantum electrodynamics on the low energy physical vacuum. This demonstrates that an extended standard model of electroweak interactions can support such a theory with the addition of new physics at high energy.

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