APPENDIX TWO: BREAKDOWN OF THE CONDITION FOR NO FIELD (SINGLE BEAM)

Checking on the Whittaker's equations:

$$\boldsymbol{E} = c\nabla\times(\nabla\times\boldsymbol{f})+\nabla\times\dot{\boldsymbol{g}}; \qquad \boldsymbol{B} = \frac{1}{c}\nabla\times\dot{\boldsymbol{f}} - \nabla\times(\nabla\times\boldsymbol{g}) \tag{1}$$

$$\boldsymbol{B} = \nabla\times\boldsymbol{A}; \qquad \boldsymbol{E} = -\nabla\times\boldsymbol{S} \tag{2}$$

$$\therefore \boldsymbol{A} = -\nabla\times\boldsymbol{g}+\frac{1}{c}\dot{\boldsymbol{f}} \tag{3}$$

$$S = -c\nabla\times\boldsymbol{f} - \dot{\boldsymbol{g}} \tag{4}$$

$$A_Z = \frac{1}{c}\dot{F}; \qquad S_Z = -\dot{G}$$

These are **longitudinal** components of A_Z and S_Z which do not exist in vacuo in the received view.

For circularly polarized transverse plane waves:

$$A = \frac{A^{(0)}}{\sqrt{2}}(i\boldsymbol{i}+\boldsymbol{j})e^{i(\omega t-\kappa Z)}$$

$$\boldsymbol{B} = \frac{B^{(0)}}{\sqrt{2}}(i\boldsymbol{i}+\boldsymbol{j})e^{i(\omega t-\kappa Z)}$$

$$\boldsymbol{S} = icA = cA^{(0)}\left(-\boldsymbol{i}+i\boldsymbol{j}\right)e^{i(\omega t-\kappa Z)}$$

$$\boldsymbol{E} = \frac{E^{(0)}}{\sqrt{2}}(\boldsymbol{i}-i\boldsymbol{j})e^{i(\omega t-\kappa Z)}$$

For plane waves in general (whether circularly polarized or not), the condition $E = -iB$ prevails, which requires that:

$$-c\nabla\times\boldsymbol{f}-\dot{\boldsymbol{g}} = -ic\nabla\times\boldsymbol{g}+i\dot{\boldsymbol{f}}$$

This equality is satisfied by the following relationship between $\boldsymbol{f}$ and $\boldsymbol{g}$:

$$\boldsymbol{f} = i\boldsymbol{g}$$

upon which these corollary relations are based:

$$-\dot{g} = i\dot{f}$$
$$\dot{g} = -i\dot{f}$$
$$\dot{f} = i\dot{g}$$

Cross-check

Based on the foregoing plane wave requirement that $f = ig$ and the definitions that $f = F\boldsymbol{k}$ and $g = G\boldsymbol{k}$, the scalar potentials F and G are related as:

$$F = iG$$

Let:

$$G = \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

so that

$$F = \frac{iA^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

$$\dot{G} = -i\dot{F}; \qquad \dot{F} = i\dot{G}$$

$$\boldsymbol{A}_L = \frac{i}{c}\dot{G}\boldsymbol{k} = -\kappa\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}\boldsymbol{k}$$

If $\boldsymbol{A}_L$ is physical, we have to prove that $\dot{G}\boldsymbol{k}$ is physical. Using the Lorenz condition:

$$\nabla \bullet \boldsymbol{A}_L + \frac{1}{c^2}\frac{\partial \phi_L}{\partial t} = 0$$

$$\phi_L = -\omega\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)} = \dot{F} = i\dot{G}$$
$$= cA_L$$

Checking

$$\nabla \bullet \boldsymbol{A}_L = i\kappa^2\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

$$\frac{1}{c^2}\frac{\partial \phi_L}{\partial t} = -i\kappa^2\frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)}$$

This checks the paper titled "Inconsistencies of U(1) Gauge Field Theory in Electrodynamics: the Inverse Faraday Effect".

Next, checking the paper titled "On the Representation of the Electromagnetic Field in Terms of Two Whittaker Potentials".

$$B = -\nabla \times (\nabla \times g) + \frac{1}{c}\nabla \times \dot{f}$$

$$E = c\nabla \times (\nabla \times f) + \nabla \times \dot{g}$$

Let

$$g \to g + \nabla a; \qquad \nabla \times g \to \nabla \times g + \nabla b;$$

$$f \to f + \nabla c; \qquad \nabla \times f \to \nabla \times f + \nabla d$$

then ***B*** and ***E*** are **unchanged**. So are *f* and ***g*** physical or not?

Due to the mathematical identity that $\nabla \times \nabla a = 0$, we therefore have :

$$\nabla \times g \to \nabla \times g + \nabla \times (\nabla a)$$
$$= \nabla \times g$$

$$\boxed{A_T = -(\nabla \times g) \to A_T}$$

The transverse vector potential is **physical. There is no gauge freedom in Maxwell-Heaviside theory**. This still leaves open the question of whether ***g*** and *f* are physical.

We know that $\nabla \times g$ and $\nabla \times f$ are physical under $g \to g + \nabla a$ and $f \to g + \nabla c$.

Now using:

$$\boxed{A = -\nabla \times g + \frac{i}{c}\dot{g}}$$

$$A_T = -\nabla \times g; \qquad A_L = \frac{i}{c}\dot{g}$$

$$A \cdot A = A_T \cdot A_T + A_L \cdot A_L$$
$$= (\nabla \times g)\cdot(\nabla \times g) - \frac{1}{c^2}\dot{g}\cdot\dot{g}$$

If

$$(\nabla \times g)\cdot(\nabla \times g) = \frac{1}{c^2}\dot{g}\cdot\dot{g} \tag{5}$$

then

$$A \bullet A = 0$$

$$A = (A \bullet A)^{1/2} = 0$$

$$B = (B \bullet B)^{1/2} = 0$$

$$E = (E \bullet E)^{1/2} = 0$$

Eqn (5) is one example of a condition under which **g** (and ∴ **f**) is **physical. Under condition (5), there is no vector potential, no magnetic field, and no electric field.** The only thing present is:

$$G = \frac{A^{(0)}}{\sqrt{2}}(X - iY)e^{i(\omega t - \kappa Z)} \tag{6}$$

$$\Box G = 0 \tag{7}$$

The real part of G is the physical part,

$$\mathrm{Re}(G) = \frac{A^{(0)}}{\sqrt{2}}\left[X\cos(\omega t - \kappa Z) + Y\sin(\omega t - \kappa Z)\right] \tag{8}$$

i.e.

$$G = \frac{A^{(0)}}{\sqrt{2}}\left[X\cos(\omega t - \kappa Z) + Y\sin(\omega t - \kappa Z)\right]$$

This is a **propagating magnetic flux** with units of weber. After canonical quantization of the Klein-Gordon equation:

$$\Box G = 0$$

it is found that G generates the **energy:**

$$H = \frac{1}{\mu_0}\int B^2 dV$$

and produces **photons** with energy:

$$En = \hbar\omega$$

which are spin one bosons, with eigen values -1, 0, +1.

$$\frac{\dot{G}}{c} = \kappa\frac{A^{(0)}}{\sqrt{2}}(Y\cos\phi - X\sin\phi)$$

where $\phi \equiv \omega t - \kappa Z$.

An example of how fieldless G-waves can be generated is shown in Figure 1.

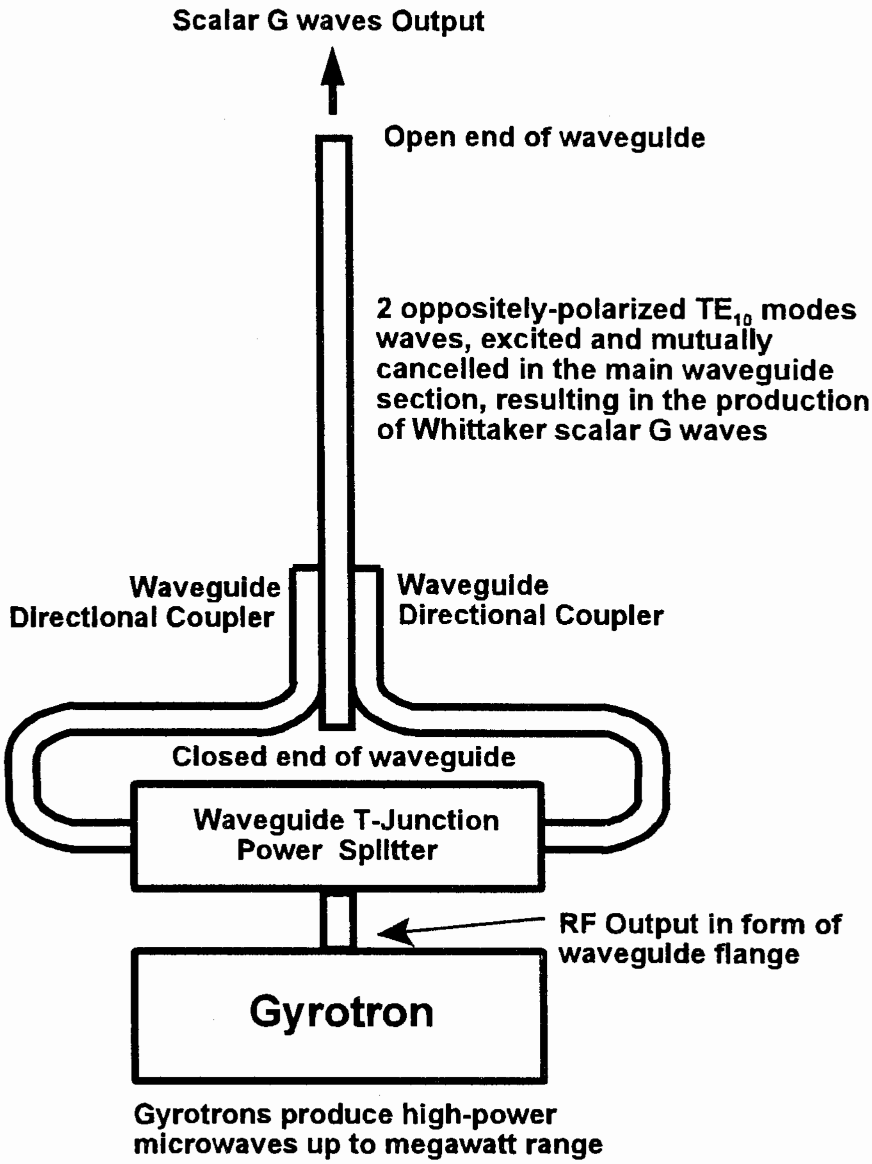

Figure 1: Practical conception for a source of scalar G waves.