5 70

Optik International Journal for Light and Electron Optics

Equations of the Yang-Mills theory of classical electrodynamics

P. K. Anastasovski[1], T. E. Bearden[2], C. Ciubotariu[3], W. T. Coffey[4], L. B. Crowell[5], G. J. Evans[6], M. W. Evans[7,8], R. Flower[9], S. Jeffers[10], A. Labounsky[11], B. Lehnert[12], M. Mészáros[13], P. R. Molnár[13], J. P. Vigier[14], S. Roy[15]

[1] Faculty of Technology and Metallurgy, Department of Physics, University of Skopje, Republic of Macedonia
[2] CEO, CTEC Inc, 2311 Big Cove Road, Huntsville, AL 35801-1351, USA
[3] Institute for Information Technology, Stuttgart University, Stuttgart, Germany
[4] Department of Microelectronics and Electrical Engineering, Trinity College, Dublin 2, Ireland
[5] Department of Physics and Astronomy University of New Mexico, Albuquerque, New Mexico
[6] Trinity College, Carnarthen, SA3 3EP, Great Britain
[7] Former Edward Davies Chemical Laboratories, University College of Wales, Aberystwyth SY32 1NE, Wales, Great Britain
[8] Sometime JRF, Wolfson College, Oxford, Great Britain
[9] CEO, Applied Science Associates and Temple University, Philadelphia, Pennsylvania, USA
[10] Department of Physics and Astronomy, York University, Toronto, Canada
[11] The Boeing Company, Huntington Beach, California
[12] Alfven Laboratory, Royal Institute of Technology, Stockholm, S-100 44, Sweden
[13] Alpha Foundation, Institute of Physics, 11 Rutafa Street, Building H, Budapest, H-1165, Hungary
[14] Laboratoire de Gravitation et Cosmologie Relativistes, Université Pierre et Marie Curie, Tour 22-12, 4ème étage, BP 142, 4 Place Jussieu, 75252 Paris, Cedex 05, France
[15] George Mason University, Virginia, and Indian Statistical Institute, Calcutta, India

Abstract: Two homomorphic versions of the non-Abelian equations of electrodynamics are developed and their advantages discussed over the received Maxwell-Heaviside equations. The internal gauge field symmetry in these equations is respectively SU(2) and O(3), signifying, on the quantized level, three electromagnetic bosons, and on the classical level, three components of the electromagnetic field, right and left circularly polarized ((1) and (2)) and longitudinally polarized (3). The reduction of these equations to the Maxwell-Heaviside equations is discussed in terms of the coupling constant of the non-Abelian covariant derivatives. The development of these equations follows the original intent and logic of Yang and Mills in 1955 to generalize pure classical electrodynamics.

1. Introduction

The origin of non-Abelian gauge theory can be traced to a paper by Yang and Mills [1] in 1955 whose intention was to generalize pure classical electrodynamics. The non-Abelian theory of electrodynamics was developed by Barrett [2–5] and Harmuth [6–8]. Following the logic of the Yang-Mills equations, Barrett produced [2] field equations which generalize the received Maxwell-Heaviside equations, whose gauge symmetry is U(1) [9]. Later, Evans et al. [10–20] developed a set of field equations, again following the logic of Yang-Mills, but in the homomorphic O(3) gauge symmetry. In this paper, some advantages of these equations are discussed over the received U(1) view of classical electrodynamics which leads to the Maxwell-Heaviside equations if the coupling constant in the U(1) covariant derivative is set to zero. This procedure means that, in the Maxwell-Heaviside equations, there can be free-fields, i.e. fields decoupled from their source. This is cause without effect, (field without source), so the concept of free-field (zero coupling constant) is a violation of causality. It is shown that in the non-Abelian, generalized, form of electrodynamics developed by Barrett [2–5] and by Evans et al. [10–20], the coupling constant of the covariant derivative is always present in the field equations. This means that the SU(2) or O(3) symmetry electromagnetic fields described by these equations are always coupled to their source, an electron, even though this be distant from the field to quasi-infinity. If the coupling constant is set to zero in both the Barrett and Evans et alia equations, the Maxwell-Heaviside equations are recovered. In general, however, the structure of the equations both in SU(2) and O(3) form is non-linear and non-Abelian, with non-zero coupling constant. The only case in which the coupling constant disappears is when there is no radiation, in which case we recover the familiar equations of electrostatics and magnetostatics, both from the equations of Barrett and those of Evans et al.

In section 2, we discuss the structure of the Barrett field equations and the way in which they reduce to the Maxwell-Heaviside equations. In section 3, the discussion is repeated for the homomorphic equations developed by Evans et al. Finally, a discussion is provided of the question of causality in pure classical electrodynamics.

Received 9 April 1999; accepted 7 August 1999.

Correspondence to: M. W. Evans, AIAS, 82 Lois Lane, Ithaca, NY 14850, USA
E-mail: Fishn Chips@compuserve.com

http://www.urbanfischer.de/journals/optik

0030-4026/2000/111/02-53 $ 12.00/0

2. Barrett field equations

The conventional U(1) symmetry Maxwell-Heaviside equations were generalized to SU(2) gauge symmetry Yang-Mills field equations for pure classical electromagnetism by Barrett [2]. In Barrett's notation, the field equations of electromagnetism become:

$$\nabla \cdot \boldsymbol{E} = J_0 - iq\,(\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{E} \cdot \boldsymbol{A}) \tag{1}$$

$$\frac{\partial \boldsymbol{E}}{\partial t} - \nabla \times \boldsymbol{B} + \boldsymbol{J} + iq\,[A_0, \boldsymbol{E}] - iq\,(\boldsymbol{A} \times \boldsymbol{B} - \boldsymbol{B} \times \boldsymbol{A}) = 0 \tag{2}$$

$$\nabla \cdot \boldsymbol{B} + iq\,(\boldsymbol{A} \cdot \boldsymbol{B} - \boldsymbol{B} \cdot \boldsymbol{A}) = 0 \tag{3}$$

$$\nabla \times \boldsymbol{E} + \frac{\partial \boldsymbol{B}}{\partial t} + iq\,[A_0, \boldsymbol{B}] + iq\,(\boldsymbol{A} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{A}) = 0. \tag{4}$$

which are the SU(2) gauge symmetry versions of the Coulomb, Ampère-Maxwell, Gauss and Faraday laws respectively. Here. Barrett uses condensed tensor notation for the magnetic field strength $\boldsymbol{E}$ and magnetic flux density $\boldsymbol{B}$. The symbol J_0 is the charge density ad $\boldsymbol{J}$ is the current density. The symbol q is the coupling constant in the SU(2) covariant derivative [2–5] used to generate the above equations. The $SU(2)$ symmetry gauge theory developed by Barrett [2] means classically that there are three polarizations, left and right circular, and longitudinal. After quantization, there are three gauge bosons. Detailed arguments for adopting this structure for electrodynamics are given by Barrett [2, 3]. We wish to point out in this section that these equations allow for the existence of a topological magnetic monopole in the SU(2) version of the Gauss Law, a monopole given by $iq\,(\boldsymbol{B} \cdot \boldsymbol{A} - \boldsymbol{A} \cdot \boldsymbol{B})$ where $\boldsymbol{A} \cdot \boldsymbol{B} - \boldsymbol{B} \cdot \boldsymbol{A}$ is a Noether current [2]. The extra charge density in the Coulomb Law is also the coupling constant q multiplied by a Noether current $-i\,(\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{E} \cdot \boldsymbol{A})$.

The other two Noether currents, $\boldsymbol{A} \times \boldsymbol{B} - \boldsymbol{B} \times \boldsymbol{A}$, and $\boldsymbol{A} \times \boldsymbol{E} - \boldsymbol{E} \times \boldsymbol{A}$ appear in the SU(2) version of the Ampère-Maxwell and Faraday Laws. The other extra terms that appear are the commutators $[A_0, \boldsymbol{E}]$ and $[A_0, \boldsymbol{B}]$ of a scalar potential A_0 and magnetic flux density $\boldsymbol{B}$ and electric field strength $\boldsymbol{E}$. Barrett carefully develops his theory from a number of physical arguments based on physical effects in nature [2, 3], and applies it successfully to the Sagnac effect with platform in motion. The development of equations (1) to (4) is therefore carefully reasoned on physical grounds. The Barrett field equations are developed on the classical level, and are equations of pure electromagnetism, so the indices of the SU(2) vector potential imply three polarizations as argued already. After quantization, three gauge bosons appear.

There are two ways in which the SU(2) field equations reduce to the U(1) field equations of Heaviside and Maxwell: if the coupling constant q is zero, or if the Noether currents and commutators $[A_0, \boldsymbol{E}]$ and $[A_0, \boldsymbol{B}]$ are both zero. In either case, we are left with equations in Maxwell-Heaviside form, but where the symbols $\boldsymbol{E}$ and $\boldsymbol{B}$ denote matrices. (If they denoted vectors, then $\boldsymbol{A} \cdot \boldsymbol{E} - \boldsymbol{E} \cdot \boldsymbol{A}$ would always be zero.) Therefore Barrett uses condensed matrix notation, which originates in the SU(2) form of the gauge theory he uses. Setting $q = 0$ therefore results in matrix equations which look like the familiar Maxwell equations. Inside these matrices, there are three field components, as in vector notation, but arranged in SU(2) matrix form.

It is clear that the process of setting the coupling constant q to zero is equivalent to setting the Noether currents to zero, and to setting the commutators $[A_0, \boldsymbol{E}]$ and $[A_0, \boldsymbol{B}]$ to zero, because none of these quantities exist in the Maxwell-Heaviside theory. However, the process $q = ?0$ reduces the SU(2) covariant derivative to an ordinary derivative, and this violates the principles of special relativity and gauge theory whenever there is source to field coupling, such as the coupling of an electron to a radiated classical electromagnetic field. The only limit in which $q = 0$ is the electrostatic limit, where there is an electron present, but no radiated field such as an electromagnetic field. Then the Maxwell-Heaviside equations reduce to the familiar equations of electrostatics, the Coulomb, Gauss and Ampère Laws without the Maxwell displacement current. This procedure becomes clearer and easier to understand if the Barrett equations are written in their homomorphic O(3) form, and this is the subject of the next section.

3. Non-Abelian field equations of Evans et al.

These are Yang-Mills equations applied to pure classical electromagnetism using the O(3) group for the internal gauge symmetry. They are again a development of the original Yang-Mills equations and have been tested extensively for self-consistency [10–20] and against available, data, both on the classical and quantum levels. In their most condensed form, the Yang-Mills equations of pure classical electromagnetism look like the homogeneous and inhomogeneous Maxwell-Heaviside equations (which have a U(1) internal gauge symmetry):

$$D_\mu \tilde{\boldsymbol{G}}^{\mu\nu} \equiv 0 \tag{5}$$

$$D_\mu \boldsymbol{H}^{\mu\nu} = \boldsymbol{J}^\nu. \tag{6}$$

However, the familiar field tensors $\tilde{\boldsymbol{G}}^{\mu\nu}$ and $\boldsymbol{H}^{\mu\nu}$ become vectors in the internal gauge space of O(3) symmetry, homomorphic with the internal SU(2) symmetry used by Barrett [2, 3]. The ordinary derivatives of the Maxwell-Heaviside equations are replaced by O(3) symmetry covariant derivatives denoted D_μ. The four-current is also a vector in the O(3) symmetry gauge space whose basis is ((1), (2), (3)). The indices (1) and (2) indicate right and left circular polarization, and (3) indicates longitudinal polarization on the classical level. Therefore the geometrical indices (1), (2) and (3) serve as the internal indices of the gauge space. This was not realized by Yang and Mills [1] and a similar idea led to the Barrett field equations [2, 3].

The homogeneous equation (5) can be developed by writing out the covariant derivative in terms of its coupling constant g, which has the units of the universal constant $e/\hbar$, the elementary charge divided by the Dirac constant. It is well known in contemporary gauge field theory that the universal coupling constant g, denoted q by Barrett, couples the electromagnetic field to the Dirac field of the electron. The latter is the source of the electromagnetic field, and since $e/\hbar$ is always independent of time and space, the electromagnetic field in Yang-Mills theory of pure electromagnetism remain coupled. There can be no free field or sourceless region unless the coupling constant g is set to zero. Setting g to zero is self-contradictory in Yang-Mills theory because $g = e/\hbar$ is a universal is a universal constant. Therefore causality is always preserved in Yang-Mills theory of pure classical electromagnetism, the field always emanates from a source. In the received Maxwell-Heaviside equations, there can be source-free regions where there are fields which have no source, i.e. cause without effect, a violation of causality. On this point alone, it is clear that the Yang-Mills theory is superior to the Maxwell-Heaviside theory. The presence of g in the homogeneous equation (5) does not mean that the gauge boson after quantization is charged. The electric charge e in modern gauge field theory is both a conserved quantity and a dynamical coupling constant, and it appears in the Yang-Mills field as a coupling constant. This dynamical aspect of $g = e/\hbar$ is well known to be a consequence of the gauge principle, a key ingredient of all modern gauge field theories [2, 3]. It is perfectly possible to apply the same principle to pure classical electromagnetism.

The equivalents of the Barrett equations (1) to (4) in the ((1), (2), (3)) basis are as follows.

The O(3) Symmetry Gauss law (cf. eqn. (3))

$$\nabla \cdot \boldsymbol{B}^{(1)*} \equiv ig\,(\boldsymbol{A}^{(2)} \cdot \boldsymbol{B}^{(3)} - \boldsymbol{B}^{(2)} \cdot \boldsymbol{A}^{(3)}) \tag{7}$$

$$\nabla \cdot \boldsymbol{B}^{(2)*} \equiv ig\,(\boldsymbol{A}^{(3)} \cdot \boldsymbol{B}^{(1)} - \boldsymbol{B}^{(3)} \cdot \boldsymbol{A}^{(1)}) \tag{8}$$

$$\nabla \cdot \boldsymbol{B}^{(3)*} \equiv ig\,(\boldsymbol{A}^{(1)} \cdot \boldsymbol{B}^{(2)} - \boldsymbol{B}^{(1)} \cdot \boldsymbol{A}^{(2)}). \tag{9}$$

The O(3) Symmetry Faraday Induction law (cf. eqn. (4))

$$\nabla \times \boldsymbol{E}^{(1)*} + \frac{\partial \boldsymbol{B}^{(1)*}}{\partial t} \equiv -ig\,(cA_0^{(3)} \boldsymbol{B}^{(2)} - cA_0^{(2)} \boldsymbol{B}^{(3)} + \boldsymbol{A}^{(2)} \times \boldsymbol{E}^{(3)} - \boldsymbol{A}^{(3)} \times \boldsymbol{E}^{(2)}) \tag{10}$$

$$\nabla \times \boldsymbol{E}^{(2)*} + \frac{\partial \boldsymbol{B}^{(2)*}}{\partial t} \equiv -ig\,(cA_0^{(1)} \boldsymbol{B}^{(3)} - cA_0^{(3)} \boldsymbol{B}^{(1)} + \boldsymbol{A}^{(3)} \times \boldsymbol{E}^{(1)} - \boldsymbol{A}^{(1)} \times \boldsymbol{E}^{(3)}) \tag{11}$$

$$\nabla \times \boldsymbol{E}^{(3)*} + \frac{\partial \boldsymbol{B}^{(3)*}}{\partial t} \equiv -ig\,(cA_0^{(2)} \boldsymbol{B}^{(1)} - cA_0^{(1)} \boldsymbol{B}^{(2)} + \boldsymbol{A}^{(1)} \times \boldsymbol{E}^{(2)} - \boldsymbol{A}^{(2)} \times \boldsymbol{E}^{(1)}) \tag{12}$$

The O(3) Symmetry Coulomb law (cf. eqn. (1))

$$\nabla \cdot \boldsymbol{E}^{(1)*} - \frac{\rho^{(1)*}}{\varepsilon_0} = ig\,(\boldsymbol{A}^{(2)} \cdot \boldsymbol{E}^{(3)} - \boldsymbol{E}^{(2)} \cdot \boldsymbol{A}^{(3)}) \tag{13}$$

$$\nabla \cdot \boldsymbol{E}^{(2)*} - \frac{\rho^{(2)*}}{\varepsilon_0} = ig\,(\boldsymbol{A}^{(3)} \cdot \boldsymbol{E}^{(1)} - \boldsymbol{E}^{(3)} \cdot \boldsymbol{A}^{(1)}) \tag{14}$$

$$\nabla \cdot \boldsymbol{E}^{(3)*} - \frac{\rho^{(3)*}}{\varepsilon_0} = ig\,(\boldsymbol{A}^{(1)} \cdot \boldsymbol{E}^{(2)} - \boldsymbol{E}^{(1)} \cdot \boldsymbol{A}^{(2)}). \tag{15}$$

The O(3) Symmetry Ampère-Maxwell law (cf. eqn. (2))

$$\nabla \times \boldsymbol{B}^{(1)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(1)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(1)*}}{\varepsilon_0} = -\frac{ig}{c}\,(A_0^{(2)} \boldsymbol{E}^{(3)} - A_0^{(3)} \boldsymbol{E}^{(2)} + c\boldsymbol{A}^{(2)} \times \boldsymbol{B}^{(3)} - c\boldsymbol{A}^{(3)} \times \boldsymbol{B}^{(2)}) \tag{16}$$

$$\nabla \times \boldsymbol{B}^{(2)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(2)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(2)*}}{\varepsilon_0} = -\frac{ig}{c}\,(A_0^{(3)} \boldsymbol{E}^{(1)} - A_0^{(1)} \boldsymbol{E}^{(3)} + c\boldsymbol{A}^{(3)} \times \boldsymbol{B}^{(1)} - c\boldsymbol{A}^{(1)} \times \boldsymbol{B}^{(3)}) \tag{17}$$

$$\nabla \times \boldsymbol{B}^{(3)*} - \frac{1}{c^2}\frac{\partial \boldsymbol{E}^{(3)*}}{\partial t} - \frac{1}{c^2}\frac{\boldsymbol{J}^{(3)*}}{\varepsilon_0} = -\frac{ig}{c}\,(A_0^{(1)} \boldsymbol{E}^{(3)} - A_0^{(2)} \boldsymbol{E}^{(1)} + c\boldsymbol{A}^{(1)} \times \boldsymbol{B}^{(2)} - c\boldsymbol{A}^{(2)} \times \boldsymbol{B}^{(1)}). \tag{18}$$

If the coupling constant g disappears, or if the Noether currents premultiplied by g are zero, these equations reduce to three Maxwell-Heaviside equations, one for each index (1), (2) and (3). Therefore the Maxwell-Heaviside equations are recovered from the more general Yang-Mills equations by a self-contradictory procedure: $g = ?\ 0$. This means that the O(3) covariant derivative is replaced by the ordinary derivative, and the dynamical aspect of $g = e/\hbar$ is lost., there can be free fields independent of source. This is actually a violation of special relativity and the principle of gauge transformation of the second kind [21].

Comparison of eqns. (7) to (18) in O(3) gauge symmetry and eqns. (1) to (4) in SU(2) gauge symmetry shows that they are identical in structure. In eqns. (7) to (18) however, the components are written out in full in the basis ((1), (2), (3)), so that it becomes clear that of g is set to zero, we recover three Maxwell-Heaviside equations, one each for the indices (1), (2) and (3); left and right circular polarization and longitudinal polarization. These again quantize to three gauge bosons [20] which are uncharged photons as required. It is well known that left and right circularly polarized photons exist in nature, and the longitudinal photon may also exist [21] self-consistently.

In the definition of the field tensor of the Yang-Mills theory behind eqns. (5) and (6), there appears an extra longitudinal field:

$$\boldsymbol{B}^{(3)*} = -ig\,\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}) \tag{19}$$

which [10–20] accounts from first gauge principles for the existence of the conjugate product $\boldsymbol{A}^{(1)} \times \boldsymbol{A}^{(2)}$ of complex vector potentials responsible for the fundamental existence of circular polarization, through the third Stokes parameter [20] and for the existence of magneto-optical effects such as the inverse Faraday effect [2, 3]. These phenomena add to the list supplied by Barrett [2, 3] of phenomena not described by the Maxwell-Heaviside equations. There are many other advantages [10–20] of the non-Abelian theory over the Abelian theory of pure classical electromagnetism. The evidence for the former is by now overwhelming, as deduced both by Barrett [2, 3] and by Evans et al. [10–20].

We note that if g is set to zero in eqns. (7) to (18), the Noether currents premultiplied by g also disappear, because these currents are specific properties of the non-Abelian nature of the theory, and setting $g = 0$ reduces the non-Abelian theory to three Abelian equations whose solutions are such that the Noether currents are zero. Therefore the theory is self-consistent and homomorphic with Barrett's equations (1) to (4). Significantly, the Barrett and Evans et al. equations were derived independently using entirely different routes to arrive at the same conclusion. They are also of course inherent in the Yang-Mills theory of 1955 when applied to pure classical electromagnetism.

Therefore both the equations of Barrett and of Evans et al. are logical consequences of the use of g as a dynamical coupling constant in contemporary gauge field theory. The equations are isomorphic and essentially contain the same information, O(3) being the covering group of SU(2).

In the limit of electro-statics and magneto-statics, the dynamical Noether currents premultiplied by g disappear, and the complex vector potential reduces of a real valued vector potential. the dynamical $\mathbf{B}^{(3)}$ field disappears because the conjugate product $A^{(1)} \times A^{(2)}$ disappears, and the Maxwell-Heaviside equations are recovered in standard form [20]. They reduce to the standard Gauss, Coulomb and Ampère laws of electro-statics and magneto-statics and the Maxwell displacement current vanishes.

4. Discussion

The coupling constants q and g that appear respectively in the Barrett and Evans et al. equations are a consequence of the general gauge principle which is central to contemporary particle/field physics. The charge e is conserved, and associated with it is the electromagnetic field. The coupling constants q and g are dynamical aspects of the theory and do not imply that the electromagnetic field is charged, or that the photons from the quantized field are charged. The equations of Barrett, or those of Evans et al., are infact the correct form of the Maxwell-Heaviside equations in contemporary gauge theory. There are many physical indications [2, 3, 10–20] for this conclusion, and for the incompleteness [2, 3] of the Maxwell-Heaviside equations. The coupling constants q or g appear in S.I. units of $e/\hbar$, but in general are variable. They depend on the material field upon which the covariant derivative is acting. Covariant derivatives can appear in a theory to any power when acting on a composite field, such as Ψ^2 of the Dirac field, but the coupling constant is zero when the covariant derivative acts on $\bar{\psi}\psi$ of the Dirac field.

In deriving the Barrett or Evans et al. equations, the covariant derivative acts on the electromagnetic field itself, and the coupling constants q or g are in units of $e/\hbar$. As shown elsewhere [20], this gives a satisfactory description of the g factor of the electron to within ten decimal places, and of the Lamb shift in hydrogen to within 17 Hz. The use of Yang-Mills concepts in electrodynamics enriches the subject considerably [2, 3, 10–20], allowing for example instanton solutions, and new explanations for effects such as the Sagnac effect [2, 21] which the Maxwell-Heaviside theory has difficulty in describing [2].

In this paper, we have pointed out that the Barrett equations and those of Evans et al. are homomorphic, and reduce to the equations of electro-statics and magneto-statics in the appropriate limits.

Acknowledgements. Prof. Dr. E. Kapuscik of the Jagiellonian University in Krakow is thanked for clarifying remarks on the nature of the coupling constant, denoted q by Barrett and g by Evans et al. Many colleagues are thanked for e-mail discussion, and the constituent institutes of AIAS (Alpha Foundation's Institute for Advanced Study) are thanked for funding to individual coauthors.

References

[1] Yang CN, Mills RL: Conservation of Isotopic Spin and Isotopic Gauge Invariance, Phys. Rev. **96** (1954) 191

[2] Barrett TW: In Lakhtakia A (ed.): Essays on the Formal Aspects of Electromagnetic Theory. pp. 6ff. World Scientific, Singapore 1993

[3] Barrett TW: In Barrett TW, Grimes DM (eds.): Advanced Electromagnetism. pp. 288 ff. World Scientific, Singapore 1995

[4] Barrett TW: Comments on the Harmuth Ansatz: use of magnetic current density in the calculation of the propagation velocity of signals by amended Maxwell theory. IEEE Trans. Electromagn. Compat. EMC-**30** (1988) 419

[5] Barrett TW: Maxwell's Theory extended, Part 1: empirical reasons for questioning the completeness of Maxwell's theory, effects demonstrating the physical significance of the A_μ potential. Ann Fond. Louis Broglie **14** (1989) 37

[6] Harmuth HF: Corrections for Maxwell's equations for signals 1. IEEE Trans. Electromagn. Compat. EMC-**28** (1986) 250

[7] Harmuth HF: ibid., part II, pp. 259 ff

[8] Harmuth HF: ibid., part III, pp. 267 ff

[9] Jackson JD: Classical Electromagnetism. Wiley, New York 1962

[10] Evans MW: The Elementary Static Magnetic Field of the Photon, Physica B **182** (1992) 227

[11] Evans MW, Jeffers S, Roy S: The Inverse Faraday Effect, Fermion Spin, and $B^{(3)}$. Nuovo Cimento B **110** (1995) 1473

[12] Evans MW: The Photomagneton and Photon Helicity. Physica A **214** (1995) 605

[13] Evans MW: The Magnetic Fields and Free Space Generators of Free Space Electromagnetism. Found. Phys. **24** (1994) 1519; ibid., The Charge Quantization Condition in O(3) Vacuum Electrodynamics **25** (1995) 175; ibid., Unification of Gravitation and Electromagnetism with $B^{(3)}$. **26** (1996) 1243

[14] Evans MW: The Photon's Magnetic Field. World Scientific, Singapore 1992

[15] Evans MW, Kielich S (eds.): Modern Nonlinear Optics. Wiley, New York 1992, 1993 and 1997 (paperback), part 2.

[16] Evans MW, Hasanein AA: The Photomagneton in Quantum Field Theory. World Scientific, Singapore 1994

[17] Evans MW, Vigier JP: The Enigmatic Photon. volumes 1 and 2, Kluwer Academic, Dordrecht 1994 and 1995

[18] Evans MW, Vigier JP, Roy S, Jeffers S: The Enigmatic Photon. volumes 3 and 4, Kluwer Academic, Dordrecht 1996

[19] Evans MW: The Enigmatic Photon, volume 5, Kluwer Academic, Dordrecht 1999

[20] Evans MW, Crowell LB: Classical and Quantum Electrodynamics and the $B^{(3)}$ Field. World Scientific, Singapore in prep.

[21] Ryder LH: Quantum Field Theory. Cambridge, 1987, 2nd ed.