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THE $B^{(3)}$ FIELD FROM CURVED SPACETIME EMBEDDED IN THE IRREDUCIBLE REPRESENTATION OF THE EINSTEIN GROUP

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The details of this paper build on the previous work of M. Sachs and M. Evans that describe an enhanced form of general relativity which contains an inherently non-abelian field tensor. We focus on a particular field arising from the non-abelian form of electrodynamics. In particular, the form of this field will be demonstrated in a first-order perturbation approach within the context of a simple manifold, and also the leading order contribution to this field due to the presence of matter. The enhanced form of general relativity, as detailed in [1], is that of antisymmetrized general relativity which relies on the irreducible representation of the Einstein translation group. We also discuss the possibility of an inherent energy induced by curvature.

Key words: Einstein-Sachs theory of general relativity, $O(3)$ electrodynamics, Einstein perturbation method, Einstein group of translations.

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1. INTRODUCTION

The arguments set out in this paper are based on fundamental enhancements to the currently accepted theories of general relativity and $U(1)$ electrodynamics. The modification to general relativity is set in the consideration of the irreducible representations of the underlying Einstein group. This defines the *antisymmetrized* general theory (AGR) as a quaternionic formalism. The quaternionic formulation has many advantages (with the embedded $O(3)$ group) over its contemporary and generally accepted $U(1)$ subgroup. The field equations of $O(3)$ electrodynamics are a special case of quaternion GR (Sachs theory) and are Yang Mills in nature [7].

Important and unsolved conditions in the fundamentals of physics such as the concept of charge quantization can be found to have a grounding within the AGR theory. It predicts the photon, principally understood as massless, to have a small non-zero mass. Similarly for the neutrino, which recent data collected from various groups supports. The theory is manifestly free of singularities. It has also been shown that phenomena such as the Sagnac effect [5] have a phase shift, which is not complementary in the $U(1)$ theory.

However, more significantly, it demonstrates that the unification of electromagnetism and gravity follows from this formalism in a consistent manner. Furthermore, the nature of the electromagnetic field tensor depends intimately on the structure of the spacetime manifold. In that sense, the theory of electromagnetism as formally described by the four Maxwell-Heaviside equations is manifestly an intricate artifact of gravity. In this light, electromagnetism exists as a product of the extra six degrees of freedom present in the sixteen shown to exist from the irreducible Einstein group. Furthermore, it will be seen that the non-abelian and gravitational field equations are generated from the same quaternionic structure.

This then necessarily leads to a manifestly enhanced form as compared with that which exists within the context of the Einstein group with ten degrees of freedom. As [2] demonstrates, the fields that belong to this new class of electromagnetic non-abelian fields are encoded in the group $O(3)$. This has been shown, and we will in addition, show that this has far reaching consequences.

In the case of the reducible group, this defines curvature that induces non-inertial properties of physics described on the manifold. However, this is a deterministic theory which allows the seating of fields which are to provide $SU(3) \times SU(2)_L \times U(1)_Y$ interaction physics for particles that are sources for them. In quaternion form, the electromagnetic field is not based on a structure that is deterministic in nature, yet is dependent on the constant $\hbar$. This shows the ability to describe electromagnetic quantum effects.

In contrast to the setting of superstrings, gravity is quantized by the closed spin-2 string. However, the geometry of this theory retains singular properties and has not yet been successfully described in "off-shell" interactions. In a similar situation that provided the first divergence between quantum aspects and the deterministic properties of general relativity, the measurable properties of strings rely on the explicit enforcement of commutation relations between variables. While one of the attractive features of extended objects in the form of strings led to the resolution to certain divergences of point like particles, in contrast, antisymmetric general relativity is absent of singularities. Parallel in importance is that it contains a structured vacuum, one which is required to describe observed effects outside the resolution of current theories.

In the context of an $O(3)$ gauge group, the electrodynamic equations provide an extended set of equations that, in particular, give rise to a new field, alongside those of the familiar Maxwell equations, called $B^{(3)}$. This field constitutes the longitudinal component of the photon. Additional support for the idea of longitudinal EM radiation can be found in [6]. While it is trivially appreciated that $U(1) \subset O(3)$, the original encoding of electromagnetism in the unphysical vector potential A_μ is now described in terms of cyclical relations of the enlarged set [2].

As will be described, the field has been shown to vanish in flat spacetime in accordance with the vanishing of the Ricci scalar field. The purpose of this paper will be to provide some qualitative insight to the effects and consequences that a non-zero Ricci field and higher symmetry group has on spacetime. The possibilities for experimental verification through the effect of the field surrounding a massive object will be discussed. In addition, we will show the consequences of a local non-zero $B^{(3)}$ in the effect of providing a local energy density.

The approach is a first order one, using the well known perturbation method used by Einstein except in the context of quaternion perturbation. Consequently, discussion is kept up to and including small massive objects.

1.1. Quaternion Components and $O(3)$ Algebra

The antisymmetrized form of special relativity [1] has a spacetime metric given by the enlarged structure

$$\eta^{\mu\nu} = (1/2)(\sigma^\mu \sigma^{\nu *} + \sigma^\nu \sigma^{\mu *}), \tag{1.1}$$

where σ^μ are the Pauli matrices satisfying the Clifford algebra

$$\{\sigma^\mu, \sigma^\nu\} = 2\delta^{\mu\nu},$$

which are represented as

$$\sigma_0 = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \; \sigma_1 = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \; \sigma_2 = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}, \; \sigma_3 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}. \tag{1.2}$$

The $*$ operation denotes quaternion conjugation, which translates to a spacial parity transformation. The quaternion basis which has the set form $\{1, \mathbf{i}, \mathbf{j}, \mathbf{k}\}$ is defined, in addition to the representations of (1.2), as $\mathbf{i}^2 = \mathbf{j}^2 = \mathbf{k}^2 = -1$ with the use of conjugation. From herein, when describing aspects of spacetime, we shall be referring to that which exists in the antisymmetrized form of the respective theory of relativity. It has shown in [2], that the field denoted by $B^{(3)}$, given by

$$\mathbf{B}^{(3)*} = -ig\mathbf{A}^{(1)} \times \mathbf{A}^{(2)}. \tag{1.3}$$

The quaternion dependent form of this field will be shown later, this also shows its dependency on the Ricci scalar field.

Using a perturbation method that has been used in conventional relativity, we will demonstrate the existence of such a field within the context of curvature. In enlarging (1.2) to describe an embedding in a curved spacetime, one can define

$$\sigma^{\mu} \to q(x)^{\mu}. \tag{1.4}$$

Such a generalization gives rise to a quaternion field of the form

$$q^{\mu} = (q_1^{\mu}, q_2^{\mu}, q_3^{\mu}, q_4^{\mu}), \tag{1.5}$$

where the sub-index refers to a generalized form of the corresponding bases in the representation (1.2).

The constraints that are imposed on the quaternion form q^{μ} that provide the solution for fields that have an $O(3)$ gauge group are the $O(3)$ cyclic relations (in the Pauli basis)

$$\begin{aligned} q_1^x q_2^{y*} - q_2^y q_1^{x*} &= 2iq_3^z, \\ q_2^y q_3^{z*} - q_3^z q_2^{y*} &= 2iq_1^x, \\ q_3^z q_1^{x*} - q_1^x q_3^{z*} &= 2iq_2^y. \end{aligned} \tag{1.6}$$

It will be shown how these confine a metric to a somewhat simplified set of spacial diagonal components in the cutoff of $\mathcal{O}(\epsilon)$ terms (in the next section it will become clear as to the purpose of the parameter ϵ which will prescribe the curvature of the manifold a distance ϵ from the origin of spacetime coordinates). The component g^{00} is unaffected.

To first simplify the calculation, we proceed with decomposing the quaternion elements in terms of a product of a two component spinor field. The spinor decomposition

$$q_i^{\mu} \sim (\Phi(x)_i \otimes \bar{\Phi}(x)_i)^{\mu} \tag{1.7}$$

provides such a simplification. The flat form (1.1) naturally generalizes to

$$g^{\mu\nu} = (1/2)(q^{\mu}q^{\nu\dagger} + q^{\nu}q^{\mu\dagger}). \tag{1.8}$$

The proceeding chapter will deal with the first order form of the generalized quaternion components q^{μ}. From there, the antisymmetrized theory field equations will be obtainable directly from the following arguments.

1.2. The Manifold Structure and Perturbation

To introduce the concept of spinor induced curvature on a physical manifold, one can start by constructing a sufficiently smooth manifold N^1 which has dimension $s+1$. Each composite manifold $S \otimes \bar{S}$ has a dimension s, there will be four copies of N, with this layering of s dimensional manifolds, corresponding to each direction in spacetime. So, N thus contains a continuous class of non intersecting manifolds parameterized by $\epsilon \in \mathbb{R}$ as M_{ϵ}, where ϵ is small and positive. N has the topological structure $S \otimes \bar{S} \times \mathbb{R}$ and each point in M_0 (which contains σ_{μ}) corresponds to a unique point ($\epsilon = 0$) in M_{ϵ} with the perturbed quaternion field. Such a "one to one" mapping requires a choice of gauge vector $\mathbf{X}_{\delta}(\epsilon)$ embedded in the space $S \otimes \bar{S}$ (δ denotes a continuous set which defines the class of possible paths and simply allows one to distinguish the possible path choices) which generates an integral curve $\chi_{\delta}(\epsilon)$, such that $\chi_{\delta'}(\epsilon)$ takes σ^{μ} to q'^{μ}, a distinct quaternion field which allows a different metric in the target spacetime. The neighborhood of consideration in spacetime is bounded by ϵ.

In the notation that follows, $Y \in S$ and $Y' \in \bar{S}$. Through the development of the perturbed quaternion field, it will be convenient to reduce spinor components to the tetrad representation $YY' \to y$, where y is considered to act in a vector-like way.

In perturbing, it will be necessary to define a *derivation* of a spinor field as having the form $D\psi = \eta^{y}\nabla_{y}\psi$ for some spinor field η^{y} [4]. A natural choice is to turn to a derivation involving the Lie derivative, however, this tool is not in general definable for spinor fields.

Since the operator "∇" (by definition) obeys the Leibnitz rule, it will simplify this approach in its use of the space $S \otimes \bar{S}$. The covariant derivative acts as

$$\nabla_{y} = \nabla_{YY'} : \Psi_{\dots} \to \Psi_{\dots;YY'}.$$

In expanding around a small neighborhood ϵ of M_{δ}, one then has

$$q(\epsilon)^{\mu}_{a} = \delta_{i\mu} c^{\mu}_{i} \sigma^{i}_{a} + \epsilon_{\mu,i,a} \nabla_{\mathbf{X}_{\delta}} q(0)^{\mu}_{a,i} + \mathcal{O}(\epsilon^2), \tag{1.9}$$

where the tetrad formalism $a \sim AA'$ for the matrix components is used, and again, i is the quaternion component. The factor c^{μ}_{i} is simply a constant coefficient. The notation used here is a little confusing. For

[1] By "smooth" is meant that there are no singularities local to the frame of reference.

the parameter $\epsilon_{\mu,i,a}$; the index μ is simply to label this copy of the parameter associated with the i^{th} quaternion for the spacetime direction μ. Equation (1.9) is then general form of a possible quaternion expansion, from here on, the index a (which defines the tetrad component of the spinor composite (1.7)) will be identified with i. This then implies that there is one parameter for each quaternion component. Therefore, with the index a now redundant, the form of (1.9) is now

$$q(\epsilon)^{\mu} = \delta_{i\mu} c_i^{\mu} \sigma^i + \epsilon_{\mu,i} \nabla_{\mathbf{X}_\delta} q(0)_i^{\mu} + \mathcal{O}(\epsilon^2). \tag{1.10}$$

The spacetime index is a label on all terms except q^{μ} and c_i^{μ} and is not internally summed over. However, there is explicit summation over i.

It is then easily seen that as each $\epsilon_{\mu,i} \to 0$, (1.10) then reduces to the required flat metric as given by (1.1).

From here, one has all the necessary machinery to construct a perturbative set of equations which will allow one to determine all aspects of AGR.

2. THE METRIC AND THE $B^{(3)}$ FIELD

With all the necessary perturbative formalism understood, it is now possible to set up a physical spacetime metric. One can approach the problem from either having defined a metric, or, having defined a particular gauge in the class $\chi_\delta(\epsilon)$ in $S \otimes \bar{S} \times \mathbb{R}$ first, will lead to a physical metric, subject to constraints.

2.1. Metric Construction

Having first defined a gauge, one then proceeds to find the solution of (1.9). As a general approach, the decomposition (1.7) has tetrad components

$$\begin{aligned}(\Phi(x) \otimes \Phi^*(x))^{\mu}_{AA'} &= (|\Phi_1(x)|^2)^{\mu}_1 + (\Phi_1(x)\bar{\Phi}_2(x))^{\mu}_2 \\ &+ (\Phi_2(x)\bar{\Phi}_1(x))^{\mu}_3 + (|\Phi_2(x)|^2)^{\mu}_4 = q^{\mu}_a(x).\end{aligned} \tag{2.1}$$

However, requiring that the components q_i^{μ} should conform to similar algebraic constraints to the components σ_i up to factors, then the appropriate tetrad components are assumed to be zero. For example, in the above notation, and in the case of σ_2, the $a = 1$ and $a = 4$ components are zero, while $a = 2$ and $a = 3$ values are $-i$ and i respectively, up to factors of ϵ. Similar is done for the other quaternion terms.

This then outlines the generalized construction details for defining all possible physical manifolds that are within the realm of the first order approach. The adaptation to first order curvature comes with the solution

$$q_{\mu}(\epsilon) = c_i^{\mu} \sigma^i \left(\delta_{i,\mu} + f_{i\mu}(\epsilon)\right), \tag{2.2}$$

where again, i labels the quaternion components. This, by Eq. (1.9) implies

$$f_{i\mu}(\epsilon)\sigma_a^i = \epsilon_{a,i}\nabla \mathbf{x}_\delta f_{i\mu}(0)\sigma_a^i. \tag{2.3}$$

From this, one finds for constants $k_{\mu,i}$,

$$f_{i\mu}(\epsilon) = \epsilon_{\mu,i}k_{\mu,i}. \tag{2.4}$$

Where the indices μ and i although repeated, are just labels, since there are a total of sixteen parameters in antisymmetrized GR, so each is labelled appropriately. The form of (2.1) is then

$$\begin{aligned} g^{\mu\nu} = \sigma^0 \Big[& c_0^\mu c_0^\nu \left(\delta_{0\mu} + f_{0\mu}(\epsilon)\right)\left(\delta_{0\mu} + f_{0\mu}(\epsilon)\right) \\ & - \sum_{i=1,2,3} c_i^\mu c_i^\nu \left(\delta_{i\mu} + f_{i\mu}(\epsilon)\right)\left(\delta_{i\mu} + f_{i\mu}(\epsilon)\right). \end{aligned} \tag{2.5}$$

In the limit of special relativity (2.2) must conform to the condition

$$q^\mu \to \sigma^\mu, \tag{2.6}$$

which, it is easily seen that (2.5) obeys with the help of (2.3), this is required so that $g^{\mu\nu} \to \eta^{\mu\nu}$. Some of the constants therefore have confinements of their own. By the above statements, consistency requires

$$c_0^0 c_0^0 = 1 \text{ and } c_i^\mu c_j^\nu = 1 \quad \text{for} \quad \mu = \nu \sim i = j. \tag{2.7}$$

In addition, by defining

$$K_{nml} = c_n^n / c_m^m c_l^l, \tag{2.8}$$

the cyclic constraints (1.6) are satisfied provided

$$K_{nml}(1 + f_{nn}(\epsilon)) = (1 + f_{mm}(\epsilon))(1 + f_{ll}(\epsilon)), \tag{2.9}$$

where n, m and l run from 1 to 3. This is in agreement with the SR limit (with 2.7). This then imposes the extra condition that $f_{ll} = 0$. As will be indicated later, these cyclic conditions take away first-order terms from the space-like diagonal components.

The identity

$$\{q_\gamma, q^{\kappa *}\} = 2\sigma_0 \delta_\gamma^\kappa \tag{2.10}$$

leads to

$$c_{\gamma i} c_j^\kappa \left(\delta_{\gamma i} + f_{\gamma i}(\epsilon)\right)\left(\delta_j^\kappa + f_j^\kappa(\epsilon)\right)\{\sigma^i, \sigma^{j*}\} = 2\sigma^0 \delta_\gamma^\kappa. \tag{2.11}$$

In taking only the first-order terms, (2.10) while a non-perturbative identity, holds only for a condition on the functions $f_{\gamma l}(\epsilon)$. The label referring to the spacetime component on f has to be taken seriously with respect to that which it refers to (i.e., contravariant or covariant) with

$$f_{\gamma l}(\epsilon) = -f_l^{\gamma}(\epsilon). \tag{2.12}$$

However, we remind the reader of the nature of this index as a label only, the function $f_{\gamma l}(\epsilon)$ behaves as a scalar under spacetime transformations.

2.2. The General $B^{(3)}$ Field from a Curved Manifold

As has been demonstrated rigorously by Sachs [1], the Ricci scalar defined for a spacetime manifold contained in AGR, has the form

$$\sigma_0 R = (1/2)(K_{\rho\lambda} q^{\lambda} q^{\rho\dagger} - q^{\rho} q^{\lambda\dagger} K_{\rho\lambda} + q^{\lambda} K^{\dagger}_{\rho\lambda} q^{\rho\dagger} - q^{\rho} K^{\dagger}_{\rho\lambda} q^{\lambda\dagger}). \tag{2.13}$$

The spin curvature tensor K which arises from the commutator of derivatives of the spinor components in (1.7) (as described in [1]), is set out in terms of the spin connection

$$\Omega_{\rho} = (1/4)\left(\partial_{\rho} q^{\mu} + \Gamma^{\mu}_{\tau\rho} q^{\tau}\right) q^{\dagger}_{\mu} \tag{2.14}$$

as

$$K_{[\lambda\rho]} = 2\left(\partial_{[\lambda}\Omega_{\rho]} + \Omega_{[\lambda}\Omega_{\rho]}\right), \tag{2.15}$$

where the spacetime connection is a metric one.

The electromagnetic force tensor has a general form

$$F_{\mu\nu} = (Q/4)\big[(K_{\mu\lambda} q^{\lambda} q^{*}_{\nu} + q_{\nu} q^{\lambda *} K_{\mu\lambda} + q^{\lambda} K^{\dagger}_{\mu\lambda} q^{*}_{\nu} + q_{\nu} K^{\dagger}_{\mu\lambda} q^{\lambda *})$$
$$+ (1/2)(q_{\mu} q^{*}_{\nu} - q_{\nu} q^{*}_{\mu}) R\big], \tag{2.16}$$

as shown in [1], with R the Ricci field of $g^{\mu\nu}$ († here denotes Hermitian adjoint). This tensor field then contains the $B^{(3)}$ field as the component

$$B^{(3)} = (Q/8) R (q^1 q^{2*} - q^2 q^{1*}). \tag{2.17}$$

This, therefore, allows a physical $B^{(3)}$ field as

$$B^{(3)} = -(Q/4) R \Bigg\{ \sum_k \Big[c^1_k c^2_0 (\delta^1_k + f^1_k(\epsilon)) f^2_0(\epsilon) - c^1_0 c^2_k (\delta^2_k + f^2_k(\epsilon)) f^1_0(\epsilon) \Big] \sigma_k$$
$$+ \sum_k c^1_k c^2_k (\delta^1_k + f^1_k(\epsilon))(\delta^2_k + f^2_k(\epsilon)) \sigma_0$$
$$+ \sum_{i,j\neq 0} c^1_i c^2_j (\delta^1_i + f^1_i(\epsilon))(\delta^2_j + f^2_j(\epsilon)) \sigma_j \sigma_i \Bigg\}. \tag{2.18}$$

The analysis so far thus sets the tools needed to define first order manifolds and the manifest properties of non-abelian fields defined on them. It is now prudent to illustrate physically relevant models to demonstrate how the $B^{(3)}$ field manifests itself in a measurable manner.

3. $B^{(3)}$ FROM A SIMPLE CURVED MANIFOLD

From (2.18) it follows that the $B^{(3)}$ field is zero in a metric with vanishing Ricci scalar field. What follows is the simple calculation of a metric that only has diagonal components and gives rise to a non-zero Ricci scalar field, and so a measurable $B^{(3)}$ field. The metric components are

$$g^{00} = (1 + f_0^0(\epsilon))^2 - \sum_{i=1,2,3} (c_i^0)^2 f_i^0(\epsilon)^2,$$

$$g^{jj} = -1 + f_0^j(\epsilon)^2 - \sum_{\{i=1,2,3\}/j} (c_i^j)^2 f_i^j(\epsilon)^2. \tag{3.1}$$

This shows a solution which is confined to the $O(3)$ cyclic conditions (1.6), which describes circularly polarized radiation, superimposed with a Schwarzschild like curvature. The conditions that constrain some of the c_j^μ terms (2.7) have been used, in this case, we choose $c_0^0 = +1$.

The conventional form of the gamma matrices is

$$\Gamma^\sigma_{\lambda\mu} = (1/2) g^{\nu\sigma} \{g_{\mu\nu,\gamma} + g_{\lambda\nu,\mu} - g_{\mu\lambda,\nu}\}, \tag{3.2}$$

with

$$\frac{\partial}{\partial x^\mu} = \frac{\partial \epsilon_{\mu,i}}{\partial x^\mu} \frac{\partial}{\partial \epsilon_{\mu,i}} \tag{3.3}$$

for summation over i only. For a calculation of the $B^{(3)}$ field, we use a simplified form of (3.1) that has all $c_j^i = 0$ except $c_1^2 = i$. The metric components now become

$$g^{00} = (1 + f_0^0(\epsilon))^2,$$

$$g^{ii} = -1 + f_0^i(\epsilon)^2 + \delta_2^i f_1^i(\epsilon)^2. \tag{3.4}$$

In the limit of taking only the terms up to and including order ϵ, this metric has the form

$$g^{\mu\nu} = \begin{pmatrix} 1 + 2f_0^0(\epsilon) & 0 & 0 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 0 & 0 & -1 \end{pmatrix}. \tag{3.5}$$

Here it is seen how the $O(3)$ cyclic conditions remove the first order terms in the diagonal space like components of the metric. The non zero Christoffel symbols are

$$\Gamma^0_{00} = \frac{1}{2}c^4_{00}(1+f_{00}(\epsilon))^2\partial_0\epsilon\partial_\epsilon(1+f_{00}(\epsilon))^2,$$
$$\Gamma^0_{0i} = \frac{1}{2}c^4_{00}(1+f_{00}(\epsilon))^2\partial_i\epsilon\partial_\epsilon(1+f_{00}(\epsilon))^2,$$
$$\Gamma^i_{00} = \frac{1}{2}c^2_{00}\partial_i\epsilon\partial_\epsilon(1+f_{00}(\epsilon))^2. \tag{3.6}$$

These connections imply that the spin tensor $K_{\rho\lambda}$ and the spin connection vector Ω_ρ have non zero components. Those of the spin curvature are

$$K_{[0j]} = \frac{1}{8}c_{00}\partial_j f_{00}\Gamma^i_{00}q^0q^*_iq^*_0\sigma^0 + \frac{1}{2}\partial_{[0}\Gamma^0_{j]0}q^0q^*_0, \tag{3.7}$$

$$K_{[kj]} = \frac{1}{4}c_{00}(\partial_{[k}f_{00}\Gamma^0_{j]0}q^0(q^0_0)^2\sigma^0 + \frac{1}{8}\Gamma^0_{0[k}\Gamma^0_{j]0}(q^0q^*_0)^2 + \frac{1}{2}\partial_{[k}\Gamma^0_{j]0}q^0q^*_0).$$

It is then a matter of cumbersome calculation to see that the above results and (2.13) yield a nonzero, real valued scalar field for this metric. The function $R(\epsilon)$ from the above results, yields a non-zero simple form where $B^{(3)} \propto QR(\epsilon)\sigma_3$ and so only varies with a change in scalar curvature and is a static phase solution.

3.1. Discussion

Using the first order approach in the context of AGR, first devised by Einstein for use in the reducible representation, it has been shown that a measurable quantity in the form of (2.18) is obtainable from the components of a realistic manifold with curvature. The simple example set out in Sec. 3 shows how a spacetime with scaled temporal component gives rise to a form similar to that obtained for the flat case, but with a Ricci scalar curvature that is dependent on such a scaling. The next appropriate step is to demonstrate the existence of the field in a manifold with matter content.

4. $B^{(3)}$ FROM A MANIFOLD WITH GRAVITATING SMALL MASS

The study of a non-abelian form of electrodynamics coupling to matter in a simple universe with one electron far from other interaction with other fields will yield interesting results that can be applied practically. The phenomena that such an investigation should shed light on are the Inverse Faraday Effect IFE and Radiative Fermion Resonance RFR.

The examination of RFR will provide a consistent underpinning as to the existence of the $B^{(3)}$ field. The details of IFE and RFR are discussed extensively in [2], with further elaboration in the last section.

In the 10 component form of general relativity [3], the well-known tensor field equations are [3]

$$R_{\mu\nu} - \frac{1}{2} R g_{\mu\nu} = k T_{\mu\nu}. \tag{4.1}$$

As set out in [1], the 16 component quaternion equivalent to this given by

$$\begin{aligned}(K_{\mu\gamma} q^{\gamma} q^{*}_{\nu} - h q_{\nu} q^{*\gamma} K_{\mu\gamma} + q^{\gamma} K^{\dagger}_{\mu\gamma} q^{*}_{\nu} - h q_{\nu} K^{\dagger}_{\mu\gamma} q^{\gamma *}) + \frac{1}{2}(q_{\mu} q^{*}_{\nu} + h q_{\nu} q^{*}_{\mu}) R \\ = 4 \begin{pmatrix} k \\ k' \end{pmatrix} (T_{\mu} q^{*}_{\nu} + h q_{\nu} T^{*}_{\mu}),\end{aligned} \tag{4.2}$$

whereby h takes the values ± 1. For $h = -1$, and k' as the proportionality constant, there is a direct one to one correspondence with the 10 component symmetric field equation (4.1). The choice $h = +1$, and constant k, the above is exactly that of the 6 components of the field tensor (2.16).

The electro-gravitic equation is given by

$$R q_{\rho} = 8 k T_{\rho} - 2(K_{\rho\lambda} q^{\lambda} + q^{\lambda} K^{\dagger}_{\rho\lambda}), \tag{4.3}$$

where T_{ρ} contains energy momentum information of both the gravitational and electromagnetic components. The $B^{(3)}$ field defined for non-zero energy momentum components is then

$$\begin{aligned}B^{(3)} = Q\big[k(T^{1} q^{2*} - T^{2} q^{1*}) - \frac{1}{4}(K^{1}_{\lambda} q^{\lambda} q^{2*} \\ - K^{2}_{\lambda} q^{\lambda} q^{1*}) + q^{\lambda} K^{\dagger 1}_{\lambda} q^{2*} - q^{\lambda} K^{\dagger 2}_{\lambda} q^{1*}\big],\end{aligned} \tag{4.4}$$

which loosely translates into

$$B^{(3)} = T_{\text{Matter}} + T_{\text{Spin}}. \tag{4.5}$$

In the case of the irreducible group representation, the conventional tensor is the sixteen-component object

$$T^{\mu}(T^{\mu}_{0}, T^{\mu}_{1}, T^{\mu}_{2}, T^{\mu}_{3}) \tag{4.6}$$

or

$$T^{\mu} = T^{\mu}_{j} \sigma_{j} (\delta^{\mu}_{j} + f^{\mu}_{j}(\epsilon)). \tag{4.7}$$

One then finds that T_{matter} now is

$$T_{\text{matter}} = Qk\Big[\sum_j (T_j^1 c_0^2 - T_j^2 c_0^1) f_j^1(\epsilon) f_0^2(\epsilon)\sigma_j$$
$$- \sum_{i\in\{1,2,3\} j} (T_j^1 c_i^2 - T_j^2 c_i^1)(\delta_j^1 + f_j^1(\epsilon))(\delta_i^2 + f_i^2(\epsilon))\sigma_j\Big]. \qquad (4.8)$$

The leading order term here is the one dependent on the diagonal contributions from δ_i^2 and δ_j^1, it has the form

$$B^{(3)}_{\text{matter}} \sim i(\hbar k/e)\sigma_3(T_1^1 c_1^1 - T_1^2 c_2^1), \qquad (4.9)$$

where $Q = \hbar/e$ and e is the charge of an electron. The first term in (4.8) is of second order and so is neglected. In the case of choosing the simple metric form (3.5), then (4.9) is the result. This is the approximation of a weak Schwarzschild solution for a charged particle with the first order terms in the space-like diagonal components absent by virtue of the $O(3)$ cyclic relations. In this case, the spin curvature tensor components already computed (3.7), result in (4.5) having an additional background component, which for brevity, will not be stated here.

Equation (4.9) gives the magnetization observed in the inverse Faraday effect [8] and resonance between the states of σ_3 gives radiatively induced fermion resonance.

5. ENERGY INHERENT IN CURVATURE

From (2.18) we can calculate the classical electromagnetic energy density due to $B^{(3)}$ in the Schwarzschild metric devoid of matter fields:

$$\frac{E}{V} = \frac{(B^{(3)})^2}{\mu_0} = \frac{Q^2 R^2}{\mu_0}, \qquad (5.1)$$

where μ_0 is the vacuum permeability in SI units. Q is a fundamental constant, a primordial magnetic flux in the universe. After quantization it is $\hbar/e$ for one photon. Therefore $B^{(3)}$ exists in curved spacetime free of radiating electrons, a spacetime with no matter fields present, a spacetime which is described by the Schwarzschild metric used to derive (2.18). The concept of a $B^{(3)}$ field generated by choice of metric in free space, devoid of source (accelerated electron), does not exist in the Maxwell-Heaviside theory, and (5.1) can be understood as electromagnetic energy density produced by curved spacetime, "the structured vacuum." (5.1) is therefore the simplest equation that describes electromagnetic energy density from the vacuum in AGR. It describes how electromagnetic energy density can be generated simply by the expansion of the universe, without radiating (or source) electrons being present. In Maxwell-Heaviside theory there must always be source electrons present for an electromagnetic field to be present. It is clear from (1.8) and (2.16) that the symmetric and antisymmetric fields embedded in spacetime are generated by the q^μ_a components. In the flat case of $q^\mu_a \rightarrow \sigma^m u$, one has a manifold in which all $\mathbf{B}^{(i)}$ components are zero, as provided by

$$\mathbf{B}^{(1)} \times \mathbf{B}^{(2)} = iB^{(0)}\mathbf{B}^{(3)*},$$
$$\mathbf{B}^{(2)} \times \mathbf{B}^{(3)} = iB^{(0)}\mathbf{B}^{(1)*},$$
$$\mathbf{B}^{(3)} \times \mathbf{B}^{(1)} = iB^{(0)}\mathbf{B}^{(2)*}.$$

In addition, the relations

$$\mathbf{E}^{(1)} \times \mathbf{E}^{(2)} = ic^2B^{(0)}\mathbf{B}^{(3)*},$$
$$\mathbf{B}^{(3)} \times \mathbf{E}^{(1)} = icB^{(0)}\mathbf{E}^{(2)*},$$
$$\mathbf{B}^{(3)} \times \mathbf{B}^{(2)} = -icB^{(0)}\mathbf{E}^{(1)*},$$

demonstrate that is is only possible to define electromagnetic fluctuations by introducing a non-zero Ricci scalar field. This therefore implies a more far reaching notion of a point charge placed on a background which provides a source of electromagnetic fluctuations.

The scalar or Ricci curvature R does not exist in Maxwell-Heaviside theory, or in its quantized equivalent following canonical quantization, or in quantum electrodynamics. This is because in Maxwell-Heaviside theory the electromagnetic field is an entity superimposed on flat (Euclidean) spacetime. In AGR and in (5.1) the electromagnetic field and energy density from the $B^{(3)}$ field are manifestations of the structured vacuum (curved, or Riemannian, spacetime itself). The concept of primordial magnetic flux, embedded in the Sachs constant Q, is not present in the Maxwell-Heaviside theory. In AGR, every photon carries the elementary magnetic fluxon $\hbar/e$ and this is present in the background radiation of the universe, and whenever there are photons present. Therefore the electromagnetic energy density from the $B^{(3)}$ field for an expanding universe is proportional to $(QR)^2$. In AGR both Q and R must always be non-zero if the electromagnetic field from AGR is to be non-zero. So electromagnetism is a manifestation of the curvature of spacetime, of the structured vacuum itself on the classical level. In AGR there is no longer a requirement of radiating (accelerated) electrons to generate the electromagnetic field. This is clear from (2.18) because that equation is derived for free space (a manifold devoid of matter fields and thus containing no radiating electrons). Moreover the charge current density of the electromagnetic field in AGR can be similarly generated by the expansion of the universe in a manifold free of radiating electrons, and the charge current density is also a manifestation of the structured vacuum itself. The charge current density of electromagnetism in AGR depends on the gravitational constant k, a clear sign that AGR is a unified field theory of gravitation and electromagnetism on the classical level. Therefore there is always a source present (nonzero charge current density depending on k) when the electromagnetic field propagates through curved spacetime in AGR, a spacetime free of matter fields. Self consistently this source charge current density also depends on curvature and on a non-zero R, and vanishes in Euclidean spacetime (unstructured vacuum).

So on the simplest level, electromagnetic energy density from the structured vacuum is always available if the product QR is non-zero.

As additional discussion, in AGR, the electron is a classical matter field without singularities, and so AGR solves the paradox of infinite electron self energy, which plagues Maxwell-Heaviside field theory and quantum electrodynamics. It is generally true of the AGR framework that it is absent of singularities. It provides an elegant way of providing a structure that can encompass fundamental interactions. Alongside the well understood interactions between matter and gauge fields, the $B^{(3)}$ field interacts with one electron to produce the inverse Faraday effect (magnetization due to circularly polarized electromagnetic radiation). While this effect has been measured [8] already, new measurements are now called for in the light of this new explanatory framework.

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