Numerical evaluation of claim by Shapiro.

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The correct evaluation of the Shapiro claim is as follows:

Define firstly:

$$f(r) = \left(1 - \frac{r_0}{r}\right)^{-1}\left(1 - \left(1 - \frac{r_0}{r}\right)\left(\frac{R_0{}^2}{r^2}\right)\right)^{-1/2}. \tag{1}$$

The time delay is:

$$\Delta t = t_3 - t_0 \ , \tag{2}$$

where

$$t_3 = \frac{2}{c}\left(\int_{R_0}^{R_E} f\ (r)\, dr + \int_{R_0}^{R_P} f\ (r)\, dr \right) \tag{3}$$

$$t_0 = \frac{2}{c}\left(\int_{R_0}^{R_E}\left(1 - \left(\frac{R_0{}^2}{r^2}\right)\right)^{-1/2} dr + \frac{2}{c}\left(\int_{R_0}^{R_P}\left(1 - \left(\frac{R_0{}^2}{r^2}\right)\right)^{-1/2} dr \right.\right.$$

$$= \frac{2}{c}\left(r_1 + r_2\right) \quad . \tag{4}$$

Wald in his equation (6.3.45) gives an expression for Δt . Firstly, note that Wald's notation is:

$$M(\text{Wald}) \longrightarrow \frac{MG}{c^2} \quad \text{(S.I.)} \tag{5}$$

So Wald gives, in S.I. units:

$$\Delta t = \frac{2}{c}\left[\left(R_E{}^2 - R_0{}^2\right)^{1/2} + \left(R_P{}^2 - R_0{}^2\right)^{1/2} \right]$$

$$+ \frac{2MG}{c^3}\left[2 \log_e\left(\frac{R_E + (R_E{}^2 - R_0{}^2)^{1/2}}{R_0} \right)\right.$$

$$+\ 2\log_e\left(\frac{R_P+(R_P{}^2-R_0{}^2)^{1/2}}{R_0}\right)$$

$$+\left(\frac{R_E-R_0}{R_E+R_0}\right)^{1/2}+\left(\frac{R_P-R_0}{R_P+R_0}\right)^{1/2}\Big]\ . \qquad (6)$$

The first part of Eq. (6) is our Eq. (4):

$$t_0=\frac{2}{c}\left(r_1+r_2\right)=\frac{2}{c}\left[\left(R_E{}^2-R_0{}^2\right)^{1/2}+\left(R_P{}^2-R_0{}^2\right)^{1/2}\right]\ , \qquad (7)$$

which is obtained analytically from the condition:

$$\frac{r_0}{R_0}=0\ . \qquad (8)$$

It is important to note that Shapiro and Wald give Δt as an expression adding to t_0 . i.e.

$$\boxed{\Delta t\ (\text{Wald})=t_0+t_3} \qquad (9)$$

so the so called "time delay" is a time increase .

Therefore, the claim by Shapiro repeated by Wald is:

$$t_3=\frac{2MG}{c^3}\Big[2\log_e\left(\frac{R_E+(R_E{}^2-R_0{}^2)^{1/2}}{R_0}\right)$$

$$+\ 2\log_e\left(\frac{R_P+(R_P{}^2-R_0{}^2)^{1/2}}{R_0}\right)$$

$$+\left(\frac{R_E-R_0}{R_E+R_0}\right)^{1/2}+\left(\frac{R_P-R_0}{R_P+R_0}\right)^{1/2}\Big]\ . \qquad (10)$$

Check :

This is to evaluate Eq. (3) numerically to machine precision, and compare with Eq. (10).

Input parameters:

These are r_0 , R_0 , R_E and R_P , but for numerical purposes, any input parameters can be used. Use:

MG = 1.327581035 x 10^{20} m^3 s^{-2}

$c =$ 2.997925 x 10^8 m s^{-1}

so

$$\frac{2MG}{c^3} = 9.8543672 \times 10^{-6} \text{ s}$$

= 9.8543672 microseconds.