

100(34) : Relation Between Field and Potential is the Coulomb Law.

The potential is more conveniently defined by raised index as follows:

$$F^{\kappa\mu} = \partial^\mu A^{\kappa\nu} - \partial^\nu A^{\kappa\mu} + \frac{1}{4} \left(\omega^{\kappa\mu}_{\lambda} A^{\lambda\nu} - \omega^{\kappa\nu}_{\lambda} A^{\lambda\mu} \right) \quad \lambda=0 \quad (1)$$

The electric field components are:

$$\begin{aligned} E^{010} &= c \partial^1 A^{00} - c \partial^0 A^{01} + \frac{c}{4} \left(\omega^{01}_{\lambda} A^{\lambda 0} - \omega^{00}_{\lambda} A^{\lambda 1} \right) \\ E^{020} &= c \partial^2 A^{00} - c \partial^0 A^{02} + \frac{c}{4} \left(\omega^{02}_{\lambda} A^{\lambda 0} - \omega^{00}_{\lambda} A^{\lambda 2} \right) \\ E^{030} &= c \partial^3 A^{00} - c \partial^0 A^{03} + \frac{c}{4} \left(\omega^{03}_{\lambda} A^{\lambda 0} - \omega^{00}_{\lambda} A^{\lambda 3} \right) \end{aligned} \quad (2)$$

The vector notation eq (2) is:

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} + \underline{\phi} \underline{\omega} - \underline{\omega} \underline{A} \quad (3)$$

where:

$$\underbrace{\phi = c A^{00}}_{\text{energy}}, \quad \underbrace{\underline{A} = A^{01} \underline{i} + A^{02} \underline{j} + A^{03} \underline{k}}_{\text{momentum}} \quad (4)$$

and where:

momentum

$$\omega = \frac{1}{4} c \omega_0^{\omega}, \quad \underline{\omega} = \frac{1}{4} (\omega_0^{\omega 1} \underline{i} + \omega_0^{\omega 2} \underline{j} + \omega_0^{\omega 3} \underline{k})$$

- (5)

This has a clear interpretation because ϕ is proportional to energy density and \underline{A} to momentum density. The potential tensor $A^{\mu\nu}$ is proportional to the energy-momentum density tensor.

The Coulomb law is a law of forces directed between the line of two charges.

For engineering the vector structure (3) can be used while noting that the quantities have an internal structure defined by eqs. (4) and (5).

Finally the electric field of the Coulomb law is identified as an orbital angular energy momentum density tensor with a proportionality. FAAP is the lab. orbit is the straight line limit between two charges.
