100(10)

Homogeneous ECE Flow Chart

Definition → $D\wedge T := R\wedge q$ Bianchi identity → $D\wedge(D\wedge T) := D\wedge(R\wedge q)$ Second Bianchi identity, paper 88

↓

$D\wedge T^a := R^a{}_b\wedge q^b$

↓

$d\wedge T^a = R^a{}_b\wedge q^b - \omega^a{}_b\wedge T^b$ → $d\wedge T^a = 0$, pure rotation. → $R\wedge q = \omega\wedge T$

↓

$d\wedge T^a = -(q^b\wedge R^a{}_b + \omega^a{}_b\wedge T^b)$

↓

$$\partial_\rho T^a{}_{\mu\nu} + \partial_\nu T^a{}_{\rho\mu} + \partial_\mu T^a{}_{\nu\rho} = -\left(R^a{}_{\rho\mu\nu} + R^a{}_{\nu\rho\mu} + R^a{}_{\mu\nu\rho} + \omega^a{}_{b\rho}T^b{}_{\mu\nu} + \omega^a{}_{b\nu}T^b{}_{\rho\mu} + \omega^a{}_{b\mu}T^b{}_{\nu\rho}\right)$$

→ Sums of antisymmetric rank two tensors in four dimensions on both sides → Hodge dual operation both sides.

↓

$$\partial_\mu \tilde{T}^{a\mu\nu} = -\left(\tilde{R}^a{}_\mu{}^{\mu\nu} + \omega^a{}_{b\mu}\tilde{T}^{b\mu\nu}\right)$$

↓

$$\boxed{\partial_\mu \tilde{F}^{a\mu\nu} = -A^{(0)}\left(\tilde{R}^a{}_\mu{}^{\mu\nu} + \omega^a{}_{b\mu}\tilde{T}^{b\mu\nu}\right)}$$

↓

$\partial_\mu \tilde{F}^{a\mu\nu} = 0$, $R\wedge q = \omega\wedge T$ → Free e/m Field

↓

$\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0$ → $\underline{\nabla}\cdot\underline{B} = 0$, $\underline{\nabla}\times\underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0}$

Inhomogeneous ECE Flow Chart

$$\boxed{D\wedge\tilde{T} := \tilde{R}\wedge q} \longrightarrow \boxed{D\wedge(D\wedge\tilde{T}) := D\wedge(\tilde{R}\wedge q)}$$

$$\downarrow$$

$$\boxed{D\wedge\tilde{T}^a := \tilde{R}^a{}_b\wedge q^b}$$

$$\downarrow$$

$$\boxed{d\wedge\tilde{T}^a = -\left(q^b\wedge\tilde{R}^a{}_b + \omega^a{}_b\wedge\tilde{T}^b\right)}$$

$$\downarrow$$

$$\partial_\rho\tilde{T}^a_{\mu\nu} + \partial_\nu\tilde{T}^a_{\rho\mu} + \partial_\mu\tilde{T}^a_{\nu\rho} = -\left(\tilde{R}^a_{\rho\mu\nu} + \tilde{R}^a_{\nu\rho\mu} + \tilde{R}^a_{\mu\nu\rho} + \omega^a{}_{b\rho}\tilde{T}^b_{\mu\nu} + \omega^a{}_{b\nu}\tilde{T}^b_{\rho\mu} + \omega^a{}_{b\mu}\tilde{T}^b_{\nu\rho}\right)$$

$$\downarrow$$

$$\boxed{\partial_\mu T^{a\mu\nu} = -\left(R^a{}_\mu{}^{\mu\nu} + \omega^a{}_{b\mu}T^{b\mu\nu}\right)}$$

$$\downarrow$$

$$\boxed{\partial_\mu F^{a\mu\nu} = -A^{(0)}\left(R^a{}_\mu{}^{\mu\nu} + \omega^a{}_{b\mu}T^{b\mu\nu}\right)}$$

$$\downarrow$$

$$\boxed{\partial_\mu F^{a\mu\nu} = -A^{(0)}\left(R^a{}_\mu{}^{\mu\nu}\right)_{EH}} \longrightarrow \boxed{\text{Ricci flat vacuum}}$$

$$\uparrow$$

$$\partial_\mu F^{\kappa\mu\nu} = 0, \quad R^\kappa{}_\mu{}^{\mu\nu} = 0.$$

$$\uparrow$$

$$\nabla\cdot E = 0, \quad \nabla\times B = \frac{1}{c^2}\frac{\partial E}{\partial t}; \quad \text{EH}: G_{\mu\nu} = 0$$

$$\uparrow$$

No sources for the e/m and gravitational fields, unphysical.

$$\downarrow$$

$$\boxed{\partial_\mu F^{\kappa\mu\nu} = -A^{(0)}\left(R^\kappa{}_\mu{}^{\mu\nu}\right)_{EH}} \longrightarrow \boxed{\begin{array}{l}\nabla\cdot E = \rho/\epsilon_0 \\ \nabla\times B - \dfrac{1}{c^2}\dfrac{\partial E}{\partial t} = \mu_0 J\end{array}}$$

3) Tensor Expansion of the Bianchi Identity.
(GCUFT1, Chap. 17, Appendix C)

$$\boxed{D\wedge T \underset{\text{Bianchi identity}}{:=} R\wedge q}$$

$$\downarrow$$

$$R^{\lambda}{}_{\rho\mu\nu} + R^{\lambda}{}_{\mu\nu\rho} + R^{\lambda}{}_{\nu\rho\mu}$$

$$:= \partial_\mu \Gamma^{\lambda}_{\nu\rho} - \partial_\nu \Gamma^{\lambda}_{\mu\rho} + \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\nu\rho} - \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\mu\rho}$$

$$+ \partial_\nu \Gamma^{\lambda}_{\rho\mu} - \partial_\rho \Gamma^{\lambda}_{\nu\mu} + \Gamma^{\lambda}_{\nu\sigma}\Gamma^{\sigma}_{\rho\mu} - \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\nu\mu}$$

$$+ \partial_\rho \Gamma^{\lambda}_{\mu\nu} - \partial_\mu \Gamma^{\lambda}_{\rho\nu} + \Gamma^{\lambda}_{\rho\sigma}\Gamma^{\sigma}_{\mu\nu} - \Gamma^{\lambda}_{\mu\sigma}\Gamma^{\sigma}_{\rho\nu}$$

$\neq 0$ in general; $= 0$ only in EH theory

$$\uparrow$$

$$\boxed{T^{\lambda}{}_{\mu\nu} = \Gamma^{\lambda}_{\mu\nu} - \Gamma^{\lambda}_{\nu\mu}}$$

$$\uparrow$$

$$\boxed{T^{a} = d\wedge q^{a} + \omega^{a}{}_{b}\wedge q^{b}}$$

$$\uparrow$$

First Cartan
structure Equation

4)

Hodge Dual Bianchi Identity

Definition

$$D \wedge \tilde{T} := \tilde{R} \wedge q$$

$$\begin{aligned}
&\tilde{R}^{\lambda}{}_{\rho\mu\nu} + \tilde{R}^{\lambda}{}_{\mu\nu\rho} + \tilde{R}^{\lambda}{}_{\nu\rho\mu} \\
&:= \partial_\mu \Delta^{\lambda}{}_{\nu\rho} - \partial_\nu \Delta^{\lambda}{}_{\mu\rho} + \Delta^{\lambda}{}_{\mu\sigma}\Delta^{\sigma}{}_{\nu\rho} - \Delta^{\lambda}{}_{\nu\sigma}\Delta^{\sigma}{}_{\mu\rho} \\
&\quad + \partial_\nu \Delta^{\lambda}{}_{\rho\mu} - \partial_\rho \Delta^{\lambda}{}_{\nu\mu} + \Delta^{\lambda}{}_{\nu\sigma}\Delta^{\sigma}{}_{\rho\mu} - \Delta^{\lambda}{}_{\rho\sigma}\Delta^{\sigma}{}_{\nu\mu} \\
&\quad + \partial_\rho \Delta^{\lambda}{}_{\mu\nu} - \partial_\mu \Delta^{\lambda}{}_{\rho\nu} + \Delta^{\lambda}{}_{\rho\sigma}\Delta^{\sigma}{}_{\mu\nu} - \Delta^{\lambda}{}_{\mu\sigma}\Delta^{\sigma}{}_{\rho\nu} \\
&\neq 0 \text{ in general, } \Delta := \text{Hodge dual connection}
\end{aligned}$$

$$\tilde{T}^{\lambda}{}_{\mu\nu} := \Delta^{\lambda}{}_{\mu\nu} - \Delta^{\lambda}{}_{\nu\mu}$$

$$R^{\lambda}{}_{\kappa}{}^{\rho\sigma} = g^{\rho\mu} g^{\sigma\nu} R^{\lambda}{}_{\kappa\mu\nu}$$

$$\tilde{R}^{\lambda}{}_{\kappa\mu\nu} = \frac{1}{2}\bar{\epsilon}_{\mu\nu\rho\sigma} R^{\lambda}{}_{\kappa}{}^{\rho\sigma}$$

The Hodge dual Riemann tensor

$$\bar{\epsilon}_{\mu\nu\rho\sigma} = \|g\|^{1/2} \epsilon_{\mu\nu\rho\sigma}$$

5)

Role of the Tetrad Postulate : HECE

$$T^a := d \wedge q^a + \omega^a{}_b \wedge q^b$$

First Cartan structure equation

$$T^a{}_{\mu\nu} = \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b} q^b{}_\nu - \omega^a{}_{\nu b} q^b{}_\mu$$

$D_\mu q^a{}_\nu = 0$; tetrad postulate $\rightarrow$ Links Cartan and Riemann geometry

$$\partial_\mu q^a{}_\nu = q^a{}_\lambda \Gamma^\lambda{}_{\mu\nu} - q^b{}_\nu \omega^a{}_{\mu b}$$

$$\partial_\nu q^a{}_\mu = q^a{}_\lambda \Gamma^\lambda{}_{\nu\mu} - q^b{}_\mu \omega^a{}_{\nu b}$$

$$T^a{}_{\mu\nu} = q^a{}_\lambda \left(\Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \right)$$

$$= q^a{}_\lambda T^\lambda{}_{\mu\nu}$$

Torsion tensor of Riemann geometry

6) Role of the Tetrad Postulate : IECE

$$\boxed{\tilde{T}^a := d \wedge q^a + \omega^a{}_b \wedge q^b}$$

$$\boxed{\tilde{T}^a{}_{\mu\nu} := \partial_\mu q^a{}_\nu - \partial_\nu q^a{}_\mu + \omega^a{}_{\mu b}\, q^b{}_\nu - \omega^a{}_{\nu b}\, q^b{}_\mu}$$

$$\boxed{D_\mu q^a{}_\nu = 0}$$

$$\boxed{\begin{aligned} \partial_\mu q^a{}_\nu &= q^a{}_\lambda \Delta^\lambda{}_{\mu\nu} - q^b{}_\nu\, \omega^a{}_{\mu b} \\ \partial_\nu q^a{}_\mu &= q^a{}_\lambda \Delta^\lambda{}_{\nu\mu} - q^b{}_\mu\, \omega^a{}_{\nu b} \end{aligned}}$$

$$\boxed{\begin{aligned} \tilde{T}^a{}_{\mu\nu} &= q^a{}_\lambda \left(\Delta^\lambda{}_{\mu\nu} - \Delta^\lambda{}_{\nu\mu} \right) \\ &= q^a{}_\lambda\, \tilde{T}^\lambda{}_{\mu\nu} \end{aligned}}$$

$$\boxed{\begin{aligned} \tilde{T}^a{}_{\mu\nu} &= \frac{1}{2} \bar{\epsilon}_{\mu\nu\rho\sigma}\, T^{a\rho\sigma}, \\ T^{a\rho\sigma} &= g^{\rho\mu} g^{\sigma\nu}\, T^a{}_{\mu\nu} \end{aligned}}$$