1) **Notes 107(1): Resonance in the Homopolar Generator**

The homopolar generator or Faraday disk was first developed with ECE theory in papers 43 and 44. They were based on the fundamental idea of ECE theory

$$F = A^{(0)} T \quad -(1)$$

In the case of the Faraday disk the torsion $T$ is set up by mechanical rotation. So the basic equations of the generator are:

$$F = A^{(0)} T \,(\text{mechanical}) \quad -(2)$$

$$= d \wedge A + \omega \wedge A$$

and the field equations:

$$D \wedge F = R \wedge A \quad -(3)$$

$$D \wedge \tilde{F} = \tilde{R} \wedge A. \quad -(4)$$

These lead to the vector equations

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(5)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(6)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(7)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(8)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega} \times \underline{A} \quad -(9)$$

$$\underline{E} = -\underline{\nabla} \phi - \frac{\partial \underline{A}}{\partial t} + \phi \underline{\omega} - \omega \underline{A}. \quad -(10)$$

2) For more details of eqns (5) to (10) refer to the review paper 100 and paper 94, plus new paper by Horst Eckardt on various resonance designs in electrical engineering.

In paper 44 a complex circular basis was used to define a rotating potential set up by mechanical rotation at frequency $\Omega$:

$$\underline{A}^{(1)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\Omega t} \quad -(11)$$

$$\underline{A}^{(2)} = \frac{A^{(0)}}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{-i\Omega t} \quad -(12)$$

This is the key concept. It is based on:

$$A = A^{(0)} q \quad -(13)$$

So: $$\underline{q}^{(1)} = \underline{q}^{(2)*} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)e^{i\Omega t} \quad -(14)$$

This is rotational relativity, the mechanical spin sets up a rotational tetrad, i.e. a rotation of space-time itself.

In paper 44 a special case of eq. (10) was used:

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \omega \underline{A} \quad -(15)$$

3) in which $\phi$ is assumed to be zero and where $\underline{A}$ is generated by the magnet of the Faraday disk. The scalar spin connection $\omega$ was assumed to be proportional to $\Omega$, so:

$$\underline{E}^{(2)} = \underline{E}^{(1)*} = -\left(\frac{\partial}{\partial t} + i\Omega\right)\underline{A}^{(2)} \quad (16)$$

and:

$$\mathrm{Real}\left(\underline{E}^{(2)}\right) = \mathrm{Real}\left(\underline{E}^{(1)}\right) = \frac{2}{\sqrt{2}} A^{(0)}\Omega\left(\underline{i}\sin\Omega t - \underline{j}\cos\Omega t\right) \quad (17)$$

This electric field strength spins around the rim of the rotating disk. As observed experimentally it is proportional to the product of $A^{(0)}$ and $\Omega$. An emf is set up between the centre and rim of the rotating disk, and this is measured by a voltmeter at rest with respect to the disk.

Recently there have been reports of a homopolar generator like this, exhibiting a powerful resonance of emf as a well defined RPM (i.e. $\Omega$) is approached. The

4) onset of resonance was reported to be sudden, and resulted in an explosion of the generator. In ECE theory this is explained by spin connection resonance.

Therefore we search for the appropriate resonance structure following the methods of papers 63, 94 and the new paper on resonance designs by Dr. Horst Eckardt. The simplest structure is found from:

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho}{\epsilon_0}, \quad - (18)$$

$$\underline{E} = -\frac{\partial \underline{A}}{\partial t} - \omega \underline{A}, \quad - (19)$$

$$\underline{A}^{(1)} = \underline{A}^{(2)*} = \frac{A^{(0)}}{\sqrt{2}} \left( \underline{i} - i\underline{j} \right) e^{i\Omega t} \quad - (20)$$

$$\underline{E}^{(2)} = \underline{E}^{(1)*} = -\left( \frac{\partial}{\partial t} + i\Omega \right) \underline{A}^{(2)} \quad - (21)$$

$$\underline{E} = \frac{2}{\sqrt{2}} A^{(0)} \Omega \left( \underline{i} \sin \Omega t - \underline{j} \cos \Omega t \right) \quad - (22)$$

A second possibility is to use eq. (8) with:

$$\underline{\nabla} \times \underline{B} = \underline{0} \quad - (23)$$

because

$$\underline{B} = B^{(0)} \underline{k} \quad - (24)$$

5)

So:

$$\frac{\partial \underline{E}}{\partial t} = -c\mu_0 \underline{J} = -\epsilon_0 \underline{J} \quad -(25)$$

$$\underline{E} = \frac{\partial}{\sqrt{2}} A^{(0)} \Omega \left( \underline{i} \sin \Omega t - \underline{j} \cos \Omega t \right), \quad -(26)$$

$$\underline{E}^{(2)} = -\frac{\partial \underline{A}^{(2)}}{\partial t} - i\Omega \underline{A}^{(2)}, \quad -(27)$$

$$\frac{\partial \underline{E}^{(2)}}{\partial t} = -\frac{\partial^2 \underline{A}^{(2)}}{\partial t^2} - i\Omega \frac{\partial \underline{A}^{(2)}}{\partial t} = -\epsilon_0 \underline{J}$$

i.e

$$\boxed{\frac{\partial^2 \underline{A}^{(2)}}{\partial t^2} + i \frac{\partial}{\partial t}\left( \Omega \underline{A}^{(2)} \right) = \epsilon_0 \underline{J}} \quad -(28)$$

This is a resonance equation if we assume that

$$\boxed{\frac{d\Omega}{dt} \neq 0} \quad -(29)$$

i.e <u>that the angular frequency of rotation</u> is <u>variable</u>. Then:

$$\boxed{\frac{\partial^2 \underline{A}^{(2)}}{\partial t^2} + i\Omega \frac{\partial \underline{A}^{(2)}}{\partial t} + i\left(\frac{\partial \Omega}{\partial t}\right) \underline{A}^{(2)} = \epsilon_0 \underline{J}} \quad -(30)$$

6)

This is an undamped Euler-Bernoulli resonance equation if $\underline{J}$ is periodic:

$$\underline{J} = \underline{J}^{(0)} \cos(\Omega_0 t)\, \underline{e}^{(2)} \quad (31)$$

Finally, if the design is such that

$$\frac{d\Omega}{dt} \gg \Omega \quad (32)$$

then:

$$\boxed{\frac{d^2 \underline{A}^{(2)}}{dt^2} + i\left(\frac{d\Omega}{dt}\right)\underline{A}^{(2)} = \underline{J}^{(0)}\cos(\Omega_0 t)\, \underline{e}^{(2)}} \quad (33)$$

and this means that at resonance:

$$\boxed{\underline{A}^{(2)} \to \infty} \quad (34)$$

The explosion of the Faraday generator may be explained in this way, i.e. by a rapidly varying $\Omega$ and a periodic current density. The latter comes from the emf set up between centre and rim of the disk, or may be an additional input.