1)

108(10): Approximately Circular Orbits

To develop this method use:

$$\frac{\partial V}{\partial r} = \frac{m}{r^2}\left(c^2 r_s - \frac{L^2}{r} + 3L^2\frac{r_s}{r^2}\right) \quad -(1)$$

$$= \epsilon$$

where ϵ is a small quantity that is approximately constant. For precisely circular orbits it vanishes.

Eq. (1) is a quartic in r:

$$\frac{\epsilon}{m} r^4 - c^2 r_s r^2 + L^2 r - 3L^2 r_s = 0 \quad -(2)$$

This may be solved using Maxima and r may be expressed as a function of ϵ, L and r_s. In the limit:

$$\epsilon = 0 \quad -(3)$$

eq. (2) reduces to a quadratic in r, and it has already been shown that in the limit (3) of circular orbits, $r \rightarrow 0$ if $r_s \rightarrow 0$. This occurs in a well behaved monotonic manner. Therefore we already know from inspection of eq (2) that this will also occur in ~~the~~ the more general case of a small ϵ assumed to be a constant.

2) The potential energy being considered is:

$$V = mc^2\left(\frac{1}{2} - \frac{r_s}{r}\right) + \frac{mL^2}{2r^2} - mL^2\frac{r_s}{r^3} \quad -(4)$$

where

$$L = r^2\frac{d\phi}{d\tau} \quad -(5)$$

is a constant of motion. In the standard model:

$$r_s = \frac{2MG}{c^2} \quad -(6)$$

but in ECE theory:

$$r_s = -\frac{T}{R}(r, \theta, \phi) \quad -(7)$$

The problem can be considered as one where r_s is a function of r only, so V is a function of a function:

$$V = mc^2\left(\frac{1}{2} - \frac{r_s(r)}{r}\right) + \frac{mL^2}{2r^2} - mL^2\frac{r_s(r)}{r^3} \quad -(8)$$

i.e.

$$V = V(r, r_s(r)) \quad -(9)$$

It is known that if r_s is constant as in eq. (6), the orbit is a precessing ellipse, i.e. an ellipse with precessing perihelion. The mass M is that of the attracting object.

3)

For example, the perihelia of Mercury precess by a few arc-seconds per century and the attracting mass is that of the sun. The Pioneer / Cassini anomalies show that eq. (6) is not quite correct. There is an ~~increasing~~ excess but very small force of attraction towards the sun. An orbit in a binary pulsar is one where the precessing ellipse slowly decreases in radius. In a binary ellipse the potential energy V slowly decreases per revolution. There is therefore an extra functional dependence of V on r which is not present in eq. (4). This functional dependence is given in eq. (9), where r_s is assumed to be a function of r. If it is assumed that M is a constant, then G is slowly increasing with r.

Therefore as r_s increases, r decreases.

If we assume that r_s is proportional to r:

$$r_s = ar \quad — (10)$$

where a is a constant, then:

$$V = mc^2\left(\frac{1}{2} - a\right) + \frac{mL^2}{2r^2} - \frac{mL^2 a}{r^2}$$

$$= mc^2\left(\frac{1}{2} - a\right) + \frac{mL^2}{r^2}\left(\frac{1}{2} - a\right)$$

$$= -m\left(a - \frac{1}{2}\right)\left(c^2 + \frac{L^2}{r^2}\right) \quad — (11)$$

4)

A more realistic assumption is:

$$r_s = \frac{2mG}{c^2} + \frac{a}{r} \quad -(12)$$

where a is very small. This means that:

$$\frac{\partial r_s}{\partial r} = -\frac{a}{r^2} \quad -(13)$$

Eq (12) means that as r_s increases, r decreases. In qualitative terms this can be understood as being due to an increased gravitational attraction per revolution. The orbit of the binary pulsar slightly decreases in average diameter per revolution.

In the standard model:

$$a = 0 \quad -(14)$$

The parameter a also accounts for the slight increased attraction of the Pioneer / Cassini anomaly. From eq. (13):

$$\frac{\partial r}{\partial r_s} = -ar^2 \quad -(15)$$