110(3): Curvature of the ECE Lemma for a Spherically Symmetric Spacetime.

As in notes 110(2) calculate the curvature for:

$$q^0_0 = e^\alpha, \quad \Gamma^0_{00} = \partial_0 \alpha \quad -(1)$$

The tetrad postulate is:

$$\partial_\mu q^a_\sigma + \omega^a_{\mu b} q^b_\sigma - \Gamma^\lambda_{\mu\sigma} q^a_\lambda = 0 \quad -(2)$$

which simplifies to:

$$\partial_\mu q^0_0 + \omega^0_{\mu 0} q^0_0 - \Gamma^0_{\mu 0} q^0_0 = 0 \quad -(3)$$

with:

$$\mu = 0 \quad -(4)$$

and

$$\square = \partial^0 \partial_0 = \frac{1}{c^2} \frac{\partial^2}{\partial t^2} \quad -(5)$$

So:

$$\partial_0 e^\alpha = (\partial_0 \alpha - \omega^0_{00}) e^\alpha \quad -(6)$$

and

$$\square e^\alpha = \partial^0 ((\partial_0 \alpha - \omega^0_{00}) e^\alpha) \quad -(7)$$

The RHS of eq. (7) is defined as:

$$R e^\alpha := \partial^0 ((\partial_0 \alpha - \omega^0_{00}) e^\alpha) \quad -(8)$$

$$= (\partial^0 (\partial_0 \alpha - \omega^0_{00})) e^\alpha + (\partial_0 \alpha - \omega^0_{00}) \partial^0 e^\alpha$$

Now use:

$$\partial^0 = \partial_0 = \frac{1}{c} \frac{\partial}{\partial t} \quad -(9)$$

in eq (6) to find:

$$\partial^0 e^\alpha = (\partial_0 \alpha - \omega^0_{00}) e^\alpha \quad -(10)$$

i.e.

$$\boxed{R = \partial^0 (\partial_0 \alpha - \omega^0_{00}) + (\partial_0 \alpha - \omega^0_{00})^2} \quad -(11)$$

$$\square e^\alpha = R e^\alpha \quad -(12)$$

2)

Discussion

If R is defined as in eq. (11) then eq (12) follows. This means that for a given periodic function e^{α} there are various "energy levels" R defined by

$$R = -kT \quad -(13)$$

and these energy levels are:

$$T = -\frac{1}{k}\left(\partial^{\circ}\left(\partial_{\circ}\alpha - \omega^{\circ}_{\circ\circ}\right) + \left(\partial_{\circ}\alpha - \omega^{\circ}_{\circ\circ}\right)^{2}\right) \quad -(14)$$

These are energy levels of the generally covariant unified field. If we isolate the gravitational sector these are quantum gravitational energy levels.