1) Note 111(4): The Schwarzschild Parameter μ in ECE Theory.

Starting from the Hodge dual of the Bianchi identity:

$$D \wedge \widetilde{T} := \widetilde{R} \wedge q \quad -(1)$$

it is found that:

$$D_\mu T^{a\mu\nu} = R^{a\ \ \mu\nu}_{\ \mu} , \quad -(2)$$

a particular solution of which is:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \mu} \quad -(3)$$

Eq. (3) may be written as:

$$\partial_\mu T^{\kappa\mu\nu} + 4\omega^{\kappa}_{\ \mu\lambda} T^{\lambda\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \mu} , \quad -(4)$$

i.e. as:

$$\partial_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \mu} - 4\omega^{\kappa}_{\ \mu\lambda} T^{\lambda\mu\nu} . \quad -(5)$$

As in paper 105 this equation splits into two vector equations. One of these may be written as:

$$\underline{\nabla} \cdot \underset{\sim}{T} = R - \omega T \quad -(6)$$

or

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 (R - \omega T)} \quad -(7)$$

Here:

$$\underset{\sim}{T} = T^{010}\,\underline{i} + T^{020}\,\underline{j} + T^{030}\,\underline{k} \quad -(8)$$

$$R = R^{0\ \ 10}_{\ 1} + R^{0\ \ 20}_{\ 2} + R^{0\ \ 30}_{\ 3} \quad -(9)$$

$$\omega T = 4\left(\omega^{0}_{\ 1\lambda} T^{\lambda 10} + \omega^{0}_{\ 2\lambda} T^{\lambda 20} + \omega^{0}_{\ 3\lambda} T^{\lambda 30}\right) \quad -(10)$$

and

$$\underline{g} = c^2 \underset{\sim}{T} \quad -(11)$$

2)

is the acceleration due to gravity.

It was shown in paper 105 that the ~~ze~~ time metric element is:

$$g_{00} = -c^2 (R + \omega T) / ((\partial\phi / \partial\tau)^2 \quad -(12)$$

in spherical polar coordinates. In 1916, Karl Schwarzschild showed that the vacuum solution of the Einstein field equation is:

$$g_{00} = -\left(1 + \frac{\mu}{r}\right). \quad -(13)$$

Eq. (12) is derived directly from Cartan geometry and makes no assumption that the Ricci tensor is zero. Eq. (13) is derived without considering torsion and assumes that the Ricci tensor is zero. This was Schwarzschild's original derivation of eq. (13). However, the mathematical form of eq. (13) can be derived directly from the most general line element of a spherically symmetric spacetime:

$$ds^2 = -e^{2\alpha} c^2 dt^2 + e^{2\beta} dr^2 + r^2 d\Omega^2 \quad -(14)$$

but still assuming:

$$R_{\mu\nu} = 0, \quad -(15)$$

and also assuming that the connection is symmetric.

In view of the Bianchi identity (1) this is

3) an incorrect assumption, because it leads in general to:

$$T^{\kappa\mu\nu} = 0, \quad R^{\kappa}{}_{\mu}{}^{\mu\nu} \neq 0. \quad -(16)$$

<u>Empirical Data</u>

These data are the true source of eq. (13). Satellite data show that eq. (13) is satisfactory. <u>The reason for this</u> is <u>not</u> the <u>Einstein equation</u>. The reason is that the Bianchi identity leads to eq. (12). A particular solution of eq. (12) is eq. (13) with:

$$c^2 R = \left(\frac{d\phi}{d\tau}\right)^2, \quad c^2 \omega T = \frac{\mu}{r}\left(\frac{d\phi}{d\tau}\right)^2 \quad -(17)$$

i.e.

$$\boxed{\frac{\mu}{r} = \omega \frac{T}{R}} \quad -(18)$$

Therefore:

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 R\left(1 - \frac{\mu}{r}\right)} \quad -(19)$$

In the $r \rightarrow \infty$ limit eq. (19) becomes:

$$\underline{\nabla} \cdot \underline{g} \doteq c^2 R \quad -(20)$$

and from eq. (18):

$$\omega \rightarrow 0. \quad -(21)$$

4) Using eq. (11) in the $r \rightarrow \infty$ limit:

$$\underline{\nabla} \cdot \underset{\sim}{T} = R \quad -(22)$$

and in the radial direction:

$$\boxed{R = \frac{\partial T}{\partial r}} \quad -(23)$$

As in ~~eq.~~ paper (105):

$$\underline{g} = -(\underline{\nabla} + \underline{\omega}) \, \underline{\Phi} \quad -(24)$$

and assuming for simplicity:

$$\underline{\omega} = \omega \, \underline{e}_r \quad -(25)$$

then:

$$\nabla^2 \Phi = -c^2 R \quad -(26)$$

where

$$R = k \rho_m. \quad -(27)$$

So the Newtonian potential Φ is defined by the $r \rightarrow \infty$ limit. Here k is the Einstein constant and ρ_m the mass density. Thus in general:

$$\boxed{\underline{\nabla} \cdot \underline{g} = c^2 k \rho_m \left(1 - \frac{\mu}{r} \right)} \quad -(28)$$

which is the gravitational equivalent of the Coulomb law:

$$\underline{\nabla} \cdot \underline{E} = \rho_E / \epsilon_0 \quad -(29)$$

5) Eq. (28) is true in general, and so the most general form of the Coulomb law is also:

$$\boxed{\underline{\nabla} \cdot \underline{E} = \rho_E \left(1 - \frac{\mu}{r}\right) / \epsilon_0} \quad - (30)$$

In order to reproduce precise satellite data on orbits, it is found <u>empirically</u> that:

$$\mu = -\frac{2mG}{c^2}. \quad - (31)$$

In the laboratory:

$$2mG << c^2 \quad - (32)$$

For one kilogram, $(m = 1)$:

$$G = 6.6726 \times 10^{-11} \, N m^2 kg^{-2} \quad - (33)$$

$$c = 3 \times 10^8 \, m s^{-1} \quad - (34)$$

so:

$$\boxed{\frac{2mG}{c^2} \sim 10^{-27}}. \quad - (35)$$

The relativistic correction of the Coulomb law due to mass is:

$$\underline{\nabla} \cdot \underline{E} = \frac{\rho_E}{\epsilon_0} \left(1 + \frac{2mG}{rc^2}\right) \quad - (36)$$

<u>so is completely negligible in the laboratory.</u>

However for very large M the correction may become observable. Alternatively if r is very small i.e. the laboratory it may be observable.

If nothing is assumed other than spherically symmetric geometry, then:

$$\boxed{e^{2\alpha} = c^2(R + \omega T) / (\partial\phi / \partial\tau)^2} \quad -(37)$$

If we assume:

$$e^{2\alpha} \sim 1 + 2\alpha + \ldots \quad -(38)$$

then eq. (38) is a particular solution of eq. (37):

$$c^2 R = \left(\frac{\partial\phi}{\partial\tau}\right)^2, \quad c^2 \omega T = 2\alpha\left(\frac{\partial\phi}{\partial\tau}\right)^2 \quad -(39)$$

so:

$$\omega T = 2\alpha R \quad -(40)$$

and

$$\boxed{\alpha = \frac{\omega T}{2R}} \quad -(41)$$

Empirically we find that:

$$\alpha = -\frac{MG}{c^2 r}. \quad -(42)$$