1) Note 112(1): Topology of Orbits

Given the spherically symmetric line element:

$$ds^2 = -mc^2 dt^2 + n dr^2 + r^2 d\Omega^2 \quad -(1)$$

it is found that almost all orbital theory is described by:

$$\int dr = mr = \frac{r}{n} \quad -(2)$$

$$= r + \mu \quad -(3)$$

Therefore:

$$\boxed{r = \frac{\mu}{m-1} = \frac{\mu}{\frac{1}{n} - 1}} \quad -(4)$$

Experimentally:

$$\mu = -r_s = -\frac{2MG}{c^2} \quad -(5)$$

Therefore:

$$\frac{r}{r_s} = \frac{1}{1-m} = \frac{1}{1-\frac{1}{n}} \quad -(6)$$

For binary pulsar orbits:

$$\mu \rightarrow \mu - \frac{a}{r} \quad -(7)$$

where a is a very small perturbation. Therefore all known orbits are described by the spherical symmetry of spacetime.

Using eq. (7) in eq. (4):

2)

$$(m-1)r = \mu - \frac{a}{r},$$

i.e. $$(m-1)r^2 - \mu r + a = 0 \quad -(8)$$

and $$\boxed{r = \frac{1}{2(m-1)}\left(\mu \pm \left(\mu^2 - 4a(m-1)\right)^{1/2}\right)} \quad -(9)$$

Therefore the most general structure of spacetime responsible for orbits appears to be eq. (9), i.e. which:

$$m(r) = 1 + \frac{\mu}{r} - \frac{a}{r^2} \quad -(10)$$

$$\boxed{m(r) = 1 - \frac{2MG}{rc^2}\left(1 + \frac{a}{r}\right)}$$

and $$n(r) = \frac{1}{m(r)}. \quad -(11)$$

The mathematical origin of $2MG/c^2$ is therefore a constant of integration μ in eq (2), in other words $2MG/c^2$ is a geometrical consequence of the spherical symmetry of spacetime. In well defined limits it follows that:

$$g = -\frac{MG}{r^2} \quad -(12)$$

and that:

$$F = mg = -\frac{mMG}{r^2} \quad (13)$$

which is the weak equivalence principle. In orbital theory the mass m orbits the mass M. The spherically symmetric line element is:

$$ds^2 = -\left(1 + \frac{\mu}{r}\right)c^2 dt^2 + \left(1 + \frac{\mu}{r}\right)^{-1} dr^2 + r^2 d\Omega^2 \quad (14)$$

This line element produces all that is known experimentally about orbits, and reduces to the Newtonian orbits (13). Rotating the line element (14) produces rotational frame dragging effects.

The simple equation:

$$\boxed{\int dr = nr = r + \mu} \quad (15)$$

must therefore have a profound significance in physics, and similarly the equation:

$$\boxed{\int dr = \frac{r}{n} = r + \mu} \quad (16)$$

In order to develop the topology of eqs. (15) and (16) it is possible to develop the simple integral into a contour integral of the kind used in ECE theory to develop the Aharonov Bohm effects. In that theory the field is defined in terms of the potential and spin connection:

$$F = d \wedge A + \omega \wedge A \quad -(17)$$

in shorthand notation. The potential may be non-zero when the field is zero if:

$$d \wedge A = - \omega \wedge A. \quad -(18)$$

The class of AB effects occurs when the experimental arrangement is such that eq (18) is true.

Integrating eq. (17) over a surface..

$$\int_S F = \int_S d \wedge A + \int_S \omega \wedge A. \quad -(19)$$

Using the Stokes Theorem:

$$\int_S d \wedge A = \oint A \quad -(20)$$

so the AB effects are described by:

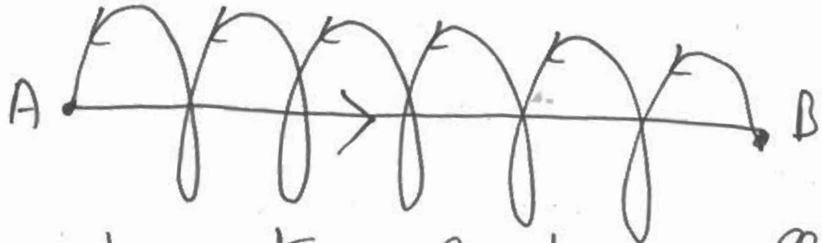
$$\boxed{\oint A = - \int_S \omega \wedge A} \quad -(21)$$

in areas where:

$$F = 0. \quad -(22)$$

5)

If we consider integration around a helix:



Then the contour integral becomes the integral from A to B, because the contour integral along the helix is zero. In this case:

$$\oint A = \int A = \int_S d \wedge A \quad -(23)$$

In three dimensions and in vector notation:

$$\int \underline{A} \cdot d\underline{r} = \int_S \underline{\nabla} \times \underline{A} \cdot d\underline{Ar} \quad -(24)$$

If we define: $\underline{B} = \underline{\nabla} \times \underline{A} \quad -(25)$

then:

$$\int \underline{A} \cdot d\underline{r} = \int_S \underline{B} \cdot d\underline{Ar} := \Phi \quad -(26)$$

In the standard electrodynamics, the spin connection is missing and Φ is the magnetic flux, $\underline{B}$ being the magnetic flux density and $\underline{A}$ the vector potential. Therefore:

$$\boxed{\int dr = \kappa \int_S dAr} \quad -(27)$$

6)

where κ is a wavenumber. If the surface of a sphere is considered:

$$\int_S dA_r = 4\pi r^2 \quad -(28)$$

Therefore:

$$\int dr = 4\pi r^2 \kappa \quad -(29)$$

Using this result in eq. (2):

$$\int dr = nr = r + \mu = 4\pi r^2 \kappa \quad -(30)$$

so:

$$n = 1 + \frac{\mu}{r} = 4\pi r \kappa \quad -(31)$$

and

$$\boxed{\kappa = \frac{1}{4\pi r}\left(1 + \frac{\mu}{r}\right)} \quad -(32)$$

This is a <u>characteristic</u> <u>wavenumber</u> of all <u>orbits</u>. If we write:

$$\kappa_0 = \frac{1}{4\pi r} \quad -(33)$$

then

$$\kappa = \kappa_0 + \frac{\mu}{4\pi r^2}$$

$$\kappa = \kappa_0 - \frac{2MG}{4\pi c^2} \cdot \frac{1}{r^2} \quad -(34)$$

Therefore:

$$2\pi(\kappa - \kappa_0)c^2 = -\frac{mG}{r^2} \quad -(35)$$

The Newton inverse square law is therefore:

$$F = 2\pi c^2(\kappa - \kappa_0)\, m \quad -(36)$$

and the acceleration due to gravity is:

$$g = 2\pi c^2(\kappa - \kappa_0). \quad -(37)$$

In ECE theory:

$$g = c^2 T \quad -(38)$$

where T is a well-defined scalar torsion, so:

$$\boxed{T = 2\pi(\kappa - \kappa_0)} \quad -(39)$$

Therefore in eq. (30):

$$\int dr = mr = r + \mu = A r T_0 \quad -(41)$$

and:

$$\boxed{m = 1 + \frac{\mu}{r} = \frac{Ar}{r} \cdot T_0} \quad -(42)$$

Orbits are due to a spacetime torsion T_0, and there are gravitational AB effects (which have been observed already).