1) 112(5): Proofs of the Field Equations of ECE

The field equations originate in the commutator of covariant derivatives:

$$[D_\mu, D_\nu] = -[D_\nu, D_\mu]. \quad -(1)$$

It has been proven in all detail in the ECE papers that:

$$[D_\mu, D_\nu] V^\rho = D_\mu (D_\nu V^\rho) - D_\nu (D_\mu V^\rho)$$

$$= R^\rho{}_{\sigma\mu\nu} V^\sigma - T^\lambda{}_{\mu\nu} D_\lambda V^\rho \quad -(2)$$

where $R^\rho{}_{\sigma\mu\nu}$ is the curvature tensor and $T^\lambda{}_{\mu\nu}$ is the torsion tensor. This result is true irrespective of any assumption on the metric and connection. The torsion tensor is:

$$T^\lambda{}_{\mu\nu} = \Gamma^\lambda{}_{\mu\nu} - \Gamma^\lambda{}_{\nu\mu} \quad -(3)$$

and the curvature tensor is:

$$R^\rho{}_{\sigma\mu\nu} = \partial_\mu \Gamma^\rho{}_{\nu\sigma} - \partial_\nu \Gamma^\rho{}_{\mu\sigma} + \Gamma^\rho{}_{\mu\lambda} \Gamma^\lambda{}_{\nu\sigma} - \Gamma^\rho{}_{\nu\lambda} \Gamma^\lambda{}_{\mu\sigma}. \quad -(4)$$

Given the tetrad postulate:

$$D_\mu q^a{}_\nu = 0 \quad -(5)$$

it follows from eqns. (3) and (4) that:

$$D \wedge T^a := R^a{}_b \wedge q^b \quad -(6)$$

which is the Bianchi identity.

Eqs. (3) - (5) are necessary and sufficient to prove eq. (6), (see the ECE papers).

2) Therefore eq. (2) is the fundamental origin of the Bianchi identity (6). The latter is true for all metrics and all connections. In ECE theory eq. (6) is the homogeneous field equation of dynamics.

Defining the Hodge dual operator:

$$[D^\alpha, D^\beta]_{HD} := \frac{1}{2}\|g\|^{1/2}\epsilon^{\mu\nu\alpha\beta}[D_\mu, D_\nu] \quad -(7)$$

it follows that:

$$[D^\alpha, D^\beta]_{HD} = \tilde{R}^\rho{}_\sigma{}^{\alpha\beta}V^\sigma - \tilde{T}^{\lambda\alpha\beta}D_\lambda V^\rho \quad -(8)$$

where:

$$\tilde{R}^\rho{}_\sigma{}^{\alpha\beta} = \frac{1}{2}\|g\|^{1/2}\epsilon^{\mu\nu\alpha\beta}R^\rho{}_{\sigma\mu\nu} \quad -(9)$$

and

$$\tilde{T}^{\lambda\alpha\beta} = \frac{1}{2}\|g\|^{1/2}\epsilon^{\mu\nu\alpha\beta}T^\lambda{}_{\mu\nu}. \quad -(10)$$

Lowering indices:

$$[D_\alpha, D_\beta]_{HD} = \tilde{R}^\rho{}_{\sigma\alpha\beta}V^\sigma - \tilde{T}^\lambda{}_{\alpha\beta}D_\lambda V^\rho \quad -(11)$$

where:

$$[D_\alpha, D_\beta]_{HD} = g_{\alpha\mu}g_{\beta\nu}[D^\mu, D^\nu]_{HD}, \quad -(12)$$

$$\tilde{R}^\rho{}_{\sigma\alpha\beta} = g_{\alpha\mu}g_{\beta\nu}\tilde{R}^\rho{}_\sigma{}^{\mu\nu}, \quad -(13)$$

$$\tilde{T}^\lambda{}_{\alpha\beta} = g_{\alpha\mu}g_{\beta\nu}\tilde{T}^{\lambda\alpha\beta}. \quad -(14)$$

Now write two examples of eq. (2):

3)

$$[D_0, D_1] V^\rho = R^\rho{}_{\sigma 01} V^\sigma - T^\lambda{}_{01} D_\lambda V^\rho \quad -(15)$$

and

$$[D_2, D_3] V^\rho = R^\rho{}_{\sigma 23} V^\sigma - T^\lambda{}_{23} D_\lambda V^\rho, \quad -(16)$$

and it is seen that eq. (16) is:

$$[D_0, D_1]_{HD} V^\rho = \tilde{R}^\rho{}_{\sigma 01} V^\sigma - \tilde{T}^\lambda{}_{01} D_\lambda V^\rho. \quad -(17)$$

The Hodge dual equation (17) is an example of the original equation, with different indices.

It follows that:

$$D \wedge \tilde{T}^a := \tilde{R}^a{}_b \wedge q^b \quad -(18)$$

is an example of

$$D \wedge T^a := R^a{}_b \wedge q^b. \quad -(19)$$

Some Details:

$$[D^2, D^3]_{HD} = \|g\|^{1/2} [D_0, D_1] \quad -(20)$$

$$\tilde{R}^\rho{}_\sigma{}^{23} = \|g\|^{1/2} R^\rho{}_{\sigma 01} \quad -(21)$$

$$\tilde{T}^{\lambda 23} = \|g\|^{1/2} T^\lambda{}_{01} \quad -(22)$$

so:

$$[D^2, D^3]_{HD} V^\rho = \tilde{R}^\rho{}_\sigma{}^{23} V^\sigma - \tilde{T}^{\lambda 23} D_\lambda V^\rho \quad -(23)$$

is the same as:

$$[D_0, D_1] V^\rho = R^\rho{}_{\sigma 01} V^\sigma - T^\lambda{}_{01} D_\lambda V^\rho \quad -(24)$$

4) Lowering indices on eq. (23) gives:

$$[D_2, D_3]_{HO} \tilde{V}^{\rho} = \tilde{R}^{\rho}{}_{\sigma 23} V^{\sigma} - \tilde{T}^{\lambda}{}_{23} D_{\lambda} \tilde{V}^{\rho} \quad -(25)$$

which is the same equation as (24).

<u>Inhomogeneous</u> <u>Field Equation</u>

Eq. (18) in tensor notation is:

$$D_{\mu} \tilde{T}^{a}{}_{\nu\rho} + D_{\rho} \tilde{T}^{a}{}_{\mu\nu} + D_{\nu} \tilde{T}^{a}{}_{\rho\mu} := \tilde{R}^{a}{}_{\mu\nu\rho} + \tilde{R}^{a}{}_{\rho\mu\nu} + \tilde{R}^{a}{}_{\nu\rho\mu} \quad -(26)$$

which may be rewritten as:

$$D_{\mu} T^{a\mu\nu} = R_{\mu}{}^{a\mu\nu} \quad -(27)$$

An example of eq. (27) is:

INHOMOGENEOUS $$\boxed{D_{\mu} T^{\kappa\mu\nu} = R_{\mu}{}^{\kappa\mu\nu}} \quad -(28)$$

where:

$$T^{a\mu\nu} = q^{a}{}_{\kappa} T^{\kappa\mu\nu} \quad -(29)$$

$$R_{\mu}{}^{\kappa\mu\nu} = q^{a}{}_{\kappa} R_{\mu}{}^{\kappa\mu\nu} \quad -(30)$$

Similarly, eq. (19) is:

HOMOGENEOUS $$\boxed{D_{\mu} \tilde{T}^{\kappa\mu\nu} = \tilde{R}_{\mu}{}^{\kappa\mu\nu}} \quad -(31)$$

It can be seen that eqns (28) and (31) have

5) similar structures to the tensorial form of the Maxwell Heaviside equations:

$$\partial_\mu F^{\mu\nu} = J^\nu / \epsilon_0 \quad -(32)$$

$$\partial_\mu \tilde{F}^{\mu\nu} = 0 \quad -(33)$$

In differential form notation eqns. (32) and (33) are:

$$d \wedge \tilde{F} = J / \epsilon_0 \quad -(34)$$

$$d \wedge F = 0 \quad -(35)$$

The Fundamental Flaw in the Einstein Equation.

This occurs in eq. (28). The Einstein equation is incorrectly given:

$$T^{\kappa\mu\nu} = 0, \; R^{\kappa\mu\nu} \neq 0 \quad -(36)$$

because it uses the Christoffel connection:

$$\Gamma^\kappa_{\mu\nu} = \Gamma^\kappa_{\nu\mu} \quad -(37)$$

for which:

$$T^\kappa_{\mu\nu} = \Gamma^\kappa_{\mu\nu} - \Gamma^\kappa_{\nu\mu} = 0. \quad -(38)$$

EQNS (28) and (31) ARE THE DUALITY INVARIANT EQUATIONS OF DYNAMICS AND ELECTRODYNAMICS.