1)

Notes 113(4) : Classes of orbits

1 Class One Orbits

These are described by the Frobenius Theorem and the theorem of orbits:

$$nr = \frac{r}{m} = \int dr = r + \mu \quad -(1)$$

i.e.

$$n = 1 + \frac{\mu}{r}, \quad m = \left(1 + \frac{\mu}{r}\right)^{-1} \quad -(2)$$

giving:

$$ds^2 = -\left(1+\frac{\mu}{r}\right)c^2dt^2 + \left(1+\frac{\mu}{r}\right)^{-1}dr^2 + r^2 d\Omega^2 \quad -(3)$$

in spherical polar coordinates.

If:

$$\mu = -\frac{2MG}{c^2} \quad -(4)$$

and

$$\Gamma^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\nu\mu} \quad -(5)$$

then the complete Bianchi identity is obeyed fortuitously as follows:

$$D \wedge T := R \wedge q = 0 \quad -(6)$$

$$D \wedge \tilde{T} := \tilde{R} \wedge q = 0. \quad -(7)$$

The great majority of orbits are described in this way, and are class one orbits, i.e. they are described by eqs. (1) to (4), without the need for a field equation at all.

Class Two Orbits (Binary Pulsars)

These orbits are describable by:

$$\mu = -\left(\frac{2MG}{c^2} + \frac{a}{r} \right) \quad — (8)$$

where:

$$a << r. \quad — (9)$$

If the connection is assumed to be the symmetric
a Christoffel connection, eq. (5), then:

$$D \wedge T := R \wedge q = 0 \quad — (10)$$

but

$$D \wedge \tilde{T} = 0, \quad \tilde{R} \wedge q \neq 0. \quad — (11)$$

Therefore these orbits need:

$$\boxed{\Gamma^{\kappa}_{\mu\nu} \neq \Gamma^{\kappa}_{\nu\mu}}. \quad — (12)$$

The Self-Consistent Description of Orbits

In note 113(3) it was shown that the
orbital torsion and spin connection can be described
self-consistently and simply by Cartan geometry.
This means that:

$$\boxed{\Gamma^{\kappa}_{\mu\nu} \neq \Gamma^{\kappa}_{\nu\mu}} \quad — (13)$$

and

$$T^{\kappa}_{\mu\nu} \neq 0. \quad — (14)$$

3)

The self-consistent description of all orbits requires the orbital and Frobenius theorems and a non-zero torsion. This eliminates the use of the Einstein field equation and the Christoffel connection.

If we try to use the Christoffel connection, the class one orbits give the fortuitous result (6) and (7), but the Christoffel connection eliminates torsion incorrectly because:

$$T^{\kappa}_{\mu\nu} = \Gamma^{\kappa}_{\mu\nu} - \Gamma^{\kappa}_{\nu\mu} = 0 \quad - (15)$$

if eq (5) is used. There is no a priori reason for assuming eq. (5). Eq. (4) is based on experimental observation. The orbits of binary pulsars are not described by eq. (4), but are described by eq. (8). If we try to use assumption (5) with eq. (8), the Bianchi identity and dual identity are not both obeyed. The correct field equations are:

$$D \wedge T := R \wedge q \quad - (16)$$

$$D \wedge \tilde{T} := \tilde{R} \wedge q \quad - (17)$$

solved simultaneously.