1.) 115(14): General Coordinate Transformation of the ECE Field Equation

The ECE field equations are based on the geometry:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu} \quad -(1)$$

and

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu}. \quad -(2)$$

This geometry transforms covariantly under the general coordinate transformation. So in a frame K' moving arbitrarily with respect to frame K:

$$\left(D_\mu \tilde{T}^{\kappa\mu\nu}\right)' = \left(\tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu}\right)' \quad -(3)$$

and

$$\left(D_\mu T^{\kappa\mu\nu}\right)' = \left(R^{\kappa\ \ \mu\nu}_{\ \ \mu}\right)'. \quad -(4)$$

This geometry ensures automatic general covariance of the ECE field equations of both dynamics and electrodynamics. The dynamical field equation are based directly on eqs. (1) and (2) and the general covariance of the electro-dynamical equations is given by:

$$\left(D_\mu \tilde{F}^{\kappa\mu\nu} = A^{(0)}\tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu}\right) \rightarrow \left(D_\mu \tilde{F}^{\kappa\mu\nu} = A^{(0)}\tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu}\right)' \quad -(5)$$

$$\left(D_\mu F^{\kappa\mu\nu} = A^{(0)}R^{\kappa\ \ \mu\nu}_{\ \ \mu}\right) \rightarrow \left(D_\mu F^{\kappa\mu\nu} = A^{(0)}R^{\kappa\ \ \mu\nu}_{\ \ \mu}\right)'. \quad -(6)$$

For engineering purposes eqs. (5) and (6) reduce to:

$$\left(\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0\right) \rightarrow \left(\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0\right)' \quad -(7)$$

$$\left(\partial_\mu F^{\kappa\mu\nu} = \frac{J^\kappa}{\epsilon_0}\right) \rightarrow \left(\partial_\mu F^{\kappa\mu\nu} = \frac{J^\kappa}{\epsilon_0}\right)' \quad -(8)$$

2)

Transformation of the Homogeneous Equation

$$\partial_\mu' \tilde{F}^{\kappa\mu\nu\prime} = \left(\frac{\partial x^\mu}{\partial x^{\mu\prime}}\right)\partial_\mu\left(\frac{\partial x^{\mu\prime}}{\partial x^\mu}\frac{\partial x^{\nu\prime}}{\partial x^\nu}\frac{\partial x^{\kappa\prime}}{\partial x^\kappa}\tilde{F}^{\kappa\mu\nu}\right) \quad -(9)$$

$$= \frac{\partial x^{\nu\prime}}{\partial x^\nu}\frac{\partial x^{\kappa\prime}}{\partial x^\kappa}\partial_\mu\tilde{F}^{\kappa\mu\nu} + \tilde{F}^{\kappa\mu\nu}\partial_\mu\left(\frac{\partial x^{\mu\prime}}{\partial x^\mu}\frac{\partial x^{\nu\prime}}{\partial x^\nu}\frac{\partial x^{\kappa\prime}}{\partial x^\kappa}\right) \quad -(10)$$

Gauss Law

Assume:

$$\kappa' = \kappa = 0 \quad -(11)$$

then:

$$\frac{\partial x^{\kappa\prime}}{\partial x^\kappa} = 1 \quad -(12)$$

and

$$\partial_\mu' \tilde{F}^{0\mu\nu\prime} = \left(\frac{\partial x^{\nu\prime}}{\partial x^\nu}\right)\partial_\mu\tilde{F}^{0\mu\nu} + \tilde{F}^{0\mu\nu}\partial_\mu\left(\frac{\partial x^{\mu\prime}}{\partial x^\mu}\frac{\partial x^{\nu\prime}}{\partial x^\nu}\right) \quad -(13)$$

Rotation About the Z Axis

$$\frac{\partial x^{\nu\prime}}{\partial x^\nu} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad -(14)$$

and

$$\begin{bmatrix} V^{0\prime} \\ V^{1\prime} \\ V^{2\prime} \\ V^{3\prime} \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \cos\theta & \sin\theta & 0 \\ 0 & -\sin\theta & \cos\theta & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}\begin{bmatrix} V^0 \\ V^1 \\ V^2 \\ V^3 \end{bmatrix} \quad -(15)$$

3) So:

$$\left.\begin{aligned} V^{0'} &= V^{0} \\ V^{1'} &= V^{0}\cos\theta + V^{1}\sin\theta \\ V^{2'} &= -V^{1}\sin\theta + V^{2}\cos\theta \\ V^{3'} &= V^{3} \end{aligned}\right\} \quad -(16)$$

If θ is not t or $\underline{r}$ dependent:

$$\partial_\mu \left(\frac{\partial x^{\mu'}}{\partial x^{\mu}} \frac{\partial x^{\nu'}}{\partial x^{\nu}} \right) = 0, \quad -(17)$$

so under these assumptions eq. (13) simplifies to:

$$\partial_\mu' \tilde{F}^{a\mu\nu'} = \left(\frac{\partial x^{\nu'}}{\partial x^{\nu}} \right) \partial_\mu \tilde{F}^{a\mu\nu} \quad -(18)$$

The Gauss Law therefore transforms as:

$$\boxed{(\underline{\nabla} \cdot \underline{B} = 0) = (\underline{\nabla} \cdot \underline{B}' = 0)} \quad -(19)$$

where:

$$\underline{B} = B^{001}\,\underline{i} + B^{002}\,\underline{j} + B^{003}\,\underline{k} \quad -(20)$$

and is invariant under a Z axis rotation.

Similar considerations apply to the transformation of the Faraday law in ECE as follows.

4) <u>Faraday Law of Induction</u>

For:
$$\kappa = \kappa' = 1 \quad -(21)$$

the transformation is:
$$\partial_\mu' \tilde{F}^{1\mu\nu\prime} = \left(\frac{\partial x^{\nu\prime}}{\partial x^\nu}\right) \partial_\mu \tilde{F}^{1\mu\nu} \quad -(22)$$

For
$$\kappa = \kappa' = 2$$

the transformation is:
$$\partial_\mu' \tilde{F}^{2\mu\nu\prime} = \left(\frac{\partial x^{\nu\prime}}{\partial x^\nu}\right) \partial_\mu \tilde{F}^{2\mu\nu} \quad -(23)$$

For
$$\kappa = \kappa' = 3$$

the transformation is:
$$\partial_\mu' \tilde{F}^{3\mu\nu\prime} = \left(\frac{\partial x^{\nu\prime}}{\partial x^\nu}\right) \partial_\mu \tilde{F}^{3\mu\nu} \quad -(24)$$

Using the table of definition in paper 100:
$$\nabla^1 = \left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t}\right)_X \quad -(25)$$

$$\nabla^2 = \left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t}\right)_Y \quad -(26)$$

$$\nabla^3 = \left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t}\right)_Z \quad -(27)$$

where the field components are <u>spin</u> components.

5) Under a Z axis rotation:

$$\left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \right) = \left(\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \right)' \quad -(28)$$

i.e. the Faraday law in ECE physics is invariant under a Z axis rotation.

Similarly:

$$\left(\underline{\nabla} \cdot \underline{E} = 0 \right) = \left(\underline{\nabla} \cdot \underline{E} = 0 \right)' \quad -(29)$$

$$\left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \right) = \left(\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} \right)' \quad -(30)$$

The four free field laws are invariant under a Z axis rotation. This means that the rotating tetrad potential is needed to explain the Sagnac and Faraday disk effects.

Finally the field/matter laws are covariant under a Z axis rotation:

$$\left(\underline{\nabla} \cdot \underline{D} = \rho \right) \rightarrow \left(\underline{\nabla} \cdot \underline{D} = \rho \right)' \quad -(31)$$

$$\left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \right) \rightarrow \left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \right)' \quad -(32)$$