1) 115(15): Invariance and Covariance of the ECE Field Equations.

The field equations are based on the geometry:

$$D_\mu T^{\kappa\mu\nu} = R^{\kappa\ \ \mu\nu}_{\ \ \mu} \quad -(1)$$

and the Hodge dual:

$$D_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{R}^{\kappa\ \ \mu\nu}_{\ \ \mu}. \quad -(2)$$

These equations can be written as:

$$\partial_\mu T^{\kappa\mu\nu} = j^{\kappa\nu} \quad -(3)$$

$$\partial_\mu \tilde{T}^{\kappa\mu\nu} = \tilde{j}^{\kappa\nu} \quad -(4)$$

The electrodynamic field equations are:

$$\partial_\mu F^{\kappa\mu\nu} = A^{(0)} j^{\kappa\nu} \quad -(5)$$

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = A^{(0)} \tilde{j}^{\kappa\nu} \quad -(6)$$

Experimentally:

$$j^{\kappa\nu} \gg \tilde{j}^{\kappa\nu} \quad -(7)$$

because a magnetic monopole has never been observed. Claims to have observed a magnetic monopole are met with counter claims. So we can only assume that $\tilde{j}^{\kappa\nu}$ is very small. Therefore:

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0 \quad -(8)$$

$$\partial_\mu F^{\kappa\mu\nu} = A^{(0)} j^{\kappa\nu}. \quad -(9)$$

Vacuum Electromagnetic Field

This is defined by:

2)

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0 \quad -(10)$$

$$\partial_\mu F^{\kappa\mu\nu} = 0 \quad -(11)$$

As shown in paper 100, these can be written as:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(12)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(13)$$

$$\underline{\nabla} \cdot \underline{E} = 0 \quad -(14)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \underline{0} \quad -(15)$$

After general coordinate transformation, eqs. (10) and (11) become:

$$\left(\partial_\mu \tilde{F}^{\kappa\mu\nu} = 0\right)' \quad -(16)$$

$$\left(\partial_\mu F^{\kappa\mu\nu} = 0\right)' \quad -(17)$$

so:

$$\partial_\mu \tilde{F}^{\kappa\mu\nu} = \partial_\mu' \tilde{F}^{\kappa\mu\nu\,\prime} = 0 \quad -(18)$$

$$\partial_\mu F^{\kappa\mu\nu} = \partial_\mu' F^{\kappa\mu\nu\,\prime} = 0 \quad -(19)$$

This means that the equations (12) to (15) remain the same to an observer in frame K' moving arbitrarily w.r.t. respect to an observer ~~moving~~ in frame K. This should however be regarded as pure mathematics because there can never be a field without a source. Eqs. (14) and (15) should always be:

3)

$$\underline{\nabla} \cdot \underline{D} = \rho \qquad -(20)$$

$$\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \qquad -(21)$$

and these equations are generally covariant, not generally invariant. So in frame K':

$$\left(\underline{\nabla} \cdot \underline{D} = \rho \right)' \qquad -(22)$$

$$\left(\underline{\nabla} \times \underline{H} - \frac{\partial \underline{D}}{\partial t} = \underline{J} \right)' \qquad -(23)$$

They retain their format, but:

$$\underline{D} \neq \underline{D}', \; \rho \neq \rho', \; \underline{H} \neq \underline{H}', \; \underline{J} \neq \underline{J}' \qquad -(24)$$

Also:

$$\underline{\nabla} \neq \underline{\nabla}', \; \frac{\partial}{\partial t} \neq \left(\frac{\partial}{\partial t} \right)' \qquad -(25)$$

So eqns. (24) and (25) introduce new physical effects.

Transformation of Fields

It is to be noted that the vacuum equations (12) to (15) are generally invariant, but the fields themselves are not invariant. They transform $\underline{E}$ and $\underline{B}$ as part of:

4)

$$T^{\kappa\mu\nu\,\prime} = \left(\frac{\partial x^{\kappa\prime}}{\partial x^{\kappa}}\right)\left(\frac{\partial x^{\mu\prime}}{\partial x^{\mu}}\right)\left(\frac{\partial x^{\nu\prime}}{\partial x^{\nu}}\right) T^{\kappa\mu\nu} \quad - (26)$$

Whereas:

$$\left(\partial_\mu T^{\kappa\mu\nu}\right)' = 0. \quad - (27)$$

The right hand side of eq. (27) is a null tensor, all of whose components are zero. A null tensor transforms as a null tensor, because, for a null tensor $A^{\kappa\nu}$:

$$A^{\kappa\nu} = 0 \rightarrow \left(\frac{\partial x^{\kappa\prime}}{\partial x^{\kappa}} \frac{\partial x^{\nu\prime}}{\partial x^{\nu}} A^{\kappa\nu} = A^{\kappa\nu\prime} = 0\right)$$

$$- (28)$$

For this reason, the vacuum equations (18) and (19) are generally invariant, but the fields $\underline{E}$ and $\underline{B}$ are generally covariant. In the Lorentz limit:

$$\underline{E}' = \underline{E} + \underline{v} \times \underline{B} \quad - (29)$$

$$\underline{B}' = \underline{B} - \frac{1}{c^2} \underline{v} \times \underline{E} \quad - (30)$$

So $\underline{E}'$ is not the same as $\underline{E}$ and $\underline{B}'$ is not the same as $\underline{B}$, even in the vacuum.