1) 116(6): Gravitomagnetic Field

The ECE equations of dynamics can be written as:

$$\underline{\nabla} \cdot \underline{h} = 0 \quad -(1)$$

$$\underline{\nabla} \times \underline{g} + \frac{1}{c}\frac{\partial \underline{h}}{\partial t} = \underline{0} \quad -(2)$$

$$\underline{\nabla} \cdot \underline{g} = 4\pi G \rho_m \quad -(3)$$

$$\underline{\nabla} \times \underline{h} - \frac{1}{c}\frac{\partial \underline{g}}{\partial t} = 4\pi G \underline{J}_m \quad -(4)$$

These are directly analogous to the equations of classical electrodynamics in ECE physics when there is no polarization or magnetization:

$$\underline{\nabla} \cdot \underline{B} = 0 \quad -(5)$$

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(6)$$

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(7)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2}\frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} \quad -(8)$$

It is well known in electrostatics and magnetostatics that:

$$\underline{E} = -\frac{e_1}{4\pi\epsilon_0}\frac{\underline{r}}{|\underline{r}|^3} \quad -(9)$$

$$\underline{B} = \frac{e_1}{4\pi\epsilon_0 c^2}\frac{\underline{v} \times \underline{r}}{|\underline{r}|^3} \quad -(10)$$

2)

Eq. (10) is the Biot - Savart law of magnetic induction $\underline{B}$ due to a current loop. It is seen that:

$$\underline{B} = -\frac{1}{c^2} \underline{v} \times \underline{E} \quad - (11)$$

which is a result of the inverse Lorentz transform when:

$$v << c \quad - (12)$$

Eq. (9) is the Coulomb law for attracting charges.

In direct analogy:

$$\underline{g} = - \frac{mG}{|\underline{r}|^3} \underline{r} \quad - (13)$$

is the Newtonian acceleration due to gravity, and

$$\boxed{\underline{h} = - \frac{mG}{|\underline{r}|^3} \underline{v} \times \underline{r} = \frac{G}{|\underline{r}|^3} \underline{L}} \quad - (14)$$

where:

$$\underline{L} = m \underline{r} \times \underline{v} \quad - (15)$$

is the angular momentum. Therefore:

$$\boxed{\underline{h} = \frac{1}{c} \underline{v} \times \underline{g}} \quad - (16)$$

Therefore in the static field limit at non-relativistic velocities $\underline{h}$ is the direct analogy

3) of the magnetic flux density or magnetic induction $\underline{B}$. Therefore $\underline{h}$ in this case is proportional to the angular momentum $\underline{L}$ of a spinning mass.

Therefore eq. (14) is the gravitational Biot-Savart law. It is the law governing the interaction of two gravitating mass currents. The Newton law governs the attraction between two mass densities. The force between two mass currents is:

$$\boxed{\underline{F}_1 = m\,\underline{h}} \qquad -(17)$$

This force is:

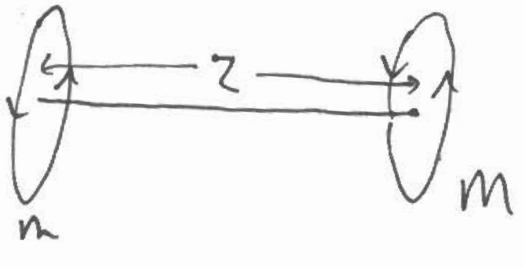
$$\underline{F}_1 = -\frac{mMG}{|\underline{r}|^3}\,\underline{v}\times\underline{r} = \frac{1}{c}\,\underline{v}\times\underline{F} \qquad -(18)$$

where:

$$\underline{F} = m\,\underline{g} \qquad -(19)$$

is the Newtonian force of attraction between m and M. If the masses are spinning in the plane perpendicular to Z, the axis of attraction of m and M, then:

4)



and:

$$\underline{F} = F_0\,\underline{k} \quad -(20)$$

$$\underline{v} = \frac{v^{(0)}}{\sqrt{2}}\left(\underline{i} - i\,\underline{j}\right)e^{i\Omega t} \quad -(21)$$

$$\underline{F}_1 = \frac{1}{c}\frac{v^{(0)}F_0}{\sqrt{2}}\begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ 1 & -i & 0 \\ 0 & 0 & 1 \end{vmatrix} e^{i\Omega t} \quad -(22)$$

$$\underline{F}_1 = \frac{1}{\sqrt{2}}\frac{v^{(0)}F_0}{c}e^{i\Omega t}\left(-i\,\underline{i} + \underline{j}\right)$$

$$\boxed{\underline{F}_1 = \frac{1}{\sqrt{2}}\frac{v^{(0)}}{c}\cdot\frac{mMG}{\cancel{4}\,r^2}e^{i\Omega t}\left(-i\,\underline{i} + \underline{j}\right)}$$

$$-(23)$$

The real part of this is:

$$\boxed{\underline{F}_1 = \frac{1}{\sqrt{2}}\frac{v^{(0)}}{c}\cdot\frac{mMG}{r^2}\left(\cos\Omega t\,\underline{j} + \sin\Omega t\,\underline{i}\right)}$$

$$-(24)$$

This is the correct description of rotational frame-dragging in the non-relativistic limit.