1)

116(8): The Generally Covariant Continuity Equation in ECE Physics.

In this note it is shown that the generally covariant continuity equation is:

$$D_\mu j^a_\nu = 0 \quad -(1)$$

where j^a_μ is a vector-valued differential one-form defined by:

$$j^a_\mu = -\epsilon_0 kT A^a_\mu. \quad -(2)$$

In the limit of Minkowski spacetime:

$$kT \rightarrow \left(\frac{mc}{\hbar}\right)^2, \quad -(3)$$

where m is the photon mass, c is the speed of light and $\hbar$ is the reduced Planck constant. The fundamental ECE postulate is:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(4)$$

therefore eq. (1) is an example of the tetrad postulate:

$$D_\mu q^a_\nu = 0. \quad -(5)$$

The structure of the inhomogeneous ECE field equation is:

$$\partial_\mu F^{\kappa\mu\nu} = j^{\kappa\ \mu\nu}_{\ \mu} / \epsilon_0. \quad -(6)$$

Therefore by index contraction the charge-current density is a rank two tensor:

$$j^{\kappa\nu} = j^{\kappa\ \mu\nu}_{\ \mu}. \quad -(7)$$

Lowering an index:

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$$j^{\kappa}{}_{\nu} = j^{\kappa}{}_{\mu}{}^{\mu}{}_{\nu}, \quad -(8)$$

and

$$j^{a}{}_{\nu} = V^{a}{}_{\kappa}\, j^{\kappa}{}_{\nu}. \quad -(9)$$

Therefore in general, eq. (6) is:

$$\boxed{\partial_\mu F^{a\mu}{}_{\nu} = j^{a}{}_{\nu} / \epsilon_0} \quad -(10)$$

Proceed now with reference to the Proca equation of the standard physics. The Proca equation is defined by:

$$\partial_\mu F^{\mu\nu} = \frac{j^\nu}{\epsilon_0} = -\left(\frac{mc}{\hbar}\right)^2 A^\nu \quad -(11)$$

where:

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu. \quad -(12)$$

Using eq. (12) in eq. (11) we obtain the Lorentz covariant Proca equation of standard physics:

$$\left(\square + \left(\frac{mc}{\hbar}\right)^2\right) A^\nu = 0 \quad -(13)$$

provided that:

$$\partial_\mu A^\mu = 0. \quad -(14)$$

The latter "Lorenz gauge" result follows from the continuity equation of standard physics:

$$\partial_\mu j^\mu = 0 \quad -(15)$$

and eq. (11).

It is well known that if the photon mass is not zero:

$$m \neq 0 \quad -(16)$$

the Proca equation is not gauge invariant, and the Lorenz equation is not an arbitrary choice. This leads to the collapse of gauge theory if the photon mass is not identically zero. In general relativity the photon mass is identically non-zero. Therefore in ECE theory the gauge principle is not used and the potential is considered to be physically meaningful.

The generally covariant Proca equation and continuity equation are obtained from a ~~of~~ development of eq. (11):

$$\boxed{j_\mu^a = -\epsilon_0 kT A_\mu^a} \quad -(17)$$

where:

$$(\Box + kT) A_\mu^a = 0 \quad -(18)$$

as a result of:

$$D_\nu A_\mu^a = 0. \quad -(19)$$

Eq. (19) is:

$$\partial_\mu q_\nu^a + \omega_{\mu b}^a A_\nu^b - \Gamma_{\mu\nu}^\lambda A_\lambda^a = 0 \quad -(20)$$

from which:

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$$\partial^\mu \left(\partial_\mu A^a_\nu + \omega^a_{\mu b} A^b_\nu - \Gamma^\lambda_{\mu\nu} A^a_\lambda \right) = 0$$

i.e.

$$\Box A^a_\nu = \partial^\mu \left(\Gamma^\lambda_{\mu\nu} A^a_\lambda - \omega^a_{\mu b} A^b_\nu \right) \quad -(21)$$

$$= -kT q^a_\nu = j^a_\nu / \epsilon_0$$

Therefore:

$$\boxed{\Box A^a_\nu = \frac{j^a_\nu}{\epsilon_0}} \quad -(22)$$

where:

$$j^a_\nu = \epsilon_0 \partial^\mu \left(\Gamma^\lambda_{\mu\nu} A^a_\lambda - \omega^a_{\mu b} A^b_\nu \right) \quad -(23)$$

Eq. (22) is a generalization of the standard model's

$$\Box A^\mu = j^\mu / \epsilon_0 \quad -(24)$$

whose solutions are the Liénard Wiechert potentials.

In ECE theory:

$$F^{a\mu}{}_\nu = \partial^\mu A^a_\nu - \partial^\nu A^a_\mu + \omega^{a\mu}{}_b A^b_\nu - \omega^{a\nu}{}_b A^b_\mu \quad -(25)$$

and

$$\partial_\mu F^{a\mu}{}_\nu = \frac{j^a_\nu}{\epsilon_0} = -kT A^a_\nu \quad -(26)$$

with

$$(\Box + kT) A^a_\nu = 0 \quad -(27)$$

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Similarly:

$$F^{a\mu\nu} = \partial^\mu A^{a\nu} - \partial^\nu A^{a\mu} + \omega^{a\mu}{}_b A^{b\nu} - \omega^{a\nu}{}_b A^{b\mu} \quad -(28)$$

and
$$\partial_\mu F^{a\mu\nu} = j^{a\nu}/\epsilon_0 = -kT A^{a\nu} \quad -(29)$$

with
$$(\Box + kT) A^{a\nu} = 0. \quad -(30)$$

So:

$$(\Box + kT) A^{a\nu} = \partial_\mu \left(\partial^\nu A^{a\mu} - \omega^{a\mu}{}_b A^{b\nu} + \omega^{a\nu}{}_b A^{b\mu} \right)$$
$$= 0 \quad -(31)$$

The "Lorenz gauge condition" in ECE is this eq. (31), but is now a geometrical requirement. From the Leibnitz theorem:

$$\partial_\mu \left(\partial^\nu A^{a\mu} \right) = \left(\partial_\mu \partial^\nu \right) A^{a\mu} + \partial^\nu \left(\partial_\mu A^{a\mu} \right)$$
$$-(32)$$

and so on.

In general, the Lorenz condition in ECE is:

$$\boxed{\partial_\mu \left(\partial^\nu A^{a\mu} - \omega^{a\mu}{}_b A^{b\nu} + \omega^{a\nu}{}_b A^{b\mu} \right) = 0} \quad -(33)$$

If eq. (33) is compared with the tetrad postulate:

$$D^{\nu} A^{a\mu} = \partial^{\mu} A^{a\nu} + \omega^{a\nu}{}_{b} A^{b\mu} - \Gamma^{\lambda\nu\mu} A^{a}_{\lambda}$$
$$= 0 \qquad -(34)$$

it is found that eq. (33) is true if:

$$\boxed{\omega^{a\mu}{}_{b} A^{b\nu} = \Gamma^{\lambda\nu\mu} A^{a}_{\lambda}} \qquad -(35)$$

Eq. (35) is the condition for eq. (2).

Discussion of the Continuity Equation

The charge current density from eq (2) may be written as:

$$j^{a}_{\mu} = j^{(0)} q^{a}_{\mu} \qquad -(36)$$

where

$$j^{(0)} = -\epsilon_0 kT A^{(0)} \qquad -(37)$$

An example of eq. (36) is:

$$j^{\kappa\nu} = j^{(0)} q^{\kappa\nu} \qquad -(38)$$

with:

$$\boxed{D_{\mu} j^{\kappa\nu} = 0} \qquad -(39)$$

This is the continuity equation associated with eq. (6).

7) Eq. (1) is:

$$D_\mu j^a_\nu = \partial_\mu j^a_\nu + \omega^a_{\mu b} j^b_\nu - \Gamma^\lambda_{\mu\nu} j^a_\lambda = 0 \quad -(40)$$

A special case of this is:

$$\partial_\mu j^{a\mu} + \omega^a_{\mu b} j^{b\mu} - \Gamma^{\lambda\mu}_\mu j^a_\lambda = 0. \quad -(41)$$

The standard continuity equation is:

$$\partial_\mu j^\mu = 0. \quad -(42)$$

which is

$$\frac{\partial \rho}{\partial t} + \underline{\nabla} \cdot \underline{J} = 0 \quad -(43)$$

with

$$j^\mu = (c\rho, \underline{J}). \quad -(44)$$

Therefore the continuity equation (1) reduces to:

$$\partial_\mu j^a_\nu = 0 \quad -(45)$$

if

$$\omega^a_{\mu b} j^b_\nu = \Gamma^\lambda_{\mu\nu} j^a_\lambda \quad -(46)$$

which is similar to eq. (35).

The replacement of the covariant derivative by the ordinary derivative may therefore be done if condition (46) is true.

8) A special case of eq. (45) is:

$$\partial_\mu j^{a\mu} = 0, \quad -(47)$$

for example:

$$\partial_\mu j^{0\mu} = 0. \quad -(48)$$

In the base manifold, eq. (47) is:

$$\boxed{\partial_\mu j^{\kappa\mu} = 0} \quad -(49)$$

In ECE theory:

$$\rho = \frac{j^{00}}{c}, \quad \underline{J} = j^{11}\underline{i} + j^{22}\underline{k} + j^{33}\underline{k}$$
$$-(50)$$

so:

$$\frac{1}{c}\frac{\partial j^{00}}{\partial t} + \frac{\partial j^{11}}{\partial X} + \frac{\partial j^{22}}{\partial Y} + \frac{\partial j^{33}}{\partial Z} = 0$$
$$-(51)$$

<u>which is eq. (43).</u>

This assumes that eq. (43) is always true experimentally, as seems to be the case, because electric charge seems to be conserved.