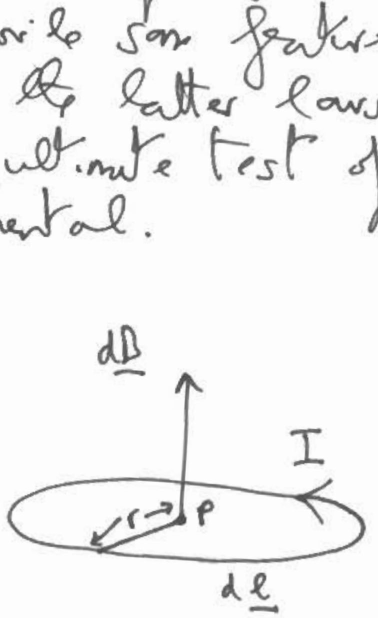
Note 117(1): Laws of Gravitomagnetism.

Some notes are given first on the basic laws of magnetism: the Biot / Savart and Ampère Laws. These provide some features of gravitomagnetism. In a unified field theory the latter laws will behave in a closely similar way. The ultimate test of the laws of gravitomagnetism must be experimental.

Law of Biot and Savart

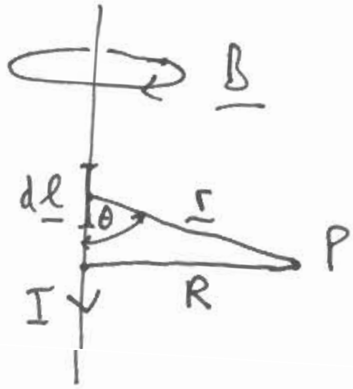
At point P:



$$d\underline{B} = \frac{\mu_0}{4\pi} I \frac{d\underline{l} \times \underline{r}}{|\underline{r}|^3} \quad -(1)$$

Here $d\underline{B}$ is the element of flux density produced by an electric current I along an element of length $d\underline{l}$. The magnetic flux density is measured at a point P a distance $\underline{r}$ from the current loop, and is perpendicular to the plane of the loop.

In a long straight wire:



$$|\underline{B}| = \frac{\mu_0}{4\pi} I R \int_{-\infty}^{\infty} \frac{dl}{(R^2 + l^2)^{3/2}} = \frac{\mu_0}{2\pi} \frac{I}{R} \quad -(2)$$

$$\boxed{|\underline{B}| = \frac{\mu_0}{2\pi} \frac{I}{R}} \quad -(3)$$

This law was discovered experimentally by Biot and Savart. The S.I. units of magnetic flux density are tesla (weber per square metre).

The ECE equations show that there is an

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analogous laws of gravitomagnetism.

Ampère's Law

This is expressed in S.I. units as:

$$\underline{\nabla} \times \underline{H} = \underline{J} \quad - (4)$$

where $\underline{H}$ is the magnetic field strength. In the absence of magnetization:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad - (5)$$

where $\underline{J}$ is the electric current density. The charge / current four vector in classical electrodynamics is defined in S.I. units by:

$$j^\mu = (c\rho, \underline{J}) \quad - (6)$$

where ρ is the electric charge density in $C\,m^{-3}$ and where $\underline{J}$ is the electric current density.

Gravitomagnetic Law

The charge / current four vector of matter is defined in analogy with eq. (6):

$$\boxed{J^\mu = (c\rho_m, \underline{J}_m)} \quad - (7)$$

where ρ_m is mass density in $kgm\,m^{-3}$ and where $\underline{J}_m$ is matter current density in $kgm\,s^{-1}\,m^{-2}$.

The static gravitomagnetic law is then:

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$$\boxed{\underline{\nabla} \times \underline{h} = \left(\frac{4\pi G}{c}\right) \underline{J}_m} \quad - (8)$$

In S.I. units:

$$G = 6.6726 \times 10^{-11} \, m^3 s^{-2} \, kgm^{-1} \quad - (9)$$

$$c = 2.998 \times 10^{8} \, m s^{-1} \quad - (10)$$

So:

$$\underline{\nabla} \times \underline{h} \sim 10^{-19} \underline{J}_m. \quad - (11)$$

Meaning of $\underline{h}$

This is the gravito-magnetic acceleration $(m s^{-2})$ that plays the role of $\underline{B}$. In electrostatics and magnetostatics in the non-relativistic limit:

$$\underline{E} = - \frac{e}{4\pi \epsilon_0} \frac{\underline{r}}{|\underline{r}|^3} \quad - (12)$$

$$\underline{B} = \frac{e}{4\pi \epsilon_0 c^2} \frac{\underline{v} \times \underline{r}}{|\underline{r}|^3} \quad - (13)$$

Eq. (13) is a re-expression of eq. (1). It is seen that:

$$\underline{B} = - \frac{1}{c^2} \underline{v} \times \underline{E} \quad - (14)$$

which is a limit of the Lorentz transform when:

$$v << c. \quad - (15)$$

In direct analogy:

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$$\underline{g} = - \frac{mG}{|\underline{r}|^3} \underline{r} \quad - (16)$$

is the Newton inverse square law, and so:

$$\boxed{\underline{h} = - \frac{mG}{c|\underline{r}|^3} \underline{v} \times \underline{r}} \quad - (17)$$

The angular momentum of a particle moving at orbital velocity $\underline{v}$ in a radius $\underline{r}$ is:

$$\underline{L} = m \underline{r} \times \underline{v} \quad - (18)$$

So:

$$\boxed{\underline{h} = \frac{G}{c} \left(\frac{\underline{L}}{V} \right)} \quad - (19)$$

where:

$$V := |\underline{r}|^3 \quad - (20)$$

Therefore:

$$\boxed{\underline{h} \sim 10^{-19} \underline{L}_D} \quad - (21)$$

where $\underline{L}_D$ is the density of orbital angular momentum.

Angular momentum theory is highly developed, and $\underline{h}$ can be related to a gravitomagnetic dipole moment. There is a gravitational interaction between angular momenta.