1)

117(4): ECE Explanation for Gravity Probe B.

This is based on the equation:

$$\underline{\nabla} \times \underline{h} = \frac{4\pi G}{c} \underline{J} \quad -(1)$$

In the dipole approximation and in the limit of vanishing spin connection this equation may be solved in direct analogy with the well known Ampère law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J}_e \quad -(2)$$

as in note 117(2). In the dipole approximation the solution of eq. (2) is:

$$\underline{B} = \frac{\mu_0}{4\pi r^3} \left(3\underline{n} (\underline{n} \cdot \underline{m}) - \underline{m} \right) \quad -(3)$$

where:

$$\underline{n} = \underline{r} / |\underline{r}| \quad -(4)$$

and as in eq. (19) of note 117(2) the solution of eq. (1) is:

$$\underline{h} = \frac{G}{c r^3} \left(\underline{L} - 3\underline{n} (\underline{n} \cdot \underline{L}) \right) \quad -(5)$$

In gravity probe B the measured quantity is an angle of 0.042 ± 0.00001 arc seconds. This is related to the angular frequency:

$$\Omega = \frac{h}{c} \quad -(6)$$

The angular momentum $\underline{L}$ in eq. (5) is calculated

2)

, to be that of a sphere:

$$L = \frac{2}{5} m R^2 \omega \quad - (7)$$

where m is the mass of the sphere and where R is its radius. Here ω is the angular velocity at which the sphere is spinning. So:

$$\frac{\underline{h}}{c} = \frac{G}{c^2 r^3}\left(\underline{L} - 3\frac{\underline{r}}{r}\left(\underline{L}\cdot\frac{\underline{r}}{r}\right)\right) - (8)$$

where $|\underline{L}|$ is given by eq. (7). So:

$$\boxed{\Omega = \frac{2}{5}\frac{G m R^2 \omega}{c^2 r^3}\left(1 - 3\left(\frac{\underline{r}}{r}\right)^2\right)} - (9)$$

This is exactly the same as the result in:

H. Pfister, http://philsci-archive.pitt.edu/archive/00002681/01/lense.pdf,

eq. (1) of that article: "On the History of the so-called Lense-Thirring Effect.", for Inst. Theoretical Physics, Tuebingen University.
