1) Notes 118(6): Hodge Transform of the Inhomogeneous Field Equation or Dual Identity.

In the standard physics the inhomogeneous field equation of electrodynamics is:

$$\partial_\mu F^{\mu\nu} = J^\nu / \epsilon_0 , \quad -(1)$$

$$J^\nu = (\rho, \underline{J} / c) . \quad -(2)$$

This translates into the Coulomb and Ampère Maxwell laws:

$$\underline{\nabla} \cdot \underline{E} = \rho / \epsilon_0 \quad -(3)$$

$$\underline{\nabla} \times \underline{B} - \frac{1}{c^2} \frac{\partial \underline{E}}{\partial t} = \mu_0 \underline{J} = \frac{1}{c^2 \epsilon_0} \underline{J} . \quad -(4)$$

For example, when:

$$\nu = 0 \quad -(5)$$

$$\partial_1 F^{10} + \partial_2 F^{20} + \partial_3 F^{30} = J^0 / \epsilon_0 . \quad -(6)$$

The Hodge transforms of the field tensors are:

$$F^{10} \rightarrow \tilde{F}_{23} , \quad F^{20} \rightarrow \tilde{F}_{31} , \quad F^{30} \rightarrow \tilde{F}_{12} \quad -(7)$$

The charge density component on the right hand side of eq. (6) may transform in three ways:

$$\tilde{J}_{123} = \epsilon_{0123} J^0 = J^0 \quad -(8)$$

$$\tilde{J}_{231} = \epsilon_{0231} J^0 = J^0 \quad -(9)$$

$$\tilde{J}_{312} = \epsilon_{0312} J^0 = J^0 \quad -(10)$$

2)

In the standard physics this Hodge Transform is never considered. Eq (1) is looked on as being derived from data, and is the standard model electrodynamics is not based on geometry. In ECE however, the structure of the dual identity shows that the Hodge dual of eq (1) must be:

$$\partial_1 \tilde{F}_{23} + \partial_2 \tilde{F}_{31} + \partial_3 \tilde{F}_{12} = \frac{1}{\epsilon_0} \left(\tilde{J}_{123} + \tilde{J}_{231} + \tilde{J}_{312} \right) \quad -(11)$$

because the dual identity is, in this case:

$$D_1 \tilde{T}^a_{23} + D_2 \tilde{T}^a_{31} + D_3 \tilde{T}^a_{12} = \tilde{R}^a_{123} + \tilde{R}^a_{231} + \tilde{R}^a_{312} \quad -(12)$$

which is equivalent to:

$$D_1 T^{a10} + D_2 T^{a20} + D_3 T^{a30} = R^a{}_1{}^{10} + R^a{}_2{}^{20} + R^a{}_3{}^{30} \quad -(13)$$

i.e.

$$\boxed{D_\mu T^{a\mu 0} = R^a{}_\mu{}^{\mu 0}} \quad -(14)$$

So ECE makes electrodynamics self-consistent and consistent with gravitation.