1)

121(3): Fundamental Conserved Quantities

The conserved quantity is a current defined by:

$$J^a_\mu = J^{(0)} q^a_\mu \quad -(1)$$

where q^a_μ is the tetrad. Therefore:

$$\boxed{D^\mu J^a_\mu = 0} \quad -(2)$$

is the fundamental conservation theorem.

In the base manifold:

$$J^\kappa_\mu = q^\kappa_a J^a_\mu \quad -(3)$$

and

$$J^{\mu\nu} = g^{\mu\kappa} J^\nu_\kappa . \quad -(4)$$

By metric compatibility:

$$D_\mu J^{\mu\nu} = 0, \quad -(5)$$

and if:

$$J^{\mu\nu} = T^{\mu\nu} \quad -(6)$$

the covariant Noether theorem is obtained for conservation of canonical energy-momentum density.

$$D_\mu T^{\mu\nu} = 0. \quad -(7)$$

In this case the energy momentum vector is

$$P_\mu = \int J^0_\mu \, d^3x \quad -(8)$$

The fundamental conserved quantity is always based on the tetrad postulate.

In electrodynamics:

$$D_\nu A^a_\mu = 0 \quad -(9)$$

where:

$$A^a_\mu = A^{(0)} q^a_\mu \quad -(10)$$

The fundamental conservation theorem (2) is equivalent to the wave equation:

$$\square J^a_\mu = R J^a_\mu \quad -(11)$$

or

$$(\square + kT) J^a_\mu = 0 \quad -(12)$$

The canonical angular momentum / energy density is proportional to the torsion $T^{\kappa\mu\nu}$, or the torsion form $T^a_{\mu\nu}$. The dual identity states that:

$$D_\mu T^{a\mu\nu} = R^{a\ \mu\nu}_{\ \mu} \quad -(13)$$

or, lowering the index ν:

$$D_\mu T^{a\mu}{}_{\nu} = R^{a\ \mu}_{\ \mu\ \nu} \quad -(14)$$

3) The conserved current in this case is:

$$\boxed{J^{a}_{\sim} := R^{a\ \mu}_{\ \mu\ \sim}} \quad -(15)$$

and

$$D_\mu J^{a}_{\sim} = 0 \quad -(16)$$

In the base manifold and in the limit of flat spacetime:

$$D_\mu T^{\kappa\mu\nu} \rightarrow \partial_\mu T^{\kappa\mu\nu} = 0 \quad -(17)$$

The angular momentum tensor is proportional to:

$$T^{\mu\nu} = \int T^{0\mu\nu} d^3x \quad -(18)$$

(see Ryder, eq. (3.46) of "Quantum Field Theory"), and in this limit:

$$\frac{d}{dt} M^{\mu\nu} = 0, \quad -(19)$$

which is the usual expression for conservation of angular energy momentum. In the translational case:

$$\frac{d}{dt} p^{\mu} = 0 \quad -(20)$$

which is conservation of energy momentum.