1) 126(1): Geodesic Theory of Galaxy Dynamics

This is based on a simple development of paper 108, i.e. on the line element:

$$-c^2 d\tau^2 = -\left(1 - \frac{r_s}{r}\right) c^2 dt^2 + \left(1 - \frac{r_s}{r}\right)^{-1} dr^2 + r^2 d\phi^2 \quad -(1)$$

In paper 108 it was shown that when:

$$r_s = \frac{2GM}{c^2} + \frac{a}{r} \quad -(2)$$

the potential energy is:

$$V = \frac{1}{2} m \left(1 - \frac{r_s}{r}\right)\left(c^2 + \frac{L^2}{r^2}\right) \quad -(3)$$

which produces an additional force of attraction towards the centre of the orbit:

$$\Delta F = -\frac{\partial \Delta V}{\partial r} = -2\frac{am}{r^3}\left(c^2 + \frac{2L^2}{r^2}\right) \quad -(4)$$

Here the angular momentum L is a constant of the motion. This additional force of attraction results in a logarithmic spiral orbit inwards of the type:

$$r = k^{1/3} \exp\left(\frac{\alpha}{3}\phi\right) \quad -(5)$$

The force of attraction in Newtonian dynamics for example is

$$\underline{F} = m\underline{g} = -\frac{mMG}{r^2}\underline{e}_r \quad -(6)$$

$$= - \underline{\nabla} V \quad -(7)$$

so: $$\underline{g} = -\frac{MG}{r^2} \underline{e}_r \quad -(8)$$

is negative. Since mass is always positive then
$\underline{F}$ is negative valued for attraction.

In Newtonian dynamics there is no repulsion between
mass m and M. This is also true in the
relativistic Kepler problem where:

$$a = 0 \quad -(9)$$

and it is also true in binary pulsars. Therefore
in these cases, if:

$$\underline{F} = - \underline{\nabla} V \quad -(10)$$

then $\underline{\nabla} V$ must be positive valued and $\underline{F}$
must be negative valued.

In galaxies however the orbit of the
stars spirals outwards from the force centre. There
is a repulsive force. Eq. (10) comes from:

$$\int_1^2 \underline{F} \cdot d\underline{r} = V_1 - V_2 \quad -(11)$$

and $$\underline{g} = - \underline{\nabla} \Phi \quad -(12)$$

where Φ is the gravitational potential of attraction.
the work per unit mass dW that must be done
by an outside agency on a particle in an

3) attractive gravitational field in order to displace the particle by a distance $d\underline{r}$ is:

$$dW = -\underline{g} \cdot d\underline{r} = \underline{\nabla} \Phi \cdot d\underline{r}$$

$$= d\Phi \qquad -(13)$$

The work done is equal to the difference in potential. The work done is against the gravitational pull, and work must be done

$$W_1 - W_2 = \Phi_1 - \Phi_2 > 0 \qquad -(14)$$

In eqn. (11), for attraction, $\underline{F}$ is negative and so V_2 is less negative than V_1:

$$V_2 > V_1 \qquad -(15)$$

e.g. $V_1 = -1$, $V_2 = -1/2$.

For a repulsive force however, the sign changes:

$$-\int_1^2 \underline{F} \cdot d\underline{r} = V_1 - V_2 \qquad -(16)$$

i.e.

$$\underline{F} = \underline{\nabla} V \qquad -(17)$$

The force in eq. (4) becomes positive, and the orbit of eq. (5) spirals outwards, with:

$$\frac{d^2}{d\theta^2}\left(\frac{1}{r}\right) + \frac{1}{r} = \frac{\mu r^2}{L^2} F(r) \qquad -(18)$$

1) 126(2): Gravitomagnetic Theory of Galactic Dynamics

This is based on the dynamical ECE equation equivalent to the Ampère Maxwell law:

$$\underline{\nabla} \times \underline{S} - \frac{1}{c} \frac{\partial \underline{L}}{\partial t} = \underline{p} \quad -(1)$$

where $\underline{S}$ and $\underline{L}$ are spin and orbital angular momenta and where $\underline{p}$ is linear momentum. For planar orbits:

$$\frac{\partial \underline{L}}{\partial t} = \underline{0} \quad -(2)$$

so:

$$\boxed{\underline{\nabla} \times \underline{S} = \underline{p}} \quad -(3)$$

where:

$$\underline{p} = m \underline{v} . \quad -(4)$$

In plane polar coordinates:

$$\underline{v} = v_r \underline{e}_r + v_\theta \underline{e}_\theta \quad -(5)$$
$$= \dot{r} \underline{e}_r + r \dot{\theta} \underline{e}_\theta ,$$

where:

$$\left.\begin{aligned} \underline{e}_r &= \underline{i} \cos\theta + \underline{j} \sin\theta \\ \underline{e}_\theta &= -\underline{i} \sin\theta + \underline{j} \cos\theta . \end{aligned}\right\} -(6)$$

Therefore:

$$\underline{\nabla} \times \underline{S} = m \left(\left(v_r \cos\theta - v_\theta \sin\theta \right) \underline{i} + \left(v_r \sin\theta + v_\theta \cos\theta \right) \underline{j} \right) \quad -(7)$$

2)

$$= m\left(v_x \underline{i} + v_y \underline{j}\right) \quad -(8)$$

If it is assumed that:

$$\underline{S} = S_Z \underline{k} \quad -(9)$$

then: $\nabla \times \underline{S} = \frac{\partial S_Z}{\partial Y}\underline{i} - \frac{\partial S_Z}{\partial X}\underline{j} \quad -(10)$

So:

$$\boxed{\begin{aligned} \frac{\partial S_Z}{\partial Y} &= m v_x \quad -(11) \\ \frac{\partial S_Z}{\partial X} &= -m v_y \quad -(12) \end{aligned}}$$

Here:

$$v_x = v_r \cos\theta - v_\theta \sin\theta \quad -(13)$$
$$= \dot{r}\cos\theta - r\dot{\theta}\sin\theta$$

$$v_y = v_r \sin\theta + v_\theta \cos\theta \quad -(14)$$
$$= \dot{r}\sin\theta + r\dot{\theta}\sin\theta$$

In a non-viscous incompressible fluid moving in a vortex, it may be shown that:

$$\frac{v_r}{v_\theta} = \frac{1}{\theta - \theta_0}\log_e \frac{r}{r_0} \quad -(15)$$

is a constant of the motion. The ratio of

3) azimuthal to radial velocity is:

$$A = V_\theta / V_r \quad \text{— (16)}$$

Eq. (15) follows from the conservation of mass and angular momentum

(www.iac.es/folleto/research/preprints/files/PP08021.pdf)

As $A \to 0$ the logarithmic spiral (15) becomes a straight line and as $A \to \infty$ it becomes a circle.

Therefore a whirlpool galaxy could be modelled by eqs. (11), (12) and (15), in terms of the spacetime angular momentum. This is a gravitomagnetic explanation.

Standard Model Reference

J. Binney and S. Tremaine, "Galactic Dynamics" (1989).

1) 126(3) : Spin Angular Momentum in Three Axes

Instead of eq. (9) of the last note it is assumed that:

$$\underline{S} = S_x \underline{i} + S_y \underline{j} + S_z \underline{k} \quad -(1)$$

$$\underline{\nabla} \times \underline{S} = \begin{vmatrix} \underline{i} & \underline{j} & \underline{k} \\ \partial/\partial x & \partial/\partial y & \partial/\partial z \\ S_x & S_y & S_z \end{vmatrix} \quad -(2)$$

$$= \underline{i}\left(\frac{\partial S_z}{\partial y} - \frac{\partial S_y}{\partial z}\right) - \underline{j}\left(\frac{\partial S_z}{\partial x} - \frac{\partial S_x}{\partial z}\right) + \underline{k}\left(\frac{\partial S_y}{\partial x} - \frac{\partial S_x}{\partial y}\right) \quad -(3)$$

So:

$$\frac{\partial S_z}{\partial y} - \frac{\partial S_y}{\partial z} = m v_x \quad -(4)$$

$$\frac{\partial S_z}{\partial x} - \frac{\partial S_x}{\partial z} = m v_y \quad -(5)$$

$$\frac{\partial S_y}{\partial x} - \frac{\partial S_x}{\partial y} = 0 \quad -(6)$$

In general the mathematical problem parallels Ampère's Law:

$$\underline{\nabla} \times \underline{B} = \mu_0 \underline{J} \quad -(7)$$

i.e.

$$\underline{\nabla} \times \underline{S} = \underline{p} \quad -(8)$$

1. 126(4): Some Calculations from Ren et al. on the Logarithmic Spiral Trajectory

Ren et al. (2006) found that the trajectory of an electron in the equation:

$$m\underline{a} = e\underline{v} \times \underline{B} + \alpha \underline{v} \quad -(1)$$

is a logarithmic spiral:

$$x(t) = a e^{b\theta} \cos\theta \quad -(2)$$

$$y(t) = a e^{b\theta} \sin\theta \quad -(3)$$

Therefore the gravitomagnetic evolution of a galaxy can be developed in this way with e representing the mass and $\underline{B}$ by the gravitomagnetic field of spacetime.

In this note we calculate:

$$\dot{x}(t) = a\dot{\theta} e^{b\theta} \left(b\cos\theta - \sin\theta \right) \quad -(4)$$

$$\dot{y}(t) = a\dot{\theta} e^{b\theta} \left(b\sin\theta + \cos\theta \right) \quad -(5)$$

Therefore:

$$r^2 = x^2 + y^2 = a^2 e^{2b\theta} \quad -(6)$$

$$v^2 = \dot{x}^2 + \dot{y}^2 = a^2 e^{2b\theta} \dot{\theta}^2 (1 + b^2) \quad -(7)$$

So:

$$\boxed{v = r\dot{\theta}(1 + b^2)^{1/2}} \quad -(8)$$

This is the velocity curve of the galaxy.

2) It is seen that if:

$$v \xrightarrow[r \to 0]{} v_0 \quad -(9)$$

then

$$\dot{\theta} \xrightarrow[r \to 0]{} \frac{v_0}{r}\left(1+b^2\right)^{-1/2} \quad -(10)$$

$$= 0$$

This is the constant velocity condition

Calculation of Acceleration and Force

We use:

$$\frac{d}{dt}\left(a\dot{\theta}e^{b\theta}\right) = a e^{b\theta}\left(\ddot{\theta} + b\dot{\theta}^2\right) \quad -(11)$$

$$= \dot{\theta}\left(\cos\theta - b\sin\theta\right) \quad -(12)$$

$$\frac{d}{dt}\left(b\cos\theta - \sin\theta\right) =$$

$$\dot{\theta}\left(b\cos\theta - \sin\theta\right) \quad -(13)$$

$$\frac{d}{dt}\left(b\sin\theta + \cos\theta\right) =$$

so:

$$\ddot{x}(t) = a e^{b\theta}\Big(\left(b\cos\theta - \sin\theta\right)\ddot{\theta} + \dot{\theta}^2\left(\left(1+b^2\right)\cos\theta - 2b\sin\theta\right)\Big) \quad -(14)$$

$$\ddot{y}(t) = a e^{b\theta}\Big(\left(b\sin\theta + \cos\theta\right)\ddot{\theta} + \dot{\theta}^2\left(\left(b^2-1\right)\sin\theta + 2b\cos\theta\right)\Big) \quad -(15)$$

and

$$a^2 = \ddot{x}^2 + \ddot{y}^2 \quad -(16)$$

3) The force on the star is therefore:

$$F = m\left(\ddot{x}^2 + \ddot{y}^2\right)^{1/2} \quad -(17)$$

Remarks

It is seen that the constants a and b can be evaluated from the above equation and:

$$m\ddot{x} = -q\dot{y}B + \alpha\dot{x} \quad -(18)$$

$$m\ddot{y} = q\dot{x}B + \alpha\dot{y} \quad -(19)$$

It is seen that a in eqs. (2) and (3) have the dimension of distance. If:

$$a \geq 0, \quad b \geq 0 \quad -(20)$$

then $x(t)$ and $y(t)$ increase with time. In another part of their paper, Ren et al. define:

$$\theta = \omega t. \quad -(21)$$

Therefore the dynamics can be plotted and animated in terms of a characteristic angular velocity ω.

From eq. (6), it is seen that $\boxed{r = a e^{b\theta}}$

i.e a logarithmic spiral: $r(t) = a \exp\left(b\,\theta(t)\right)$

1) 126(5): Spiral Trajectory and Force Law

Consider the logarithmic spiral trajectory:

$$r = r_0 \exp(b\theta) \quad -(1)$$

Then:

$$\dot{r} = r_0 b \dot{\theta} \exp(b\theta) \quad -(2)$$

The velocity is

$$\underline{v} = \dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta \quad -(3)$$

so

$$\boxed{v = |\underline{v}| = \left(\dot{r}^2 + r^2\dot{\theta}^2\right)^{1/2} = r\dot{\theta}\left(1+b^2\right)^{1/2}} \quad -(4)$$

The acceleration is

$$\underline{a} = \frac{d\underline{v}}{dt} = \frac{d}{dt}\left(\dot{r}\,\underline{e}_r + r\dot{\theta}\,\underline{e}_\theta\right)$$

$$= \left(\ddot{r} - r\dot{\theta}^2\right)\underline{e}_r + \left(r\ddot{\theta} + 2\dot{r}\dot{\theta}\right)\underline{e}_\theta \quad -(5)$$

and the force is

$$\underline{F} = m\underline{a}. \quad -(6)$$

The kinetic energy is

$$T = \frac{1}{2}mv^2 = \frac{1}{2}mr^2\dot{\theta}^2\left(1+b^2\right) \quad -(7)$$

In general there are radial and transverse components of the velocity and force. The magnitude of the

2)

angular momentum is:

$$J = m r v = m r^2 \dot{\theta} (1 + b^2)^{1/2} \quad - (8)$$

Therefore from eqs. (7) and (8)

$$T = \frac{1}{2} \dot{\theta} J \quad - (9)$$

$$T = \frac{1}{2} \omega J. \quad - (10)$$

where: $\omega = \dot{\theta}$ — (11)

is the angular velocity.

<u>Potential Energy from Angular Momentum</u>

For spiral orbits in a plane, such eq. (1), the angular momentum is a constant of the motion. Now define the potential energy

$$U = \frac{J^2}{m r^2} \quad - (12)$$

and effective radial force:

$$F = -\frac{\partial U}{\partial r} = \frac{J^2}{m r^3} = m r \dot{\theta}^2 (1 + b^2) \quad - (13)$$

This is the effective radial force that moves the particle of mass m on the log spiral (1).

It is an effective force because it is defined from eqs. (5) and (6) to be the radial force.

3) and transverse components. More generally:

$$\underline{F} = -\underline{\nabla} U = m(\ddot{r} - r\dot{\theta}^2)\underline{e}_r + (r\ddot{\theta} + 2\dot{r}\dot{\theta})\underline{e}_\theta \quad -(14)$$

and U is a function of $\underline{e}_r$ and $\underline{e}_\theta$.

For the sake of simplicity define:

$$F = m\frac{dv}{dt} \quad (15)$$

$$= m(\dot{r}\dot{\theta} + r\ddot{\theta})(1+b^2)^{1/2} \quad (16)$$

Comparing (13) and (16):

$$r\dot{\theta}^2 = \dot{r}\dot{\theta} + r\ddot{\theta} \quad -(17)$$

Using eq. (1)

$$\dot{\theta}^2 = b\dot{\theta}^2 + \ddot{\theta} \quad -(18)$$

i.e. $$\boxed{\ddot{\theta} = (1-b)\dot{\theta}^2} \quad -(19)$$

This is the supplementary condition needed for the force law (13) to produce the spiral trajectory (1). Here is an effective inverse cubed radial force outwards from the centre of the spiral.

1) 126(6): Description of Newtonian Orbits in Terms of Angular Momentum

As for log spiral orbits start with the conserved spacetime angular momentum:

$$L = mrv \quad -(1)$$

which generates the potential energy:

$$U = \frac{L^2}{mr^2} \quad -(2)$$

and various spacetime forces, notably the radial force:

$$F_r = -\frac{\partial U}{\partial r}. \quad -(3)$$

The initial momentum for the orbit is generated by the relevant initial event. The rotation of spacetime then ensures that the planet or star travels in a transverse as well as a radial direction. By observation, the particle, planet or star eventually reaches a stable orbit. It is carried around in this orbit by the rotating spacetime. The concept of orbit is no longer due to a central attraction balanced by a centripetal repulsion. The orbit is due purely to the conserved angular momentum of spacetime.

By observation, a Keplerian orbit is described by:

$$r(t) = \frac{\alpha}{1 + \epsilon \cos \theta(t)} \quad -(4)$$

where α and ϵ are constants of the motion. In general r and θ are functions of time. By differentiation of eqn. (4) it is possible to calculate:

$$v = (\dot{r}^2 + r^2 \dot{\theta}^2)^{1/2} \quad -(5)$$

2) and therefore the quantities (1) to (3). From eq. (4):

$$\dot r(t) = \frac{d}{dt}\left(\frac{\alpha}{1+\epsilon\cos\theta(t)}\right) \quad - (6)$$

$$= -\alpha\frac{d}{dt}\left(1+\epsilon\cos\theta(t)\right) \Big/ \left(1+\epsilon\cos\theta(t)\right)^2$$

where:

$$\frac{d}{dt}\left(1+\epsilon\cos\theta(t)\right) = -\epsilon\dot\theta\sin\theta(t) \quad - (7)$$

So:

$$\dot r(t) = \frac{\epsilon\alpha\sin\theta(t)}{\left(1+\epsilon\cos\theta(t)\right)^2} \quad - (8)$$

$$\boxed{\dot r(t) = \epsilon\dot\theta r\left(\frac{\sin\theta(t)}{1+\epsilon\cos\theta(t)}\right)} \quad - (9)$$

So:

$$v^2 = r^2\dot\theta^2\left(1+\left(\frac{\epsilon\sin\theta(t)}{1+\epsilon\cos\theta(t)}\right)^2\right) \quad - (10)$$

$$\boxed{v = V_\theta\left(1+\left(\frac{\epsilon\sin\theta(t)}{1+\epsilon\cos\theta(t)}\right)^2\right)^{1/2}} \quad - (11)$$

where the transverse velocity component is:

$$V_\theta = r\dot\theta \quad - (12)$$

Using eq. (4):

$$v = V_\theta\left(1+\overset{\sin\theta(t)}{\left(\frac{\epsilon r}{\alpha}\right)}^2\right)^{1/2} \quad - (13)$$

$$\boxed{v = V_\theta\left(1+\left(\frac{\epsilon r\sin\theta(t)}{\alpha}\right)^2\right)^{1/2}} \quad - (14)$$

3) Therefore:

$$v^2 = r^2\dot{\theta}^2\left(1 + \left(\frac{\epsilon r \sin\theta(t)}{\alpha}\right)^2\right) \quad -(15)$$

and the potential energy is:

$$U = \frac{m^2 r^4 \dot{\theta}^2}{m r^2}\left(1 + \left(\frac{\epsilon r \sin\theta(t)}{\alpha}\right)^2\right)$$

$$\boxed{U = m r^2 \dot{\theta}^2\left(1 + \left(\frac{\epsilon r \sin\theta(t)}{\alpha}\right)^2\right)} \quad -(16)$$

The potential energy of rotating spacetime needed for a Keplerian orbit is eq. (16)

The potential energy needed for a logarithmic spiral orbit is:

$$\boxed{U = m r^2 \dot{\theta}^2 (1 + b^2)} \quad -(17)$$

The conserved angular momentum leading to (16) is:

$$L = m r^2 \dot{\theta}\left(1 + \left(\frac{\epsilon r \sin\theta(t)}{\alpha}\right)^2\right)^{1/2} \quad -(18)$$

and to (17) is:

$$L = m r^2 \dot{\theta}(1 + b^2)^{1/2} \quad -(19)$$

So spiral to Kepler is:

$$\boxed{b \rightarrow \frac{\epsilon r \sin\theta(t)}{\alpha}} \quad -(20)$$

1) 126(8): Some Basic Concepts, Part 2

The check on total velocity is as follows:

$$v^2 = v_x^2 + v_y^2 = (\dot{r}\cos\theta - r\dot{\theta}\sin\theta)^2 + (\dot{r}\sin\theta + r\dot{\theta}\cos\theta)^2 \quad -(1)$$

$$= \dot{r}^2 + r^2\dot{\theta}^2$$

The total velocity contains time variation of both r and θ:

$$v = (\dot{r}^2 + r^2\dot{\theta}^2)^{1/2} = (v_r^2 + v_\theta^2)^{1/2} \quad -(2)$$

The Keplerian Orbit

We wish to work out the angular momentum:

$$L = mr^2\dot{\theta} \quad -(3)$$

$$= m(r_x v_y - r_y v_x)$$

from the Kepler orbit. The latter is defined by observation as:

$$\boxed{r(t) = \frac{\alpha}{1 + \epsilon\cos\theta(t)}} \quad -(4)$$

where α and ϵ are constants. The path, or geodesic, is therefore eq. (4). Both r and θ are functions of time.

Differentiating eq. (4):

$$\dot{r}(t) = \frac{\alpha\epsilon\dot{\theta}\sin\theta(t)}{(1 + \epsilon\cos\theta(t))^2}, \quad -(5)$$

2) so for a Keplerian orbit:

$$\boxed{\dot{r}(t) = \frac{\epsilon\,\dot{\theta}(t)\, r \sin\theta(t)}{1+\epsilon\cos\theta(t)}} \quad -(6)$$

Note that this is a purely observational result. Eq. (6) may be simplified using eq. (4):

$$\boxed{\dot{r}(t) = \frac{\epsilon}{d}\, r^2\, \dot{\theta}\, \sin\theta(t)} \quad -(7)$$

Using eq. (7) we now work out:

$$v_x = \dot{r}\cos\theta - r\dot{\theta}\sin\theta, \quad -(8)$$

$$v_y = \dot{r}\sin\theta + r\dot{\theta}\cos\theta.$$

Therefore for a Keplerian orbit:

$$v_x = \frac{\epsilon}{d}\, r^2\dot{\theta}\sin\theta\cos\theta - r\dot{\theta}\sin\theta$$

$$= r\dot{\theta}\left(\frac{\epsilon r}{d}\cos\theta - 1\right)\sin\theta$$

$$\boxed{v_x = v_\theta \sin\theta\left(\frac{\epsilon r}{d}\cos\theta - 1\right)} \quad -(9)$$

$$v_y = \frac{\epsilon}{d}\, r^2\dot{\theta}\sin^2\theta + r\dot{\theta}\cos\theta$$

$$\boxed{v_y = v_\theta\left(\frac{\epsilon}{d}\, r\sin\theta + \cos\theta\right)} \quad -(10)$$

3)

The angular momentum for the Kepler orbit is, by definition:

$$\boxed{L = m r^2 \dot{\theta}} \quad — (11)$$

where, from eq. (7):

$$r^2 \dot{\theta} = \frac{\underline{d}}{t} \frac{r(t)}{\sin \theta(t)} \quad — (12)$$

$$= \text{constant}.$$

The angular momentum L is a constant of motion. It is the constant <u>spacetime</u> angular momentum that gives the Kepler orbit (4), and indeed, <u>all</u> <u>planar orbits</u>. Therefore all that is <u>needed</u> for any planar orbit is constant spacetime angular momentum <u>(11)</u>:

$$\frac{d\underline{L}}{dt} = \underline{0} \quad — (13)$$

but:

$$\underline{\nabla} \cdot \underline{L} \neq 0. \quad — (14)$$

No other theory is needed by Ockham's Razor. The only concept used is:

$$\underline{L} = \underline{r} \times \underline{p} = \text{constant} — (15)$$

1) 126(a) : Logarithmic Spiral Orbits

In this case the orbital path is :

$$r(t) = r_0 \exp(b\,\theta(t)) \quad - (1)$$

So:

$$\dot{r}(t) = b\, r_0\, \dot{\theta}\, \exp(b\theta(t))$$

$$\boxed{\dot{r}(t) = b V_\theta \exp(b\,\theta(t))} \quad - (2)$$

Therefore:

$$V_x = \dot{r}\cos\theta - r\dot{\theta}\sin\theta \quad - (3)$$

$$V_y = \dot{r}\sin\theta + r\dot{\theta}\cos\theta \quad - (4)$$

$$\boxed{L = m r^2 \dot{\theta}} \quad - (5)$$

Thus:

$$\boxed{\begin{aligned} V_x &= V_\theta (b\cos\theta - \sin\theta) \\ V_y &= V_\theta (b\sin\theta + \cos\theta) \end{aligned}} \quad \begin{aligned} &- (6) \\ &- (7) \end{aligned}$$

and

$$V = (V_x^2 + V_y^2)^{1/2}$$

$$\boxed{V = r\dot{\theta}\,(1+b^2)^{1/2}} \quad - (8)$$

By observation

$$\left.\begin{aligned} V &\rightarrow V_0 \text{ (constant)} \\ \text{as: } r &\rightarrow \infty \end{aligned}\right] - (9)$$

So:

$$\boxed{\dot{\theta}\,(1+b^2)^{1/2} \rightarrow 0} \quad - (10)$$

1. 126(10): Relativistic Keplerian Orbits

In this case, to a good approximation:

$$r(t) = d\left(1 + \epsilon\cos\left(\left(1 - \frac{\beta}{d}\right)\theta(t)\right)\right)^{-1} \quad -(1)$$

So:

$$\dot{r}(t) = \frac{\epsilon r^2}{d}\dot{\theta}\left(1 - \frac{\beta}{d}\right)\sin\left(\left(1 - \frac{\beta}{d}\right)\theta\right) \quad -(2)$$

These are found from Marion and Thornton, eq. (7.8).

Therefore:

$$v_X = r\dot{\theta}\left(\frac{\epsilon r}{d}\left(1 - \frac{\beta}{d}\right)\sin\left(\left(1 - \frac{\beta}{d}\right)\theta\right)\cos\theta - \sin\theta\right) \quad -(3)$$

$$v_Y = r\dot{\theta}\left(\frac{\epsilon r}{d}\left(1 - \frac{\beta}{d}\right)\sin\left(\left(1 - \frac{\beta}{d}\right)\theta\right)\sin\theta + \cos\theta\right) \quad -(4)$$

and

$$L = mr^2\dot{\theta} \quad -(5)$$

$$v_\theta = r\dot{\theta} \quad -(6)$$

The orbit or path (1) is a precessing ellipse as is well known. It is caused by the constant spacetime angular momentum (5) in ECE theory.

1. 126(11): Evolution in Orbital Theory

This may be analysed using the X and Y components of velocity as follows.

Keplerian

$$v_X = v_\theta \left(\frac{\epsilon r}{\alpha} \sin\theta \cos\theta - \sin\theta \right) \quad -(1)$$

$$v_Y = v_\theta \left(\frac{\epsilon r}{\alpha} \sin^2\theta + \cos\theta \right) \quad -(2)$$

Relativistic Keplerian

$$v_X = v_\theta \left(\frac{\epsilon r}{\alpha} \left(1 - \frac{\beta}{\alpha} \right) \sin \left(\left(1 - \frac{\beta}{\alpha} \right) \theta \right) \cos\theta - \sin\theta \right) \quad -(3)$$

$$v_Y = v_\theta \left(\frac{\epsilon r}{\alpha} \left(1 - \frac{\beta}{\alpha} \right) \sin \left(\left(1 - \frac{\beta}{\alpha} \right) \theta \right) \sin\theta + \cos\theta \right) \quad -(4)$$

Logarithmic Spiral

$$v_X = v_\theta \left(b\cos\theta - \sin\theta \right) \quad -(5)$$

$$v_Y = v_\theta \left(b\sin\theta + \cos\theta \right) \quad -(6)$$

In each case angular momentum is conserved.

$$\boxed{L = m r^2 \dot{\theta} = \text{constant}} \quad -(7)$$

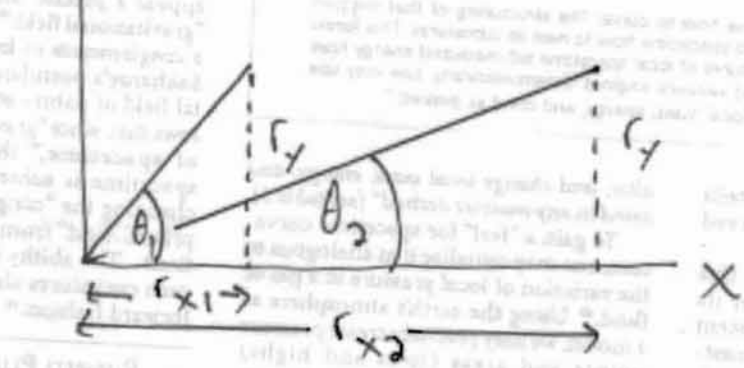
2.

The Evolution from Keplerian to logarithmic spiral occurs when:

$$\boxed{\frac{\epsilon r}{d}\sin\theta \rightarrow b} \quad -(8)$$

This means:

$$\boxed{r_y = r\sin\theta = \frac{db}{\epsilon} = \text{constant}} \quad -(9)$$



It is possible for the orbit to evolve from an ellipse to a logarithmic spiral while keeping the angular momentum conserved.

Similarly the evolution from relativistic Keplerian to logarithmic spiral occurs when:

$$\frac{\epsilon r}{d}\left(1-\frac{\beta}{d}\right)\sin\left(\left(1-\frac{\beta}{d}\right)\theta\right) \rightarrow b \quad -(10)$$

3) i.e.

$$\boxed{r \sin\left(\left(1-\frac{\beta}{\alpha}\right)\theta\right) = \frac{\alpha b}{\epsilon} = \text{constant}} \qquad (11)$$

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Part II of II

From eqs. (9) and (11) it is seen that the logarithmic spiral orbit is a special case of the relativistic Keplerian orbit. The latter is the orbital theorem of paper obtained from 111 the orbital theorem replaces the so-called solution of the now obsolete Einstein Schwarzschild field equation.

Conclusion

A rigorously self-consistent analysis of all orbits is given by ECE theory

1) 126(12) : Dynamics of the Log Spiral Trajectory

The log spiral trajectory is :

with :

$$\boxed{\begin{aligned} r &= r_0 \exp(b\theta) \quad &-(1) \\ L &= m r^2 \dot{\theta} = \text{constant} \quad &-(2) \end{aligned}}$$

From eq. (2) :

$$e^{2b\theta} \, d\theta = \left(\frac{L}{m r_0^2}\right) dt \quad -(3)$$

Integrating both sides of eq. (3) :

$$\frac{e^{2b\theta}}{2b} = \left(\frac{L}{m r_0^2}\right) t + C \quad -(4)$$

i.e.

$$e^{2b\theta} = \left(\frac{2bL}{m r_0^2}\right) t + 2bC \quad -(5)$$

$$\boxed{\theta(t) = \frac{1}{2b} \log_e \left(\frac{2bLt}{m r_0^2} + 2bC\right)} \quad -(6)$$

Similarly:

$$\boxed{r(t) = \left(\frac{2bL}{m} t + r_0^2 b C\right)^{1/2}} \quad -(7)$$

1) As t increases, θ increases (eq. (6))
2) As t increases, r increases (eq. (7))
3) As θ increases, r increases (eq. (1))

2) For the elliptical trajectory:

$$\frac{d\theta}{dt} = \frac{L}{mr^2} = \frac{L}{md^2}\left(1+\epsilon\cos\theta\right)^2 \quad -(8)$$

because:
$$\frac{1}{r} = \frac{1}{d}\left(1+\epsilon\cos\theta\right) \quad -(9)$$

so:
$$\frac{d\theta}{\left(1+\epsilon\cos\theta\right)^2} = \frac{L}{md^2}\,dt \quad -(10)$$

$$\int_0^\theta \frac{d\theta}{\left(1+\epsilon\cos\theta\right)^2} = \frac{Lt}{md^2} \quad -(11)$$

$$= \frac{1}{(1-\epsilon^2)}\left(\frac{2}{(1-\epsilon^2)^{1/2}}\tan^{-1}\left(\frac{(1-\epsilon)\tan(\theta/2)}{(1-\epsilon^2)^{1/2}}\right) - \frac{\epsilon\sin\theta}{1+\epsilon\cos\theta}\right)$$

$$:= f(\theta)$$

So:
$$\boxed{t = \frac{md^2}{L} f(\theta)} \quad -(12)$$

For a circle:
$$\epsilon = 0 \quad -(13)$$

so:
$$\boxed{\theta = \left(\frac{L}{md^2}\right)t} \quad -(14)$$

3) Animations

1) Plot r against θ from eq. (12).

2) Show how this deviates from the circle.

Reduction of Log Spiral to Circle

This occurs when:

$$b = 0, \quad r = r_0 \qquad -(15)$$

In eq. (7): $\quad C \to \infty, \ b \to 0 \qquad -(16)$

$$\qquad -(17)$$

and $\qquad r = r_0$

Velocity Components

For a log spiral:

$$v_\theta = r\dot{\theta} = \frac{L}{r} \qquad -(18)$$

so:

$$\boxed{\begin{aligned} v_x r &= b\cos\theta - \sin\theta \\ v_y r &= b\sin\theta + \cos\theta \end{aligned}} \qquad -(19)$$

For an ellipse:

$$\frac{\epsilon r_y}{d} = b = \frac{\epsilon r}{d}\sin\theta \qquad -(20)$$

Plot $v_x r$ and $v_y r$ against θ.