147(1): Rotation of Plane of Polarization in a Helical Optical Fibre, the Tomita Chiao Effect.

Let the circular helix be parameterized by:

$$X = r\cos\phi, \quad Y = r\sin\phi, \quad Z = Z_0\,\phi \quad - (1)$$

of pitch $2\pi Z_0$, the pitch of the helix being the distance along the helical axis (Z) that results in one full turn of the helix. Consider the metric of Minkowski in cylindrical polar coordinates:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad - (2)$$

Therefore for a helical optical fibre:

$$\boxed{ds^2 = c^2 dt^2 - dr^2 - (r^2 + Z_0^2)\, d\phi^2} \quad - (3)$$

for one turn of the helix. For n turns define:

$$R = n Z_0 \quad - (4)$$

so

$$ds^2 = c^2 dt^2 - dr^2 - (r^2 + R^2)\, d\phi^2 \quad - (4)$$

Consider electromagnetic radiation propagating through helical optical fibre. In the particulate epresentation this is a photon propagating at c, so:

$$ds^2 = 0 \quad - (5)$$

long the null geodesic. The radius of the fibre constant, so

$$dr^2 = 0 \quad - (6)$$

2) Therefore from these equations:

$$c^2 dt^2 = (r^2 + R^2)\, d\phi^2 \quad -(7)$$

and

$$\boxed{\omega = \frac{d\phi}{dt} = \frac{c}{(r^2 + R^2)^{1/2}}} \quad -(8)$$

The angular frequency ω defined in eq. (8) defines the phase angle:

$$\phi_R = \omega t \quad -(9)$$

in radians. It is seen from eqs. (8) and (9) that as

$$R \rightarrow \infty \quad -(10)$$

$$\omega \rightarrow 0 \quad -(11)$$

then

This means that if the helical optical fibre is drawn out into a straight line, the phase ϕ_R disappears. On the other hand, if:

$$R \rightarrow 0 \quad -(12)$$

$$\omega \rightarrow \frac{c}{r} \quad -(13)$$

then

which is the Sagnac effect when the platform is static.

Eq. (8) and the phase (9) produce a rotation of the plane of polarization of light propagating through the optical fibre. This is one way of describing the Tomita Chiao effect: the rotation of the

3) linearly polarized light can be calculated as follows. Consider the linearly polarized unit vector:

$$\underline{e}_l^{(1)} = \underline{e}^{(1)}\left(e^{i\phi_e} + e^{-i\phi_e}\right) \quad -(14)$$

$$\mathrm{Real}\left(\underline{e}_l^{(1)}\right) = 2\underline{i}\cos\phi_e \quad -(15)$$

where

$$\phi_e = \omega t - \kappa Z \quad -(16)$$

is the electromagnetic phase. The phase (9) results in:

$$\underline{e}_l^{(1)\prime} = e^{i\phi_R}\,\underline{e}_l^{(1)} \quad -(17)$$

where

$$\underline{e}^{(1)} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right) \quad -(18).$$

therefore:

$$\underline{e}_l^{(1)\prime} = \frac{1}{\sqrt{2}}\left(\underline{i} - i\underline{j}\right)\left(e^{i(\phi+\phi_R)} + e^{-i(\phi-\phi_R)}\right) \quad -(19)$$

where:

$$e^{i(\phi+\phi_R)} = \cos(\phi+\phi_R) + i\sin(\phi+\phi_R) \quad -(20)$$

$$e^{-i(\phi-\phi_R)} = \cos(\phi-\phi_R) - i\sin(\phi-\phi_R) \quad -(21)$$

Here:

$$\cos(\phi\pm\phi_R) = \cos\phi\cos\phi_R \mp \sin\phi\sin\phi_R \quad -(22)$$

$$\sin(\phi\pm\phi_R) = \sin\phi\cos\phi_R \pm \cos\phi\sin\phi_R \quad -(23)$$

Therefore:

(4) $e^{i(\phi+\phi_R)} + e^{-i(\phi-\phi_R)} = 2(\cos\phi_R - i\sin\phi_R)\cos\phi$ — (24)

and

$$\underline{e}^{(1)\prime}_{L} = \frac{2}{\sqrt{2}}(\underline{i} - i\underline{j})(\cos\phi_R - i\sin\phi_R)\cos\phi \quad -(25)$$

so

$$\boxed{\mathrm{Real}\left(\underline{e}^{(1)\prime}_{L}\right) = \frac{2}{\sqrt{2}}\left(\underline{i}\cos\phi_R - \underline{j}\sin\phi_R\right)} \quad -(26)$$

Comparing eqns. (15) and (26) it is seen that the phase ϕ_R has rotated the plane of the helical light after it has propagated through the optical fibre. This is the Tomita Chiao effect, Q.E.D. The same happens in the Sagnac effect, so the Sagnac effect is one loop of the Tomita Chiao effect.

Both these effects are manifestations of the Berry phase, so the Berry phase is produced by rotating the Minkowski metric.

147(2): High Accuracy and Compact Fibre Optic Gyro

This is made up of many thousands of turns of optical fibre wound on a drum spinning at Ω, i.e. a Sagnac interferometer with many thousands of turns. The helical metric (4) of note 147(1) is rotated so that:

$$c^2 dt^2 = (r^2 + R^2)(d\phi \mp \Omega dt)^2 \quad -(1)$$

i.e.

$$\frac{d\phi}{dt} = \frac{c}{(r^2+R^2)^{1/2}} \pm \Omega \quad -(2)$$

Define:

$$r_1 = (r^2 + R^2)^{1/2}, \quad -(3)$$

$$\cos\lambda = \frac{r}{(r^2+R^2)^{1/2}}, \quad -(4)$$

$$\omega = \frac{c}{r_1}. \quad -(5)$$

Therefore:

$$\frac{d\phi}{dt} = \frac{c}{r_1} \pm \frac{v}{r} = \frac{rc \pm r_1 v}{r r_1} \quad -(6)$$

$$\frac{dt}{d\phi} = \frac{r r_1}{rc \pm r_1 v}. \quad -(7)$$

For a 2π rotation of ϕ:

$$t = \frac{2\pi r r_1}{rc \pm r_1 v} \quad -(8)$$

$$\Delta t = 2\pi r r_1 \left(\frac{1}{rc - r_1 v} - \frac{1}{rc + r_1 v} \right)$$

$$= \frac{2\pi r r_1^2 v}{(rc - r_1 v)(rc + r_1 v)}. \quad -(9)$$

2)

$$= \frac{\Omega A_r}{\left(c - \frac{r_1}{r} v\right)\left(c + \frac{r_1}{r} v\right)}$$

where

$$\boxed{A_r = \pi r_1^2 = \pi\left(r^2 + R^2\right)} \quad -(10)$$

This result can be expressed as:

$$\boxed{\Delta t = \frac{\Omega A_r \cos\lambda}{(c\cos\lambda - v)(c\cos\lambda + v)}} \quad -(11)$$

If

$$c \gg v \quad -(12)$$

then

$$\Delta t \rightarrow \frac{\Omega A_r}{c^2} \cdot \frac{1}{\cos\lambda} \quad -(13)$$

$$\boxed{\Delta t = \frac{\Omega \pi}{c^2}\left(\frac{\left(r^2 + R^2\right)^{3/2}}{r}\right)} \quad -(14)$$

If

$$r \gg R \quad -(15)$$

this result reduces to the Sagnac effect:

$$\Delta t = \frac{\Omega}{c^2} \pi r^2 \quad -(15)$$

In these equations:

3)

$$R = n Z_0 \quad -(16)$$

and the pitch of the helix is:

$$p = 2\pi Z_0. \quad -(17)$$

The pitch is the distance along the Z axis that results in one full turn of the helix. Therefore the area of the gyro is:

$$\boxed{A_r = \pi \left(r^2 + n^2 Z_0^2 \right)} \quad -(18)$$

and as n becomes very large and Z_0 becomes large, the area is such that the instrument has very high resolution and is capable of measuring a very small Ω, even though it is a compact instrument.

This is the rotating Tomita Chiao effect.

7(3): Effect of Gravitation on the Sagnac Effect and Tomita

: Chiao Effect.

The effect of gravitation on light is most easily worked out by considering phase effects such as the Sagnac effect and Tomita Chiao effect. The simplest solution of the ECE ... is the Minkowski metric, which in ... Orbital Theorem of UFT 111 ... cylindrical polar coordinates is:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(1)$$

Another possible solution is the gravitational metric:

$$ds^2 = x^2 c^2 dt^2 - \frac{dr^2}{x^2} - r^2 d\phi^2 - dZ^2 \quad -(2)$$

By comparison with a limited sample of experimental data in the solar system, it was found empirically that

$$x = \left(1 - \frac{2GM}{c^2 R}\right)^{1/2} \quad -(3)$$

where G is Newton's constant, M is the gravitating mass, c is the speed of light and R is the distance between the two objects of mass m and M. When considering light, m is the mass of the photon, and:

$$ds^2 = 0. \quad -(4)$$

Eq. (4) is known as the null geodesic condition. By now it is known that the metric (3) describes only a very

2)

limited sample of data. It is completely unable to describe whirlpool galaxies for example. Also, it is becoming well known that Schwarzschild did not discover eq. (3). The obsolete standard model adheres to the idea that the Einstein field equation produced eq. (3). This idea is doubly incorrect: 1) the Einstein field equation was the incorrect connection symmetry, 2) eq. (3) was not derived by Schwarzschild.

For our present purposes the Sagnac effect is easily derived by considering eq. (1) in the limit of the X Y plane:

$$dr^2 = dZ^2 = 0 \quad -(5)$$

and for the null geodesic (4), so:

$$c^2 dt^2 = r^2 d\phi^2 \quad -(6)$$

the platform is rotated by $\mp \Omega$, so:

$$c^2 dt^2 = r^2 (d\phi \mp \Omega\, dt)^2 \quad -(7)$$

or

$$c dt = r (d\phi \mp \Omega\, dt) \quad -(8)$$

$$(c \pm r\Omega) dt = r\, d\phi \quad -(9)$$

and

$$\frac{d\phi}{dt} = \frac{c}{r} \pm \Omega = \omega \pm \Omega \quad -(10)$$

where

$$\omega = \frac{c}{r} \quad -(11)$$

) In the Tomita Chiao effect eq. (10) is modified by

$$\omega = \frac{c}{(r^2 + n^2 Z_0^2)^{1/2}} \quad -(12)$$

where the pitch of the helix of the Tomita Chiao effect is

$$p = 2\pi Z_0 \quad -(13)$$

and where n is the number of pitches being considered.

Therefore the static Minkowski metric is

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(14)$$

which in the Thomas precession is rotated:

$$ds'^2 = c^2 dt^2 - dr^2 - r^2 (d\phi \mp \Omega dt)^2 - dZ^2. \quad -(15)$$

The gravitational metric is:

$$ds^2 = x^2 c^2 dt^2 - \frac{dr^2}{x^2} - r^2 d\phi^2 - dZ^2 \quad -(16)$$

which when rotated gives the de Sitter precession:

$$ds'^2 = x^2 c^2 dt^2 - \frac{dr^2}{x^2} - r^2 (d\phi \mp \Omega dt)^2 - dZ^2 \quad -(17)$$

In the calibration:

$$ds' = dr = dZ = 0 \quad -(18)$$

eq. (15) gives the Sagnac effect, and eq. (17) gives the effect of gravitation on the Sagnac effect.

The static metric (14) for the helix is:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - n^2 Z_0^2 d\phi^2 \quad -(18)$$

$$= c^2 dt^2 - dr^2 - (r^2 + n^2 Z_0^2) d\phi^2$$

When $ds = dr = 0$ $\quad -(19)$

eq. (18) gives the Tomita Chiao effect and Berry phase:

$$c^2 dt^2 = (r^2 + n^2 Z_0^2) d\phi^2 \quad -(20)$$

When rotated, eq. (20) becomes:

$$c^2 dt^2 = (r^2 + n^2 Z_0^2)(d\phi \mp \Omega dt)^2 \quad -(21)$$

and gives the high accuracy fibre optic gyro — a rotating helical optical fibre.

The effect of gravitation on eq. (20) is to change it to:

$$\frac{d\phi}{dt} = \frac{xc}{(r^2 + n^2 Z_0^2)^{1/2}} \quad -(22)$$

which produces the effect of gravitation on the Berry phase:

$$\Delta\phi_B = \frac{xc\,\omega t}{(r^2 + n^2 Z_0^2)^{1/2}} \quad -(23)$$

which is the effect of gravitation on the Tomita Chiao phase change observed when light propagates through a helically wound optical fibre.

5) This effect (23) produces a gravimeter of a different type from that in paper 1145.

Finally if the gravity affected coil is spun around its Z axis at $\mp \Omega$, then:

$$\Delta \phi_B = \left(\frac{xc}{(r^2 + n^2 Z_0^2)^{1/2}} \pm \Omega \right) t \quad -(24)$$

which is a third different type of gravimeter.

147(4): Comparison of Rotating Frame and Rotating Metric Method

The rotating frame method of the ECE theory used to describe the Sagnac effect and Faraday disk consists of:

$$e^{i\Omega t} = \cos \Omega t + i \sin \Omega t, \quad -(1)$$

the rotation being expressed through the vector:

$$\underline{q}^{(1)} = \frac{1}{\sqrt{2}} \left(\underline{i} - i \underline{j} \right) e^{i\Omega t} \quad -(2)$$

where Ω is the angular frequency of the platform in the Sagnac effect and the Faraday disk.

It is first shown that this is the same method as rotating the Minkowski metric to give the Sagnac effect. The Minkowski metric in general is:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(3)$$

This is rotated with the Born coordinate method:

$$d\phi' = d\phi \mp \Omega dt \quad -(4)$$

In the ~~light~~ null geodesic limit in a plane:

$$ds = dr = dZ = 0 \quad -(5)$$

so

$$d\phi = (\omega \pm \Omega)\, dt \quad -(6)$$

where

$$\omega = \frac{c}{r} \quad -(7)$$

So the extra effect a phase angle of Ω is:

$$\Delta \phi = \Omega t \quad -(8)$$

and the phase change is:

2)

$$\Delta Y = \exp(i\Omega t) \quad -(9)$$

which is eq. (1) Q.E.D.

If the null geodesic is not used:

$$ds^2 \neq 0 \quad -(10)$$

so: $$ds^2 = c^2 dt^2 - r^2 (d\phi \mp \Omega dt)^2 \quad -(11)$$

i.e $$d\phi = \omega dt_0 \pm \Omega dt \quad -(12)$$

where $$dt_0 = (dt^2 - dt_s^2)^{1/2}, \quad -(13)$$

$$ds_s^2 = c^2 dt_s^2 \quad -(14)$$

It is seen that the phase change due to Ω is again eq. (9).

So the tetrad method (2) and the rotating metric method are the same.

This finding may now be used to calculate the effect of gravitation on the Faraday disk.

147(5): Sagnac Effect for the Electron and the Faraday Disk.

The Sagnac effect for the electron was first observed by Hasselbach et al. in the mid nineties. In this note it is derived straightforwardly by rotating the Minkowski metric:

$$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(1)$$

under the condition:

$$ds^2 \neq 0 \quad -(2)$$

in the plane defined by:

$$dr = dZ = 0 \quad -(3)$$

In order to understand this procedure consider the basics of the Lorentz transform

$$(x^\mu x_\mu)' = x^\mu x_\mu, \quad -(4)$$

i.e.

$$c^2 t'^2 - (X'^2 + Y'^2 + Z'^2) = c^2 t^2 - (X^2 + Y^2 + Z^2) \quad -(5)$$

In the Z axis

$$c^2 t'^2 - Z'^2 = c^2 t^2 - Z^2. \quad -(6)$$

The basic postulate of the Lorentz transform is:

$$Z' = \gamma (Z - vt) \quad -(7)$$

and

$$Z = \gamma (Z' + vt'). \quad -(8)$$

In the Galilean transform of classical non-relativistic physics:

$$\gamma = 1. \quad -(9)$$

Using eq. (7) in eq. (8):

2)

$$z = \gamma\left(\gamma(z - vt) + vt'\right) \quad -(10)$$

so

$$t' = \frac{(1-\gamma^2)}{\gamma v} z + \gamma t \quad -(11)$$

The Einstein postulate is that c is the same in all frames of reference, so:

$$z = ct, \quad z' = ct' \quad -(12)$$

Therefore:

$$\boxed{\begin{aligned} z' &= \gamma(z - vt) = ct' \quad -(13) \\ z &= \gamma(z' + vt') = ct \quad -(14) \end{aligned}}$$

Eqns (13) and (14) are counter-intuitive but are verified by comparison with experimental data.

From eqs. (13) and (14):

$$ct' = \gamma t(c - v) \quad -(15)$$

$$ct = \gamma t'(c + v) \quad (16)$$

From eq. (15) in eq. (16):

$$ct = \gamma^2 \frac{t}{c}(c+v)(c-v) \quad -(17)$$

i.e

$$\gamma^2 (c+v)(c-v) = c^2 \quad -(18)$$

$$\boxed{\gamma^2 = \frac{c^2}{c^2 - v^2}} \quad -(19)$$

so

$$\boxed{\gamma = \left(1 - \frac{c^2}{v^2}\right)^{-1/2}} \quad -(20)$$

3)

From eq. (20) in eq. (11):

$$t' = \gamma t + \frac{z}{\gamma v}\left(1 - \frac{1}{1 - v^2/c^2}\right)$$

$$= \gamma\left(t - \frac{v}{c^2} z\right) \quad -(21)$$

The Lorentz transform is:

$$\boxed{\begin{aligned} x' &= x \\ y' &= y \\ z' &= \gamma(z - vt) \\ t' &= \gamma\left(t - \frac{v}{c^2} z\right) \end{aligned}} \quad -(22).$$

and can be written in matrix format:

$$x^{\mu'} = \Lambda^{\mu'}{}_{\mu} x^{\mu}. \quad -(23)$$

Eq. (23) is a special case of a general coordinate transformation.

The proper time infinitesimal is defined by

$$ds^2 = c^2 d\tau^2$$
$$= c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2. \quad -(24)$$

where:

$$\underline{v} = \frac{d\underline{r}}{dt}. \quad -(25)$$

4)

Eq. (25) is:

$$ds^2 = c^2 dt^2 - |d\underline{r}|^2 = c^2 dt^2 \left(1 - \frac{v^2}{c^2}\right) \quad -(26)$$

so

$$\boxed{d\tau^2 = \gamma^{-2} dt^2} \quad -(27)$$

or

$$\boxed{d\tau = \frac{dt}{\gamma} = \left(1 - \frac{v^2}{c^2}\right)^{1/2} dt} \quad -(28)$$

The proper time τ is the time when the system being considered (for example a particle) is not moving. The proper time τ is the time in the rest frame of reference in which the particle is at rest in that frame. The proper time is the least time. In a frame of reference that moves with respect to the original frame, time ~~infinitis~~ infinitesimals are defined by:

$$dt = d\tau \Big/ \left(1 - \frac{v^2}{c^2}\right)^{1/2} \quad -(29)$$

so:

$$\boxed{dt \geq d\tau}, \quad -(30)$$

and time intervals are longer. This is known as time dilatation. This concept is counter-intuitive but is well verified experimentally to very high precision.

5) Therefore:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2 \quad -(31)$$

In the plane: $dr = dZ = 0$ $\quad -(32)$

we have $c^2 \left(dt^2 - d\tau^2 \right) = r^2 d\phi^2$ $\quad -(33)$

where $d\tau^2 = \left(1 - \frac{v^2}{c^2} \right) dt^2$ $\quad -(34)$

From eqs. (33) and (34):

$$\boxed{\omega = \frac{d\phi}{dt} = \frac{v}{r}} \quad -(35)$$

This is the Sagnac effect for an electron moving around a circle at velocity v. It causes the phase shift:

$$\alpha = \omega t = \frac{v}{r} t \quad -(36)$$

which is observable by interferometry (Hasselbach et al. mid nineties).

The Faraday disk is exactly this, an electric charge moving around the rim of a disk rotating at ω.

141(6): Sagnac Effect for Counter-Rotating Electron Beams

The wave function of the electron is:

$$\psi = \psi_0 \exp\left(\frac{i}{\hbar} p^\mu x_\mu\right) \quad -(1)$$

where:

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right) \quad -(2)$$

$$x_\mu = \left(ct, -\underline{r}\right) \quad -(3)$$

so

$$\psi = \psi_0 \exp\left(\frac{i}{\hbar}\left(Et - \underline{p}\cdot\underline{r}\right)\right) \quad -(4)$$

where

$$E = \hbar\omega, \quad \underline{p} = \hbar\underline{\kappa}. \quad -(5)$$

So

$$\boxed{\psi = \psi_0 \exp\left(i\left(\omega t - \underline{\kappa}\cdot\underline{r}\right)\right).} \quad -(6)$$

This has the same format as the wave function for the photon, but in the electron beam:

$$E^2 = c^2 p^2 + m^2 c^4 \quad -(7)$$

so

$$\omega^2 - c^2\kappa^2 = \hbar^2 m^2 c^4$$

$$\left(\omega^2 - c^2\kappa^2\right)^{1/2} = \hbar m c^2 \quad -(8)$$

and

$$\boxed{E_0 = mc^2 = \frac{1}{\hbar}\left(\omega^2 - c^2\kappa^2\right)^{1/2}} \quad -(10)$$

Therefore for the electron:

$$\omega \neq c\kappa \quad -(11)$$

but for the photon

$$\omega = c\kappa. \quad -(12)$$

2) The Sagnac effect for counter-rotating electron beams is therefore:

$$\phi \rightarrow e^{\pm i\omega_0 t}\,\phi \quad -(13)$$

where

$$\omega_0 = \frac{v}{r} \quad -(14)$$

As shown in note 147(5), eq. (15) is a result of special relativity, derived directly from the Minkowski metric. The two electron beams are counter-rotating, so in one direction:

$$\phi_1 = \phi\, e^{i\omega_0 t} \quad -(15)$$

and in another direction:

$$\phi_2 = \phi\, e^{-i\omega_0 t} \quad -(16)$$

and an interferogram is set up between ϕ_1 and ϕ_2. This is an interferogram set up between interfering electron beams, and was first observed by Hasselbach et al. at Tuebingen in the mid nineties.

If we consider an electron beam rotating in one direction, and spin the platform at $\mp\Omega$, the relevant Minkowski metric is:

$$ds^2 = c^2 d\tau^2 = c^2 dt^2 - r^2(d\phi \mp \Omega dt)^2 \quad -(17)$$

i.e.

$$r^2(d\phi \mp \Omega dt)^2 = c^2(dt^2 - d\tau^2) \quad -(18)$$
$$= v^2 dt^2$$

3)

so $r\,d\phi = (v \pm \Omega r)\,dt$ — (19)

and $\frac{d\phi}{dt} = \frac{v}{r} \pm \Omega$ — (20)

i.e $\boxed{\frac{d\phi}{dt} = \omega_0 \pm \Omega}$ — (21)

where $\omega_0 = \frac{v}{r}$ — (22)

Therefore if the platform is spun there is a fringe shift in the interferogram, exactly as in the usual photon Sagnac effect.

Effect of Gravitation

First consider the static platform, with an electron beam rotated around its rim. This situation is described by the Minkowski metric:

$ds^2 = c^2 dt^2 - dr^2 - r^2 d\phi^2 - dZ^2$ — (23)

in the XY plane and in cylindrical polar coordinates. In the electron beam: for constant radius r of the electron beam:

$dr = dZ = 0$ — (24)

so $r^2 d\phi^2 = c^2 dt^2 - ds^2$
$= c^2 (dt^2 - d\tau^2)$ — (25)
$= v^2 dt^2$

i.e. $\frac{d\phi}{dt} = \frac{v}{r} = \omega$. — (26)

4) Gravitation changes eq. (23) to:

$$ds^2 = x^2 c^2 dt^2 - \frac{dr^2}{x^2} - r^2 d\phi^2 - dZ^2 \quad -(27)$$

where

$$x = \left(1 - \frac{2GM}{c^2 R}\right)^{1/2} \quad -(28)$$

Eq. (27) is a solution of the orbital theorem of paper 111, a special case of the Frobenius theorem for spherically symmetric spacetime. Here M is the mass of a gravitating object, R the distance between the mass m and the mass M, G is Newton's constant. So using eq. (24):

$$r^2 d\phi^2 = c^2 \left(x^2 dt^2 - d\tau^2\right) \quad -(29)$$

where

$$d\tau^2 = \left(1 - \frac{v^2}{c^2}\right) dt^2 \quad -(30)$$

so

$$r^2 d\phi^2 = c^2 \left(x^2 - 1 + \frac{v^2}{c^2}\right) dt^2$$

$$= \left(v^2 - \frac{2GM}{R}\right) dt^2$$

$$\boxed{\frac{d\phi}{dt} = \left(v^2 - \frac{2GM}{R}\right)^{1/2} / r} \quad -(31)$$

1) 147(7) : Kinetic and Potential Energy

In note 147(6), it was found that the effect of gravitation on the electron Sagnac effect ~~was~~ is:

$$\omega = \frac{v_1}{r} \quad -(1)$$

where

$$v_1 = \left(v^2 - \frac{2GM}{R}\right)^{1/2} \quad -(2)$$

Here v is the tangential velocity of the electron and

$$\Phi = -\frac{GM}{R} \quad -(3)$$

is the classical gravitational potential. The classical potential energy is

$$U = m\Phi \quad -(4)$$

where m is the mass of the electron. From eq. (2):

$$v_1^2 = v^2 - \frac{2GM}{R} \quad -(5)$$

and so

$$\frac{1}{2}mv_1^2 = \frac{1}{2}mv^2 - \frac{GmM}{R}. \quad -(6)$$

The hamiltonian is:

$$H = T + U \quad -(7)$$

where

$$T = \frac{1}{2}mv^2, \quad -(8)$$

$$U = -\frac{GmM}{R}. \quad -(9)$$

The classical gravitational force between m and

2) on attracting mass M is:

$$\underline{F} = -m\underline{\nabla}\,\Phi = -\underline{\nabla}U \quad -(10)$$

$$\boxed{\underline{F} = -\frac{GmM}{R^2}\,\underline{k}} \quad -(11)$$

in the Z axis. This is the Newton inverse square law.

The kinetic energy (T) and work done (W) is related to the force by:

$$W = T = \int \frac{d\underline{F}}{dt}\cdot\underline{v}\,dt \quad -(12)$$

So

$$\boxed{\underline{F} = m\frac{d\underline{v}}{dt}} \quad -(13)$$

To see this use the fact that the work done from 1 to 2 is:

$$W_{12} = \int_1^2 \underline{F}\cdot d\underline{r} \quad -(14)$$

$$\underline{F}\cdot d\underline{r} = m\frac{d\underline{v}}{dt}\cdot\frac{d\underline{r}}{dt}\,dt = m\frac{d\underline{v}}{dt}\cdot\underline{v}\,dt$$

$$= \frac{m}{2}\frac{d}{dt}\left(\underline{v}\cdot\underline{v}\right)dt$$

$$= d\left(\frac{1}{2}mv^2\right) \quad -(15)$$

So

$$W_{12} = \frac{1}{2}mv^2\Big|_1^2 = \frac{1}{2}m\left(v_2^2 - v_1^2\right)$$

$$= T_2 - T_1 \quad -(16)$$

If:

3)

$$W_{12} < 0 \quad -(17)$$

The particle of mass m does work, and loses kinetic energy.

If: $$T_1 = 0 \quad -(18)$$

the result (14) is obtained.

The potential energy is the capacity to do work. If the force $\underline{F}$ transports m from 1 to 2 it does work on the particle. If there is no change in kinetic energy, the work required to move a particle from 1 to 2 is independent of the path. So

$$\int_1^2 \underline{F} \cdot d\underline{r} = U_1 - U_2 \quad -(19)$$

If
$$\underline{F} = -\underline{\nabla} U \quad -(20)$$

then
$$\underline{\nabla} \times \underline{F} = -\underline{\nabla} \times \underline{\nabla} U = 0 \quad -(21)$$

In this case:
$$\int_1^2 \underline{F} \cdot d\underline{r} = -\int_1^2 \underline{\nabla} U \cdot d\underline{r}$$
$$= -\int_1^2 dU = U_1 - U_2 \quad -(22)$$

So if m is raised to a height h by any path, an amount of work is done on it. In a constant $\underline{g}$ this is:

$$W_{12} = m \int_1^2 \underline{g} \cdot d\underline{r} = mg \Big|_0^h$$
$$= mgh = U_1 - U_2 \quad -(23)$$

4) and if: $U_2 = 0$ — (24)

then

$$W_{12} = mgh \quad -(25)$$

From eqs. (16) and (19)

$$T_2 - T_1 = U_1 - U_2 \quad -(26)$$

So

$$\boxed{U_1 + T_1 = U_2 + T_2} \quad -(27)$$

This means that the Hamiltonian is constant. The equivalence principle is:

$$\underline{F} = m\underline{g} = -\frac{mMG}{R^2}\underline{k} \quad -(28)$$

So all the features of classical dynamics are obtained from eq. (1). This proves that the method of deriving eq. (1) is correct.

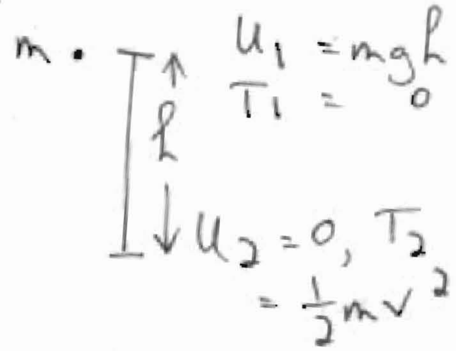


Fig (1).

Finally we:

$$\boxed{g = -\frac{MG}{R^2}} \quad -(29)$$

to obtain:

$$\boxed{\begin{aligned} V_1 &= \left(v^2 + 2Rg\right)^{1/2} \quad -(30) \\ \omega &= v_1 / r \quad -(31) \\ \alpha &= \omega t \quad -(32) \end{aligned}}$$

5) In the case of the photon of mass m:

$$v_1 = \left(c^2 + 2Rg \right)^{1/2} \quad -(31)$$

It is seen that:

$$\omega_0 = \frac{c}{r} \rightarrow \frac{1}{r}\left(c^2 + 2Rg \right)^{1/2} \quad -(32)$$

where r is the radius of the platform. Here R is the Earth's mean radius, g the magnitude of the acceleration due to gravity at the Earth's surface. Here:

$$c \sim 3 \times 10^8 \ m s^{-1}$$

$$R = 6.37 \times 10^6 \ m$$

$$g = 9.8 \ m s^{-2}$$

So for the photon:

$$c^2 = 9 \times 10^{16} \ (m/s)^2$$

$$2Rg = 1.25 \times 10^8 \ (m/s)^2$$

The instrument has to have a resolution of one part in 10^8 to see an effect. However, the instrument is already in the Earth's gravitational field, so for a Sagnac interferometer at the Earth's surface, eq. (33) is observed directly. The e/m frequency shift is

$$\alpha = \omega t = \frac{t}{r}\left(c^2 + 2Rg \right)^{1/2} \quad -(33)$$

The time taken for the light beam to go around 2π radians is:

6)

$$\boxed{t = \frac{2\pi r}{(c^2 + 2Rg)^{1/2}}} \quad -(34)$$

for the photon (ordinary Sagnac effect with static platform) and for the electron:

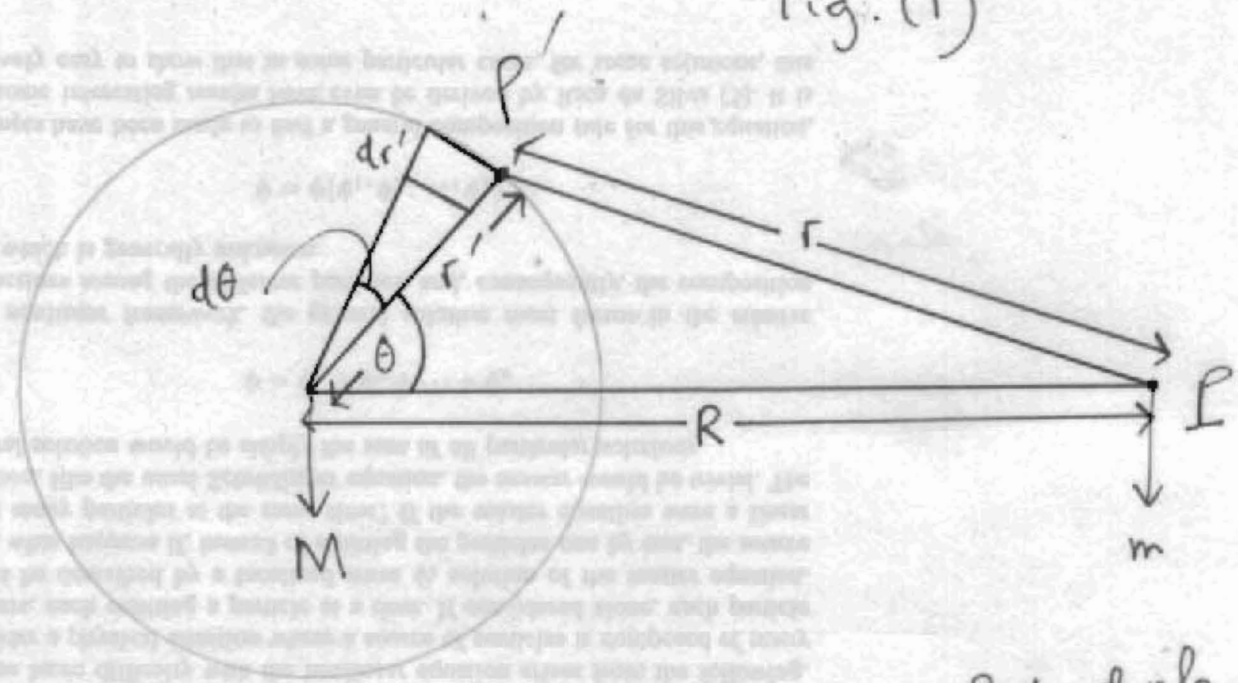
$$\boxed{t = \frac{2\pi r}{(v^2 + 2Rg)^{1/2}}} \quad -(35)$$

These times can be measured directly and measure the effect of gravitation on a photon and electron, ~~go~~ respectively, going around a circle or loop of any shape.

Therefore gravitation affects electromagnetism and electronic trajectories, for example an electron going around the rim of a spinning Faraday disk.

1)
147(8): Effect of Gravitation on the Faraday Disk

Fig. (1)



The potential at point P of a solid disk of mass M is (Marion & Thornton pp. 160 ff)

$$\Phi = -G \int \frac{\rho_s}{r} \, da' \qquad -(1)$$

where ρ_s is the surface density of mass and da' is the element of area. In the above Fig (1):

$$r^2 = r'^2 + R^2 - 2r'R\cos\theta \qquad -(2)$$

so:

$$r = \left(R^2 + a^2 - 2aR\cos\theta\right)^{1/2} \qquad -(3)$$

$$R = a\cos\theta + \left(a^2\sin^2\theta + r^2\right)^{1/2} \qquad -(4)$$

if

$$r' = a \qquad -(5)$$

where a is the radius of the disk.

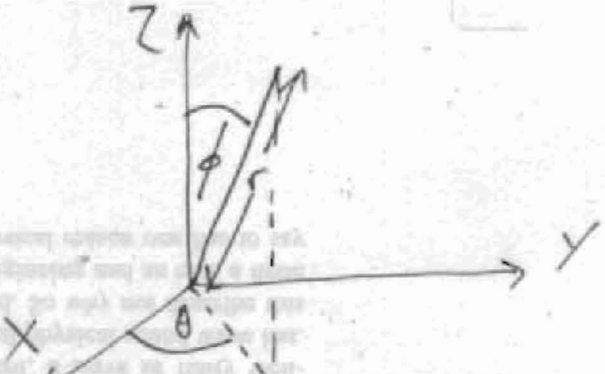
Use the results in spherical polar coordinates:

2)

$X = r \sin\phi \cos\theta$

$Y = r \sin\phi \sin\theta$

$Z = r \cos\phi$



So:

$$ds^2 = c^2 dt^2 - r^2 d\phi^2 - r^2 \sin^2\phi \, d\theta^2 \quad -(6)$$

$$dV = r^2 \sin\phi \, dr \, d\phi \, d\theta \quad -(7)$$

and

$$\pi r^2 = 2\int_0^r r' dr' \int_0^{\pi/2} d\phi \int_0^{\pi} \sin\theta \, d\theta \quad -(7)$$

Therefore:

$$\Phi = -2G\rho \int_0^a r' dr' \int_0^{\pi/2} d\phi \int_0^{\pi} \frac{\sin\theta \, d\theta}{r} \quad -(8)$$

$$\boxed{\Phi = -G\rho \int_0^a r' dr' \int_0^{\pi} \frac{\sin\theta \, d\theta}{r}} \quad -(9)$$

Frm eq. (2):

$$\frac{\sin\theta}{r} d\theta = \frac{dr}{r' R} \quad -(10)$$

So:

$$\Phi = -\frac{G\rho}{R} \int_0^a dr' \int_{R-r'}^{R+r'} dr \quad -(11)$$

3)

$$= -\frac{\pi a^2 G \rho}{R} = -\frac{mG}{R} \quad -(12)$$

where m is the mass of the solid disk. So:

$$\Phi = -\frac{mG}{R}. \quad -(13)$$

The force between m and a mass M at point P is:

$$\underline{F} = -\frac{mMG}{R^2}\,\underline{k} \quad -(14)$$

where $\underline{k}$ is along the line joining M and m. The latter is at the centre of mass of the disk.

In the Faraday disk we wish to consider a point P on the rim of a rotating disk of mass m.

So:

$$\Phi = -\frac{MG}{R} = -\frac{MG}{a\cos\theta + (a^2\sin^2\theta + r^2)^{1/2}} \quad -(15)$$

If

$$r \gg a \quad -(16)$$

then

$$\boxed{\Phi \sim -\frac{MG}{r}} \quad -(17)$$

so the problem reduces to the same mathematical form as that of the rotating photon or electron of the Sagnac effect.

147(9): Derivation of the Einstein Energy Equation From the Metric.

The metric equation is:

$$c^2 d\tau^2 = c^2 dt^2 - d\underline{r}\cdot d\underline{r}. \quad -(1)$$

In cylindrical polar coordinates:

$$d\underline{r}\cdot d\underline{r} = dr^2 + r^2 d\phi^2 + dZ^2. \quad -(2)$$

From eq. (1):

$$\left(\frac{dt}{d\tau}\right)^2 = \gamma^2 = 1 + \frac{1}{c^2}\left(\frac{dr}{d\tau}\right)^2. \quad -(3)$$

Multiply both sides of eq. (3) by m^2:

$$m^2\left(1-\frac{1}{\gamma^2}\right) = \frac{m^2}{\gamma^2 c^2}\left(\frac{dr}{d\tau}\right)^2. \quad -(4)$$

The relativistic momentum is:

$$\underline{p} = m\frac{d\underline{r}}{d\tau} \quad -(5)$$

$$= \gamma m \frac{d\underline{r}}{dt}$$

$$\boxed{\underline{p} = \gamma m \underline{v}} \quad -(6)$$

From eqs. (4) and (6)

$$\gamma^2 m^2 c^4\left(1-\frac{1}{\gamma^2}\right) = p^2c^2 \quad -(7)$$

i.e.

2)

$$E^2 = (\gamma m c^2)^2 = p^2 c^2 + E_0^2 \quad -(8)$$

where

$$E_0 = m c^2 \quad -(9)$$

Eq. (1) is:

$$x^\mu x_\mu = c^2 \tau^2 \quad -(10)$$

and eq. (8) is

$$p^\mu p_\mu = m^2 c^4 \quad -(11)$$

Here:

$$x^\mu = (ct, \underline{r}) \quad -(12)$$

$$x_\mu = (ct, -\underline{r}) \quad -(13)$$

$$p^\mu = \left(\frac{E}{c}, \underline{p}\right) \quad -(14)$$

$$p_\mu = \left(\frac{E}{c}, -\underline{p}\right) \quad -(15)$$

The relativistic Hamilton Jacobi equation is:

$$(p^\mu - eA^\mu)(p_\mu - eA_\mu) = m^2 c^4 \quad -(16)$$

where e is charge and A^μ is the electromagnetic potential:

$$A^\mu = \left(\frac{\phi}{c}, \underline{A}\right). \quad -(17)$$

Here ϕ is the scalar potential and $\underline{A}$ the vector potential.

For simplicity consider:

$$p^\mu = p_\mu = 0 \quad -(18)$$

(3) to obtain:

$$\boxed{A^{\mu} A_{\mu} = \left(\frac{mc}{e}\right)^2} \quad — (19)$$

Effect of Rotation on the Electromagnetic Potential

For simplicity consider rotation in the XY plane defined by:

$$d\underline{r} \cdot d\underline{r} = r^2 \, d\phi^2 \quad — (20)$$

i.e.

$$dr = dZ = 0, \quad — (21)$$

i.e.

$$r = \text{constant} \quad — (22)$$

Therefore

$$\left(\frac{dr}{d\tau}\right)^2 = r^2 \left(\frac{d\phi}{d\tau}\right)^2 \quad — (23)$$

and

$$\underline{p} = \gamma m v = \gamma m r \frac{d\phi}{dt} \quad — (24)$$

$$\boxed{\underline{p} = \gamma m \omega r} \quad — (25)$$

where

$$\omega = \frac{d\phi}{dt} \quad — (26)$$

Therefore in eq. (8):

$$E^2 - E_0^2 = (\gamma^2 - 1) m^2 c^4 = \gamma^2 m^2 \omega^2 c^2 r^2 = (\gamma m \omega c r)^2 \quad — (27)$$

4)

Therefore:

$$\boxed{\omega = \left(\frac{\gamma^2 - 1}{\gamma^2}\right)^{1/2} \frac{c}{r} = \frac{v}{r}} \quad -(28)$$

Therefore the energy equation is:

$$E^2 = p^2 c^2 + m^2 c^4 \quad -(29)$$
$$= (\gamma m r c)^2 \omega^2 + m^2 c^4$$

The electromagnetic vector potential is:

$$\boxed{\underline{A} = \frac{1}{e} \underline{p} = \frac{m}{e} \gamma \omega \underline{r}} \quad -(30)$$

In the limit:

$$v \ll c \quad -(31)$$

$$\boxed{\underline{A} \doteq \frac{m}{e} \omega \underline{r}} \quad -(32)$$

It is seen that mechanical rotation at Ω affects $\underline{A}$:

$$\underline{A} \rightarrow \frac{m}{e} (\omega + \Omega) \underline{r} \quad -(33)$$

147(10): Metric Theory of the Faraday Disc Generator.

The Faraday disk generator was explained in papers 107 and earlier with a rotating tetrad method. In this note it is explained using the metric method of paper 147 combined with a new derivation of the Lorentz force law:

$$\underline{E} = \underline{v} \times \underline{B} . \quad -(1)$$

The usual explanation of the Faraday disc (e.g. by Feynman) is eqs. (1), so:

$$\boxed{E = Bv = Br\omega} . \quad -(2)$$

Usually, eqs. (1) and (2) are said to be equations of special relativity. However, recent experimental and theoretical work (notably ECE) has challenged this received opinion. The main experimental characteristics of the Faraday disk are as follows. The magnet is static, so

$$\frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(3)$$

and according to the standard Faraday law of induction:

$$\underline{\nabla} \times \underline{E} + \frac{\partial \underline{B}}{\partial t} = \underline{0} \quad -(4)$$

we have

$$\underline{\nabla} \times \underline{E} = \underline{0} , \quad -(5)$$

and there is no induction of an electric field strength $\underline{E}$ (volts per metre). This is sometimes known as the Faraday paradox. From eqs. (1) and (5):

$$\underline{\nabla} \times (\underline{v} \times \underline{B}) = 0 \quad -(6)$$

in the standard explanation.

2) Using the vector identity:

$$\underline{\nabla} \times (\underline{A} \times \underline{B}) = \underline{A}\, \underline{\nabla} \cdot \underline{B} - \underline{B}\, \underline{\nabla} \cdot \underline{A} + (\underline{B} \cdot \underline{\nabla})\underline{A} - (\underline{A} \cdot \underline{\nabla})\underline{B} \qquad -(7)$$

we obtain

$$\boxed{\underline{\nabla} \times (\underline{v} \times \underline{B}) = \underline{v}\, \underline{\nabla} \cdot \underline{B} - \underline{B}\, \underline{\nabla} \cdot \underline{v} + (\underline{B} \cdot \underline{\nabla})\underline{v} - (\underline{v} \cdot \underline{\nabla})\underline{B} = \underline{0}} \qquad -(8)$$

If the disk is rotated with a uniform tangential velocity such that

$$\underline{\nabla} \cdot \underline{v} = (\underline{B} \cdot \underline{\nabla})\underline{v} = 0 \qquad -(9)$$

and if the magnetic field is uniform such that:

$$(\underline{v} \cdot \underline{\nabla})\underline{B} = \underline{0} \qquad -(10)$$

then:

$$\underline{\nabla} \times (\underline{v} \times \underline{B}) = \underline{v}\, \underline{\nabla} \cdot \underline{B} \qquad -(11)$$

i.e.

$$\underline{\nabla} \cdot \underline{B} = 0. \qquad -(12)$$

Therefore there is no paradox between eqs. (1) and (5) because eq. (12) is the usual law for magnetic flux density.

The received opinion is that rotating the magnet does not change $\underline{B}$. However, recent experiments by A. G. Kelly have shown that rotating the magnet is relevant, i.e. the magnetic field rotates with the magnet. In the received opinion the key relative motion is that between the disk and the return path (the wire). This means that the Faraday disk is a demonstration of relativity.

3) We leave aside the question of whether this is special or general relativity because this is merely a matter of definition.

From eq. (25) of note 147(5):

$$v = \gamma \omega r \quad -(13)$$

and if

$$v \ll c \quad -(14)$$

$$v = \omega r \quad -(15)$$

Eq. (15) looks like the usual result of classical dynamics, but has been obtained from the Minkowski metric. The latter is then has been obtained from the limit (14): orbital theorem of paper 111. Is

$$A = \frac{m}{e}\omega r \quad -(16)$$

from the minimal prescription:

$$p = mv = eA. \quad -(17)$$

From eqs. (2) and (16):

$$\boxed{A = \frac{m}{e}\frac{E}{B} = \frac{m}{e}v} \quad -(18)$$

In free space:

$$\frac{E}{B} = c, \quad -(19)$$

otherwise

$$\frac{E}{B} = v. \quad -(20)$$

4) In ECE theory

$$\underline{E} = -\underline{\nabla}\phi - \frac{\partial \underline{A}}{\partial t} + \phi\,\underline{\omega}_s - \omega_s \underline{A} \quad -(21)$$

$$\underline{B} = \underline{\nabla} \times \underline{A} - \underline{\omega}_s \times \underline{A} \quad -(22)$$

in general. If the potential is defined by eq. (16)

i.e.

$$\underline{A} = \frac{m}{e}\,\underline{\omega} \times \underline{r} \quad -(23)$$

and if $\underline{\omega}$, the angular velocity, and r, the disc radius, are both constant, then, in the absence of a scalar potential ~~ϕ~~ gradient $\underline{\nabla}\phi$:

$$\underline{E} = \phi\,\underline{\omega}_s - \omega_s \underline{A} \quad -(24)$$

$$= -2\omega_s \underline{A}$$

by antisymmetry.

So:

$$\boxed{\underline{E} = -2\omega_s \underline{A}} \quad -(25)$$

Antisymmetry also implies:

$$\underline{\nabla}\phi = \frac{\partial \underline{A}}{\partial t} = \underline{0} \quad -(26)$$

self consistently.

If in eq. (23)

$$\underline{\omega} = \omega_z \underline{k}, \quad \underline{r} = X\underline{i} + Y\underline{j} \quad -(27)$$

then:

$$\underline{A} = \frac{m\omega}{e}\left(-Y\underline{i} + X\underline{j}\right) \quad -(28)$$

5) and

$$\underline{\nabla} \times \underline{A} = \frac{2m\omega}{e}\,\underline{k} \quad -(29)$$

In eq. (29):

$$\underline{\nabla} \times \underline{A} = \left(\frac{\partial A_Y}{\partial X} - \frac{\partial A_X}{\partial Y} \right) \underline{k} \quad -(30)$$

where

$$\boxed{\frac{\partial A_Y}{\partial X} = -\frac{\partial A_X}{\partial Y} = 1} \quad -(30)$$

Eq. (30) is an example of the ECE antisymmetry law

$$F_{ij} = \partial_i A_j - \partial_j A_i + \omega_{Si} A_j - \omega_{Sj} A_i \quad -(31)$$

with:

$$\partial_i A_j + \omega_{Si} A_j = -(\partial_j A_i + \omega_{Sj} A_i) \quad -(32)$$

Therefore the total magnetic flux density from eq. (22) is:

$$\boxed{\underline{B} = \frac{2m}{e}\omega\,\underline{k} - \underline{\omega}_S \times \underline{A}} \quad -(33)$$

The spin connection vector $\underline{\omega}_S$ of ECE theory adds a term that is missing from the received opinion. Therefore in the Faraday disk:

$$\boxed{\begin{aligned} \underline{E} &= -2\omega_s \underline{A} \\ \underline{B} &= 2\frac{m}{e}\omega \underline{k} - \underline{\omega}_s \times \underline{A} \end{aligned}} \quad -(34)$$

In the received opinion:

$$\underline{E} = \underline{0} \quad -(35)$$

$$\underline{B} = 2\frac{m}{e}\omega \underline{k} \quad -(36)$$

As observed experimentally by Kelly, increasing ω increases $\underline{B}$, ~~both~~ in the received opinion and also in ECE.

The electric field strength $\underline{E}$ and magnetic flux density $\underline{B}$ due to the spin connection are:

$$\underline{E} = -2\omega_s \underline{A} \quad -(37)$$

$$\underline{B} = -\underline{\omega}_s \times \underline{A} \quad -(38)$$

so

$$2\omega_s \underline{B} = \underline{\omega}_s \times \underline{E} \quad -(39)$$

and

$$2\omega_s \underline{\omega}_s \times \underline{B} = \underline{\omega}_s \times (\underline{\omega}_s \times \underline{E}) = (\underline{\omega}_s \cdot \underline{E})\omega_s - \omega_s^2 \underline{E} \quad -(40)$$

Therefore:

7)

$$\underline{B} = \frac{1}{2\omega_s} \underline{\omega}_s \times \underline{E} \quad -(41)$$

$$\omega_s \times \underline{B} = \frac{1}{2}\left(\left(\underline{\omega_s} \cdot \underline{E}\right) - \omega_s \underline{E}\right). \quad -(42)$$

These are relations between $\underline{E}$ and $\underline{B}$ obtained in the limit:

$$v << c. \quad -(43)$$

However, it is known that the Minkowski metric is a statement of the Lorentz boost. The latter produces, in the limit (43):

$$\underline{E} = \underline{v} \times \underline{B} \quad -(44)$$

$$\underline{B} = -\frac{1}{v^2} \underline{v} \times \underline{E} \quad -(45)$$

It is seen that there is a structural similarity between eqs. (41) and (42) and (44) and (45), and eqs. (44) and (45) <u>are the usual explanation</u> of the <u>Faraday disc generator.</u>

In eq. (44):

$$\underline{v} \times \underline{E} = \underline{v} \times (\underline{v} \times \underline{B}) \quad -(46)$$

$$= (\underline{v} \cdot \underline{B})\underline{v} - v^2 \underline{B}$$

So the usual result (45) is obtained by assuming

$$\underline{v} \cdot \underline{B} = 0 \quad -(47)$$

8) In eq. (45):

$$\underline{v} \times \underline{B} = -\frac{1}{v^2}\,\underline{v} \times (\underline{v} \times \underline{E})$$

$$= -\frac{1}{v^2}\left((\underline{v} \cdot \underline{E})\,\underline{v} - v^2\,\underline{E}\right), \quad -(48)$$

So for self consistency

$$\boxed{\underline{v} \cdot \underline{B} = \underline{v} \cdot \underline{E} = 0} \quad -(49)$$

i.e. the usual Lorentz force laws.

In the case of the Faraday disk:

$$\underline{E} = -2\omega_s \underline{A} = 2\phi\,\underline{\omega_s} \quad -(50)$$

so:

$$\underline{\omega_s} \times \underline{A} = \underline{0} \quad -(51)$$

Therefore:

$$\boxed{\begin{aligned} \underline{E} &= 2\phi\,\underline{\omega_s} \\ \underline{B} &= 2\frac{m}{e}\,\omega\,\underline{k} \end{aligned}} \quad -(52)$$

is a complete description.

From eq. (52):

$$E = \phi\,\frac{e}{m}\,\frac{|\underline{\omega_s}|}{\omega}\,B \quad -(53)$$

$$= \left(\phi\,\frac{e}{m}\,|\underline{\omega_s}|\,\frac{r}{v}\right) B$$

9) Now use:

$$\frac{e}{m} = \frac{v}{A} \quad -(54)$$

to find

$$E = \left(\frac{\phi}{A} \left| \underline{\omega_s} \right| r \right) B \quad -(55)$$

Finally use:

$$\frac{\phi}{A} = v, \quad -(56)$$

$$\left| \underline{\omega_s} \right| = \frac{1}{r} \quad -(57)$$

to find eq. (2):

$$E = v B \quad -(58)$$

self consistently from eqs. (50), (56) and (57):

$$\omega_s = \frac{\phi}{A} \left| \underline{\omega_s} \right| = \frac{v}{r} = \omega . \quad -(59)$$

In the Faraday disk, therefore:

$$\boxed{\begin{aligned} \omega_s &= \omega \\ \underline{\omega_s} &= \frac{1}{r^2} \left(-r_y \underline{i} + r_x \underline{j} \right) \end{aligned}} \quad -(60)$$

and the metric method gives:

$$v = \omega r \quad -(61)$$

10)

SUMMARY

The Faraday disk has been described relativistically using the metric method combined with the ECE equations defining $\underline{E}$ and $\underline{B}$. The effect of gravitation and extra spin is worked out through modification of eq. (61). The spin connection scalar ω_S and the spin connection vector $\underline{\omega}_S$ are worked out exactly and given in eq. (60). The Faraday disk is described completely by eq. (52). The magnetic flux density is proportional to angular frequency ($\omega = \omega_S$) and thus to the spin connection scalar. The electric field strength $\underline{E}$ is proportional to the spin connection vector $\underline{\omega_S}$.